THE WAVE THEORY OF LIGHT AND SPECTRA

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THE WAVE THEORY OF LIGHT

Edited by Henry Crew

SCIENTIFIC MEMOIRS

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X.

THE WAVE-THEORY OF LIGHT

THE

WAVE THEORY OF LIGHT

MEMOIRS BY HUYGENS, YOUNG AND FRESNEL

EDITED BY

HENRY CREW, PH.D.

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PREFACE

THANKS to the labors of Kirchhoff, Kelvin, Huxley, and others, there is now a widespread opinion that any physical phenomenon is "explained" only when some one has devised a dynamical model which will duplicate the phenomenon. The completeness of the explanation is to be measured by the completeness with which the model will duplicate the phenomenon. Thus, for instance, a refraction model which, like that of Airy, describes only the path of the refracted ray when the incident ray is given, does not in any true sense explain how the refracted ray comes to take one path rather than another. Such a model illustrates Snell's law, but does not explain the phenomenon.

If, however, we take a large and shallow tank of water, the floor of the tank being partly covered with a false bottom, so as to give two, and only two, different depths of water, we shall find that the speed of the waves in the deeper portion of the tank bears to the speed in the shallower portion a constant ratio; hence, in passing from one depth to the other, these waves are refracted according to the sine law.

Such a model may be said to be a "partial explanation" of refraction in so far as it refers the phenomenon to change of speed which accompanies change of medium. It represents, however, only the kinematics of refraction.

If, now, we could go one step further, and make a model in which the wave-producing forces were duplicated—in other words, if we could make a model of the medium and of the disturbing forces—we should have a fairly complete "explanation" of refraction; in fact, the dynamics of refraction would be understood. This would imply not only that we knew the substance disturbed, but also that we were acquainted with the laws according to which it is disturbed.

A theory of light may be considered either from a kinematical or from a dynamical point of view. To assume, on experimental grounds, that a ray of light has a certain speed in one medium and a different speed in a different medium, and that it consists in a particular kind of motion, and *thence* to infer the laws of refraction, rectilinear propagation, and diffraction, is to construct a kinematical theory of light. But to assume a certain structure for the luminous body and for the medium, and thence to derive the motions and the different speeds assumed in the kinematical case, is to offer a dynamical explanation of light.

The wave-theory of light is used, nearly always, in the former and narrower sense to mean the kinematical explanation of light; it leaves entirely to one side the dynamical questions hinted at above. It assumes, not without strong experimental evidence, the existence of waves travelling with different speeds in different media, and proposes to explain the cardinal phenomena of optics.

To illustrate its limitations, we may cite the instance of the ordinary and extraordinary ray in crystals. How it happens that there are two rays is a problem in the dynamics of light; but, assuming these two rays, their subsequent behavior, their inability to interfere, etc., must be accounted for—in a general way, at least—by the kinematical theory of light.

It is in this narrow sense that the wave-theory of light is employed in the memoirs translated in this volume.

The first clear and unmistakable suggestion that light consists in a vibratory motion appears to be due to that brilliant but unfortunate genius, Robert Hooke (1635–1703), who, in his *Micrographia* (London, 1665), describes the three characteristic features of the motion which he believes to constitute light.

Since it has not been deemed advisable to reprint Hooke's paper in this volume, it may not be out of place here to quote what few paragraphs are necessary fairly to present his point of view. This will, perhaps, be accomplished by the following selections:

"It would be somewhat too long a work for this place *Zetetically* to examine, and positively to prove, what particular kind of motion it is that must be the efficient of Light; for though it be a motion, yet 'tis not every motion that produces

it, since we find there are many bodies very violently mov'd, which yet afford not such an effect; and there are other bodies, which to our senses, seem not mov'd so much, which yet shine. Thus Water and quick-silver, and most other liquors heated, shine not; and several hard bodies, as Iron, Silver, Brass, Copper, Wood, &c., though very often struck with a hammer, shine not presently, though they will all of them grow exceeding hot; whereas rotten Wood, rotten Fish, Sea Water, Gloworms, &c. have nothing of tangible heat in them, and yet (where there is no stronger light to affect the Sensory) they shine some of them so Vividly, that one may make a shift to read by them.

"It would be too long, I say, here to insert the discursive progress by which I inquir'd after the proprieties of the motion of Light, and therefore I shall only add the result.

"And, First, I found it ought to be exceeding *quick*, such as those motions of *fermentation* and *putrefaction*, whereby, certainly, the parts are exceeding nimbly and violently mov'd; and that, because we find those motions are able more minutely to shatter and divide the body, then the most violent heats or *menstruums* we yet know. And that fire is nothing else but such a *dissolution* of the Burning body, made by the most *universal menstruum* of all *sulphureous bodies*, namely, the Air, we shall in an other place of this Tractate endeavour to make probable. And that, in all extremely hot shining bodies, there is a very quick motion that causes Light, as well as a more robust that causes Heat, may be argued from the celerity wherewith the bodyes are dissolv'd.

"Next, it must be a *Vibrative motion*. And for this the newly mention'd *Diamond* affords us a good argument; since if the motion of the parts did not return, the Diamond must after many rubbings decay and be wasted; but we have no reason to suspect the latter, especially if we consider the exceeding difficulty that is found in cutting or wearing away a Diamond. And a Circular motion of the parts is much more improbable, since, if that were granted, and they be suppos'd irregular and Angular parts, I see not how the parts of the Diamond should hold so firmly together, or remain in the same sensible dimensions, which yet they do. Next, if they be *Globular*, and mov'd only with a *turbinated* motion, I know not any cause that can impress that motion upon the *pellucid medium*, which yet is done. Thirdly, any other *irregular* motion of the

parts one amongst another, must necessarily make the body of a fluid consistence, from which it is far enough. It must therefore be a *Vibrating* motion.

"And Thirdly, That is a very *short vibrating motion,* I think the instances drawn from the shining of Diamonds will also make probable. For a Diamond being the hardest body we yet know in the World, and consequently the least apt to yield or bend, must consequently also have its vibrations exceeding short.

"And these, I think, are the three principal proprieties of a motion, requisite to produce the effect call'd Light in the Object."—[*Micrographia,* pp. 54–56.]

The total absence of experimental evidence from the above statement of the case stands in such marked contrast with the method of modern physics as initiated by Galileo, that we cannot for a moment reckon Hooke among the founders of the wave-theory.

So important, on the contrary, have been the contributions of Huygens, Newton, Young, and Fresnel, that each has in turn been considered the founder of the modern science of optics. What justification there is for each of these views will be clearer from a brief consideration of optical theory before and after it had been modified by the work of each of these four men.

Two questions naturally arise in the consideration of any theory, *viz.*, (1) What phenomena does it explain?—and (2) How does it explain them? The answers which have been given to these two questions at various periods in the development of the wave-theory may be outlined as follows:

At the time when Huygens and Newton began their work on light, the following phenomena were demanding explanation:

1. The existence of rays and shadows, known from the earliest times.
2. The phenomenon of reflection, known from the earliest times.
3. The phenomenon of refraction, as described by Snell's law.
4. The rainbow and the production of color by the prism.
5. The colors of thin plates—Newton's rings.
6. Diffraction bands outside the geometrical shadow, described by Grimaldi, 1665.

To these might be added the two following phenomena which were discovered before the final publication of Newton's *Opticks* (1704) or Huygens's *Traité de la Lumière* (1690).

7. The polarization of light by crystals (Bartholinus, 1670).

8. The finite speed of light (Römer, 1675).

Of these eight cardinal facts, the *second*, the *third*, and the *eighth*, were explained by Huygens on the assumption—

(*a*) That a luminous disturbance consists of a wave-motion in the ether.

(*b*) That this wave-disturbance travels with a uniform finite speed through the ether in any homogeneous medium.

(*c*) That in different media it travels with speeds which are related inversely as the refractive indices of those media.

But the wave-disturbance as pictured by Huygens was a single longitudinal pulse, or blow, imparted to an elastic fluid. Since he did not have in mind either a train of waves or transverse waves, or the idea of "phase," or waves of different lengths, it is evident that he was unable to explain any of the remaining five facts.

Turning now to that portion of the work of Newton which contributed to the wave-theory, we find that the *fourth* phenomenon—prismatic colors—was explained by him in 1666, when he demonstrated that a single ray of white light contains all the colors of the spectrum, and that color is not produced at the surface of the prism, as had been hitherto supposed. This discovery made possible, for the first time, the correct explanation of the rainbow.

In Newton's ingenious, though, as we now know, incorrect explanation of the *fifth* phenomenon—colors of thin plates—we meet the earliest measurement of the wave-length of light, *viz.*, the distance traversed by a ray of light during the interval between two successive "fits" of the same kind. We meet here, also, the first evidence that, in these fits, or, as we now say, waves, there is a regular periodicity. From this point on we must consider light as travelling not only in waves, but in trains of waves.

At the close of the period of Huygens and Newton, we have then the following facts still demanding explanation:

1. The existence of rays and shadows.

5. The colors of thin plates.

6. The existence of diffraction fringes.

7. The polarization of light by crystals.

To these must now be added—

9. The phenomenon of stellar aberration, discovered by Bradley in 1727.

Considering next the work of Young, we find that he first suggested the correct explanation for the colors of thin plates, having shown by experiment that two rays of light can interfere to produce alternately bright and dark bands. From this experiment and the dark centre in Newton's rings, he concludes that light consists of series of waves which, like other wave-motions, change phase by 180° on reflection from a denser medium.

Young, at this period (1802–3), was still laboring under the impression that light-waves were longitudinal and were propagated in a fluid medium; fortunately, neither of these assumptions affects the validity of his reasoning concerning the colors of thin plates.

When Fresnel began his optical studies (1814) the following facts, *viz.*, (1) existence of rays, (6) diffraction fringes, (7) polarization, and (9) aberration, were still to be accounted for on the wave-theory. By the union of Huygens's principle with the principle of interference, Fresnel gave the first satisfactory explanation of the rectilinear propagation of light, and of the existence of diffraction fringes outside the geometrical shadow.

Fresnel's memoir, in which these discoveries are most systematically set forth, and which was "crowned" by the French Academy in 1819, is translated in the following pages. For the purpose of offering an elementary geometrical explanation of rays and diffraction bands, Fresnel invented the idea of dividing the wave-front into a certain series of zones, which in nearly all text-books are wrongly referred to as "Huygens's Zones." That this is not only unfair, but also misleading, has been pointed out by Professor Schuster.—*Phil. Mag.* vol. xxxi., p. 77 (1891). The first mention of these Fresnel Zones, as they should be called, will be found on p. 111 of the present volume.

It was in order to explain the phenomenon of polarization that Fresnel introduced the idea of transverse vibrations in the ether. The boldness of this now universally accepted hypothesis, which was then practically equivalent to supposing the

ether an elastic solid, can be fully appreciated only after one has carefully studied the views of Fresnel's contemporaries.

The evidence for the transversality of light vibrations rests upon the inability of two oppositely polarized rays to interfere. The memoir of Arago and Fresnel upon this subject is translated in the present volume.

Of the nine phenomena which we have more or less arbitrarily selected as the principal facts of optics, all, save only the last—aberration—had received a fairly complete explanation at the close of the labors of Young and Fresnel. This discovery of Bradley's, which he so easily disposed of on the corpuscular theory, has received many explanations in terms of the wave-theory; but none of these can be considered as thoroughly satisfactory. Young imagines the ether to pass through ordinary matter "as freely, perhaps, as the wind passes through a grove of trees." On this view, however, it is difficult to see how the speed of light in glass, say, should differ from its speed in a vacuum, or how the aberration constant can remain unchanged when the tube of the telescope is filled with water, as in Airy's experiment.—*Proc. Roy. Soc.*, vol. xx., p. 35 (1872).

For it will be remembered that the aberration constant is v/V radians, where

v=speed of earth in its orbit,

and V=speed of light between the objective and eye-piece of telescope employed.

Fresnel, accordingly, modified Young's hypothesis by assuming that, in their motion through space, refracting bodies carry with them only so much ether as is required to increase the density of free ether from unity to ρ, where ρ at any point in the medium is defined by the following equation:

$$\frac{\rho-1}{\rho}=\frac{\mu^2-1}{\mu^2},$$

μ being the refractive index at the same point in the body.

This is really equivalent to saying that "the luminiferous ether is entirely unaffected by the motion of the matter which it permeates." [*Amer. Jour. Sci.*, vol. cxxxi., p. 386.] And that this is the fact of nature is exactly the conclusion at which Fizeau and Michelson and Morley arrive from their experiments upon the effect of motion of the medium upon the speed of light.—*Loc cit.*, p. 377.

When, however, Michelson and Morley attempt to detect this relative motion of the earth and the ether as the earth proceeds in its orbital motion, they do not succeed in certainly finding that there is any [*Phil. Mag.*, December, 1887]; and they accordingly conclude that this relative motion is "quite small enough to refute Fresnel's explanation of aberration."

Of the two experimental facts just cited, one apparently confirms Fresnel's view, and makes possible an explanation of aberration in terms of the wave-theory; while the other leads us to think that the ether moves with the refracting medium, in which case the wave-theory appears incompetent to explain stellar aberration.

It was in the year 1850 that Fizeau and Foucault measured directly the speed of light in air and in water, and found the ratio of these speeds numerically equal to the ratio of their refractive indices. This experiment has sometimes been called the *experimentum crucis* of the wave-theory; but with scant justice we venture to think, inasmuch as no great doctrine in physics can be said to rest upon any single fact, though modification may be demanded by a single fact.

We have now followed, in merest outline, the general explanations which Huygens, Newton, Young, and Fresnel have offered for all, save one, of this group of nine cardinal facts. It is needless to remind the reader that this enumeration forms but a small fraction of the phenomena which optical science has brought to light within the last two centuries, or, indeed, since the labors of these four men were ended.

No outline of the wave-theory would be complete without mention of the important addition which was made to it in the year 1849 by Sir George Stokes. For he it was who first completely justified Huygens's principle by showing that if the primary wave be resolved as proposed by Huygens, no "back wave" will be produced provided we adopt the proper law of disturbance for the secondary wave. The discovery of this law was announced in his memoir on the *Dynamical Theory of Diffraction.* [*Trans. Camb. Phil. Soc.*, vol. ix., p. 1; *Math. and Phys. Papers*, vol. ii., p. 243.] Mathematically speaking, this contribution amounts to the introduction of the factor $1 + \cos\theta$ into the equation [*Eq.* 46, *loc. cit.*], which describes the disturbance in a secondary wave proceeding from an element of the primary wave.

While, as has been said above, the following memoirs concern themselves only with the kinematics of light-waves and not at all with the question of what is vibrating, it may not be out of place to indicate that principally during the last half of the present century at least four more cardinal facts have presented themselves and demanded explanation.

10. The speed of light in free space is numerically equal to the ratio of the electrostatic and electromagnetic units of quantity.

11. In refracting media, the speed of light varies inversely as the square root of the product of the electric and magnetic inductivities.

12. "Most transparent solid bodies are good insulators, and all good conductors are very opaque."—MAXWELL, *Treatise*, vol. ii., art. 799.

13. The plane of polarization is rotated in a magnetic field. (Faraday.)

It was to "explain" these additional phenomena that Maxwell proposed, in 1865, to modify the wave-theory of light by replacing the mechanical shear of the ether by an electric displacement. How thoroughly justified Maxwell was in this move has been amply proved mathematically by the analogy of his equations with those of the elastic solid theory, and experimentally by Hertz (1888).

Within the last decade the wave-theory has shown itself capable of explaining an entirely new group of phenomena, *viz.*, the color photography discovered by Lippmann. Wiener has shown that we have here merely two rays of light—the direct and reflected travelling in opposite directions and interfering to produce stationary light waves.

The flexibility of the wave-theory has still more recently been exemplified by the beautiful discovery of Zeeman; and Larmor and Preston have shown that by assuming a particular kind of electrical displacement, *viz.*, an orbital motion of an ion, the wave-theory is competent to predict not only the triplets and even the sextet, but also the polarization produced by placing the source of radiation in a magnetic field.—*Phil Mag.*, February, 1899.

Striking as the resemblance appears between the kinematics of wave-motion considered in this volume and the phenomena of optics, it must never be forgotten that in all probability the

vibrating atom is a structure whose motion is vastly complicated as compared with the few simple motions which the experiments of Huygens, Newton, Young, Fresnel, Maxwell, and Michelson have assigned to it.

H. C.

EVANSTON, Ill., *November*, 1899.

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TREATISE ON LIGHT

CONTAINING

THE EXPLANATION OF REFLECTION AND OF REFRACTION AND ESPECIALLY OF THE REMARKABLE REFRACTION WHICH OCCURS IN ICELAND SPAR

BY

CHRISTIAAN HUYGENS

(Leyden, 1690)

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TREATISE ON LIGHT

BY

CHRISTIAAN HUYGENS

PREFACE

THIS treatise was written during my stay in Paris twelve years ago, and in the year 1678 was presented to the Royal Academy of Sciences, to which the king had been pleased to call me. Several of this body who are still living, especially those who have devoted themselves to the study of mathematics, will remember having been at the meeting at which I presented the paper; of these I recall only those distinguished gentlemen Messrs. Cassini, Römer, and De la Hire. Although since then I have corrected and changed several passages, the copies which I had made at that time will show that I have added nothing except some conjectures concerning the structure of Iceland spar and an additional remark concerning refraction in rock-crystal. I mention these details to show how long I have been thinking about these matters which I am only just now publishing, and not at all to detract from the merit of those who, without having seen what I have written, may have investigated similar subjects: as, indeed, happened in the case of two distinguished mathematicians, Newton and Leibnitz, regarding the question of the proper figure for a converging lens, one surface being given.

It may be asked why I have so long delayed the publication of this work. The reason is that I wrote it rather carelessly in French, expecting to translate it into Latin, and, in the meantime, to give the subject still further attention. Later I

thought of publishing this volume together with another on dioptrics in which I discuss the theory of the telescope and the phenomena associated with it. But soon the subject was no longer new and was therefore less interesting. Accordingly I kept putting off the work from time to time, and now I do not know when I shall be able to finish it, for my time is largely occupied either by business or by some new investigation.

In view of these facts I have thought wise to publish this manuscript in its present state rather than to wait longer and run the risk of its being lost.

One finds in this subject a kind of demonstration which does not carry with it so high a degree of certainty as that employed in geometry; and which differs distinctly from the method employed by geometers in that they prove their propositions by well-established and incontrovertible principles, while here principles are tested by the inferences which are derivable from them. The nature of the subject permits of no other treatment. It is possible, however, in this way to establish a probability which is little short of certainty. This is the case when the consequences of the assumed principles are in perfect accord with the observed phenomena, and especially when these verifications are numerous; but above all when one employs the hypothesis to predict new phenomena and finds his expectations realized.

If in the following treatise all these evidences of probability are present, as, it seems to me, they are, the correctness of my conclusions will be confirmed; and, indeed, it is scarcely possible that these matters differ very widely from the picture which I have drawn of them. I venture to hope that those who enjoy finding out causes and who appreciate the wonders of light will be interested in these various speculations and in the new explanation of that remarkable property upon which the structure of the human eye depends and upon which are based those instruments which so powerfully aid the eye. I trust also there will be some who, from such beginnings, will push these investigations far in advance of what I have been able to do; for the subject is not one which is easily exhausted. This will be evident especially from those parts of the subject which I have indicated as too difficult for solution; and still more evident from those matters upon which I have not touched at all, such as the various kinds of luminous bodies

and the whole question of color, which no one can yet boast of having explained.

Finally, there is much more to be learned by investigation concerning the nature of light than I have yet discovered ; and I shall be greatly indebted to those who, in the future, shall furnish what is needed to complete my imperfect knowledge.

THE HAGUE, 8*th of January*, 1690.

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[*Not translated.*]

CHAPTER I

ON THE RECTILINEAR PROPAGATION OF RAYS

DEMONSTRATIONS in optics, as in every science where geometry is applied to matter, are based upon experimental facts; as, for instance, that light travels in straight lines, that the angles of incidence and reflection are equal, and that rays of light are refracted according to the law of sines. For this last fact is now as widely known and as certainly known as either of the preceding.

Most writers upon optical subjects have been satisfied to assume these facts. But others, of a more investigating turn of mind, have tried to find the origin and the cause of these facts, considering them in themselves interesting natural phenomena. And although they have advanced some ingenious ideas, these are not such that the more intelligent readers do not still want further explanation in order to be thoroughly satisfied.

Accordingly, I here submit some considerations on this subject with the hope of elucidating, as best I may, this department of natural science, which not undeservedly has gained the reputation of being exceedingly difficult. I feel myself especially indebted to those who first began to make clear these deeply obscure matters, and to lead us to hope that they were capable of simple explanations.

But, on the other hand, I have been astonished to find these same writers accepting arguments which are far from evident as if they were conclusive and demonstrative. No one has yet given even a probable explanation of the fundamental and remarkable phenomena of light, *viz.*, why it travels in straight lines and how rays coming from an infinitude of different directions cross one another without disturbing one another.

I shall attempt, in this volume, to present in accordance with

the principles of modern philosophy, some clearer and more probable reasons, first, for the rectilinear propagation of light, and, secondly, for its reflection when it meets other bodies. Later I shall explain the phenomenon of rays which are said to undergo refraction in passing through transparent bodies of different kinds. Here I shall treat also of refraction effects due to the varying density of the earth's atmosphere. Afterwards I shall examine the causes of that peculiar refraction occurring in a certain crystal which comes from Iceland. And lastly, I shall consider the different shapes required in transparent and in reflecting bodies to converge rays upon a single point or to deflect them in various ways. Here we shall see with what ease are determined, by our new theory, not only the ellipses, hyperbolas, and other curves which M. Descartes has so ingeniously devised for this purpose, but also the curve which one surface of a lens must have when the other surface is given, as spherical, plane, or of any figure whatever.

We cannot help believing that light consists in the motion of a certain material. For when we consider its production we find that here on the earth it is generally produced by fire and flame which, beyond doubt, contain bodies in a state of rapid motion, since they are able to dissolve and melt numerous other more solid bodies. And if we consider its effects, we see that when light is converged, as, for instance, by concave mirrors, it is able to produce combustion just as fire does; *i.e.*, it is able to tear bodies apart; a property that surely indicates motion, at least in the true philosophy where one believes all natural phenomena to be mechanical effects. And, in my opinion, we must admit this, or else give up all hope of ever understanding anything in physics.

Since, according to this philosophy, it is considered certain that the sensation of sight is caused only by the impulse of some form of matter upon the nerves at the base of the eye, we have here still another reason for thinking that light consists in a motion of the matter situated between us and the luminous body.

When we consider, further, the very great speed with which light is propagated in all directions, and the fact that when rays come from different directions, even those directly opposite, they cross without disturbing each other, it must be evident that we do not see luminous objects by means of matter

translated from the object to us, as a shot or an arrow travels through the air. For certainly this would be in contradiction to the two properties of light which we have just mentioned, and especially to the latter. Light is then propagated in some other manner, an understanding of which we may obtain from our knowledge of the manner in which sound travels through the air.

We know that through the medium of the air, an invisible and impalpable body, sound is propagated in all directions, from the point where it is produced, by means of a motion which is communicated successively from one part of the air to another; and since this motion travels with the same speed in all directions, it must form spherical surfaces which continually enlarge until finally they strike our ear. Now there can be no doubt that light also comes from the luminous body to us by means of some motion impressed upon the matter which lies in the intervening space; for we have already seen that this cannot occur through the translation of matter from one point to the other.

If, in addition, light requires time for its passage—a point we shall presently consider—it will then follow that this motion is impressed upon the matter gradually, and hence is propagated, as that of sound, by surfaces and spherical waves. I call these *waves* because of their resemblance to those which are formed when one throws a pebble into water and which represent gradual propagation in circles, although produced by a different cause and confined to a plane surface.

As to the question of light requiring time for its propagation, let us consider first whether there is any experimental evidence to the contrary.

What we can do here on the earth with sources of light placed at great distances (although showing that light does not occupy a sensible time in passing over these distances) may be objected to on the ground that these distances are still too small, and that, therefore, we can conclude only that the propagation of light is exceedingly rapid. M. Descartes thought it instantaneous, and based his opinion upon much better evidence, furnished by the eclipse of the moon. Nevertheless, as I shall show, even this evidence is not conclusive. I shall state the matter in a manner slightly different from his in order that we may more easily arrive at all the consequences.

Let A be the position of the sun; BD a part of the orbit or annual path of the earth; ABC a straight line intersecting in C the orbit of the moon, which is represented by the circle CD.

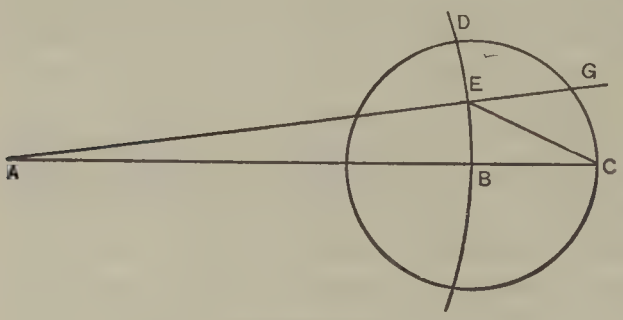

Fig. 1

If, now, light requires time — say one hour—to traverse the space between the earth and the moon, it follows that when the earth has reached the point B, its shadow, or the interruption of light, will not yet have reached the point C, and will not reach it until one hour later. Counting from the time when the earth occupies the position B, it will be one hour later that the moon arrives at the point C and is there obscured; but this eclipse or interruption of light will not be visible at the earth until the end of still another hour. Let us suppose that during these two hours the earth has moved to the position E. From this point the moon will appear to be eclipsed at C, a position which it occupied one hour before, while the sun will be seen at A. For I assume with Copernicus that the sun is fixed and, since light travels in straight lines, must always be seen in its true position. But it is a matter of universal observation, we are told, that the eclipsed moon appears in that part of the ecliptic directly opposite the sun; while according to our view its position ought to be behind this by the angle GEC, the supplement of the angle AEC. But this is contrary to the fact, for the angle GEC will be quite easily observed, amounting to about 33°. Now according to our computation, which will be found in the memoir on the causes of the phenomena of Saturn, the distance, BA, between the earth and the sun is about 12,000 times the diameter of the earth, and consequently 400 times the distance of the moon, which is 30 diameters. The angle ECB will, therefore, be almost 400 times as great as BAE, which is 5′, *viz.*, the angular distance traversed by the earth in its orbit during an interval of two hours. Thus the angle BCE amounts to almost 33°, and likewise the angle CEG, which is 5′ greater.

But it must be noted that in this argument the speed of light is assumed to be such that the time required for it to pass from here to the moon is one hour. If, however, we suppose that it requires only a minute of time, then evidently the angle CEG will amount to only 33′; and if it requires only ten seconds of time, this angle will amount to less than 6′. But so small a quantity is not easily observed in a lunar eclipse, and consequently it is not allowable to infer the instantaneous propagation of light.

It is somewhat unusual, we must confess, to assume a speed one hundred thousand times as great as that of sound, which, according to my observations, travels about 180 toises [1151 *feet*] in a second, or during a pulse-beat; but this supposition appears by no means impossible, for it is not a question of carrying a body with such speed, but of a motion passing successively from one point to another.

I do not therefore, in thinking of these matters, hesitate to suppose that the propagation of light occupies time, for on this view all the phenomena can be explained, while on the contrary view none of them can be explained. Indeed, it seems to me, and to many others also, that M. Descartes, whose object has been to discuss all physical subjects in a clear way, and who has certainly succeeded better than any one before him, has written nothing on light and its properties which is not either full of difficulty or even inconceivable.

But this idea which I have advanced only as a hypothesis has recently been almost established as a fact by the ingenious method of Römer, whose work I propose here to describe, expecting that he himself will later give a complete confirmation of this view.

His method, like the one we have just discussed, is astronomical. He proves not only that light requires time for its propagation, but shows also how much time it requires and that its speed must be at least six times greater than the estimate which I have just given.

For this demonstration, he uses the eclipses of the small planets which revolve about Jupiter, and which very often pass into its shadow. His reasoning is as follows: Let A denote the sun; BCDE, the annual orbit of the earth; F, Jupiter; and GN, the orbit of the innermost satellite, for this one, on account of its short period, is better adapted to this investi-

gation than is either of the other three. Let G represent the point of the satellite's entrance into, and H the point of its emergence from, Jupiter's shadow.

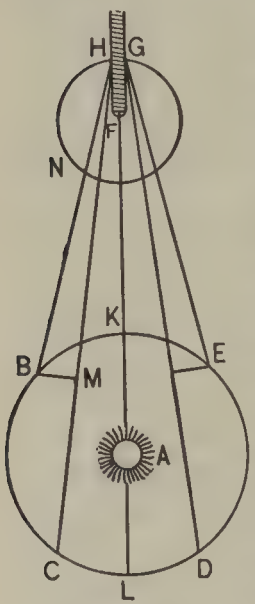

Fig. 2

Let us suppose that an emergence of this satellite has been observed while the earth occupies the position B, at some time before the last quarter. If the earth remained in this position, 42½ hours would elapse before the next emergence would occur. For this is the time required for the satellite to make one revolution in its orbit and return to opposition with the sun. If, for instance, the earth remained at the point B during 30 revolutions, then, after an interval of 30 times 42½ hours, the satellite would again be observed to emerge. But if meanwhile the earth has moved to a point C, more distant from Jupiter, it is evident that, provided light requires time for its propagation, the emergence of the little planet will be recorded later at C than it would have been at B. For it will be necessary to add to this interval, 30 times 42½ hours, the time occupied by light in passing over a distance MC, the difference of the distances CH and BH. In like manner, in the other quarter, while the earth travels from D to E, approaching Jupiter, the eclipses will occur earlier when the earth is at E than if it had remained at D.

Now by means of a large number of these eclipse observations, covering a period of ten years, it is shown that these inequalities are very considerable, amounting to as much as ten minutes or more; whence it is concluded that, for traversing the whole diameter of the earth's orbit KL, twice the distance from here to the sun, light requires about 22 minutes.

The motion of Jupiter in its orbit, while the earth passes from B to C or from D to E, has been taken into account in the computation, where it is also shown that these inequalities cannot be due either to an irregularity in the motion of the satellite or to its eccentricity.

If we consider the enormous size of this diameter, KL, which I have found to be about 24 thousand times that of the earth, we get some idea of the extraordinary speed of light.

Even if we suppose that KL were only 22 thousand diameters of the earth, a speed covering this distance in 22 minutes would be equivalent to the rate of one thousand diameters per minute, *i.e.*, 16⅔ diameters a second (or a pulse-beat), which makes more than eleven hundred times one hundred thousand toises [212,222 *kilometres*], since one terrestrial diameter contains 2865 leagues, of which there are 25 to the degree, and since, according to the exact determination made by Mr. Picard in 1669 under orders from the king, each league contains 2282 toises.

But, as I have said above, sound travels at the rate of only 180 toises [350 *metres*] per second. Accordingly, the speed of light is more than 600,000 times as great as that of sound, which, however, is a very different thing from being instantaneous, the difference being exactly that between a finite quantity and infinity. The idea that luminous disturbances are handed on from point to point in a gradual manner being thus confirmed, it follows, as I have already said, that light is propagated by spherical waves, as is the case with sound.

But if they resemble each other in this respect, they differ in several others—*viz.*, in the original production of the motion which causes them, in the medium through which they travel, and in the manner in which they are transmitted in this medium.

Sound, we know, is produced by the rapid disturbance of some body (either as a whole or in part); this disturbance setting in motion the contiguous air. But luminous disturbances must arise at each point of the luminous object, else all the different parts of this object would not be visible. This fact will be more evident in what follows.

In my opinion, this motion of luminous bodies cannot be better explained than by supposing that those which are fluid, such as a flame, and apparently the sun and stars, are composed of particles that float about in a much more subtle medium, which sets them in rapid motion, causing them to strike against the still smaller particles of the surrounding ether. But in the case of luminous solids, such as red-hot metal or carbon, we may suppose this motion to be caused by the violent disturbance of the particles of the metal or of the wood, those which lie on the surface exciting the ether. Thus the motion which produces light must also be more sudden and more rapid than that which causes sound, since we do not observe that sonorous

disturbances give rise to light any more than that the motion of the hand through the air gives rise to sound.

The question next arises as to the nature of the medium in which is propagated this motion produced by luminous bodies. I have called it ether; but it is evidently something different from the medium through which sound travels. For this latter is simply the air which we feel and breathe, and which, when removed from any region, leaves behind the luminiferous medium. This fact is shown by enclosing a sounding body in a glass vessel and removing the atmosphere by means of the air-pump which Mr. Boyle has devised, and with which he has performed so many beautiful experiments. But in trying this it is well to place the sounding body on cotton or feathers in such a way that it cannot communicate its vibrations either to the glass receiver or to the air-pump, a point which has hitherto been neglected. Then, when all the air has been removed, one hears no sound from the metal even when it is struck.

From this we infer not only that our atmosphere, which is unable to penetrate glass, is the medium through which sound travels, but also that it is different from that which carries luminous disturbances; for when the vessel is exhausted of air, light traverses it as freely as before.

This last point is demonstrated even more clearly by the celebrated experiment of Torricelli. That part of the glass tube which the mercury does not fill contains a high vacuum, but transmits light the same as when filled with air. This shows that there is within the tube some form of matter which is different from air, and which penetrates either glass or mercury, or both, although both the glass and the mercury are impervious to air. And if the same experiment is repeated, except that a little water be placed on top of the mercury, it becomes equally evident that the form of matter in question passes either through the glass or through the water or through both.

As to the different modes of transmission of sound and light, it is easy to understand what happens in the case of sound when one recalls that air can be compressed and reduced to a much smaller volume than it ordinarily occupies, and that just in proportion as its volume is diminished it tends to regain its original size. This property, taken in conjunction with its penetrability, which it retains in spite of compression,

appears to show that it is composed of small particles which float about, in rapid motion, in an ether composed of still finer particles. Sound, then, is propagated by the effort of these air particles to escape when at any point in the path of the wave they are more compressed than at some other point.

But the enormous speed of light, together with its other properties, hardly allows us to believe that it is propagated in the same way. Accordingly, I propose to explain the manner in which I think it must occur. It will be necessary first, however, to describe that property of hard bodies in virtue of which they transmit motion from one to another.

If one takes a large number of spheres of equal size, made of any hard material, and arranges them in contact in a straight line, he will find that, on allowing a sphere of the same size to roll against one end of the line, the motion is transmitted in an instant to the other end of the line. The last sphere in the row flies off while the intermediate ones are apparently undisturbed; the sphere which originally produced the disturbance also remains at rest. Here we have a motion which is transmitted with high speed, which varies directly as the hardness of the spheres.

Nevertheless, it is certain that this motion is not instantaneous, but is gradual, requiring time. For if the motion, or, if you please, the tendency to motion, did not pass successively from one sphere to another, they would all be affected at the same instant, and would all move forward together. So far from this being the case, it is the last one only which leaves the row, and it acquires the speed of the sphere which gave the blow. Besides this experiment there are others which show that all bodies, even those which are considered hardest, such as tempered steel, glass, and agate, are really elastic, and bend to some extent whether they are made into rods, spheres, or bodies of any other shape; that is, they yield slightly at the point where they are struck, and immediately regain their original figure. For I have found that in allowing a glass or agate sphere to strike upon a large, thick, flat piece of the same material, whose surface has been dulled by the breath, the point of contact is marked by a circular disk which varies in size directly as the strength of the blow. This shows that during the encounter these materials yield and then fly back, a process which must require time.

Now to apply this kind of motion to the explanation of light, nothing prevents our imagining the particles of the ether as endowed with a hardness almost perfect and with an elasticity as great as we please. It is not necessary here to discuss the cause either of this hardness or of this elasticity, for such a consideration would lead us too far from the subject. I will, however, remark in passing that these ether particles, in spite of their small size, are in turn composed of parts, and that their elasticity consists in a very rapid motion of a subtle material which traverses them in all directions and compels them to assume a structure which offers an easy and open passage to this fluid. This accords with the theory of M. Descartes, except that I do not agree with him in assigning to the pores the form of hollow circular canals. So far from there being anything absurd or impossible in all this, it is quite credible that nature employs an infinite series of different-sized molecules, endowed with different velocities, to produce her marvellous effects.

But although we do not understand the cause of elasticity, we cannot fail to observe that most bodies possess this property: it is not unnatural, therefore, to suppose that it is a characteristic also of the small, invisible particles of the ether. If, indeed, one looks for some other mode of accounting for the gradual propagation of light, he will have difficulty in finding one better adapted than elasticity to explain the fact of uniform speed. And this appears to be necessary; for if the motion slowed up as it became distributed through a larger mass of matter, and receded farther from the source of light, then its high speed would be lost at great distances. But we suppose the elasticity to be a property of the ether so that its particles regain their shape with equal rapidity whether they are struck with a hard or a gentle blow; and thus the rate at which the light moves remains the same [*at all distances from the source*].

Nor is it necessary that the ether particles should be arranged in straight lines, as was the case with our row of spheres. The most irregular configuration, provided the particles are in contact with each other, will not prevent their transmitting the motion and handing it on to the regions in front. It is to be noted that we have here a law of motion which governs this kind of propagation, and which is verified by experiment, *viz.*, when a sphere such as A, touching several other smaller ones,

CCC, is struck by another sphere, B, in such a way as to make an impression upon each of its neighbors, it transfers its motion to them and remains at rest, as does also the sphere B. Now, without supposing that ether particles are spherical (for I do not see that this is necessary), we can nevertheless understand that this law of impulses plays a part in the propagation of the motion.

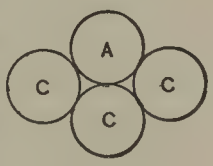

Fig. 3

Equality of size would appear to be a more necessary assumption, since otherwise we should expect the motion to be reflected on passing from a smaller to a larger particle, following the laws of percussion which I published some years ago. Yet, as will appear later, this equality is necessary not so much to make the propagation of light possible as to make it easy and intense. Nor does it appear improbable that the ether particles were made equal for a purpose so important as the transmission of light. This may be true, at least, in the vast region lying beyond our atmosphere and serving only to transmit the light of the sun and the stars.

I have now shown how we may consider light as propagated, in time, by spherical waves, and how it is possible that the speed of propagation should be as great as that demanded by experiment and by astronomical observation. It must, however, be added that although the ether particles are supposed to be in continual motion (and there is much evidence for this view), the gradual transmission of the waves is not thus interfered with. For it does not consist in a translation of these particles, but merely in a small vibration, which they are compelled to transmit to their neighbors in spite of their proper motion and their change of relative position.

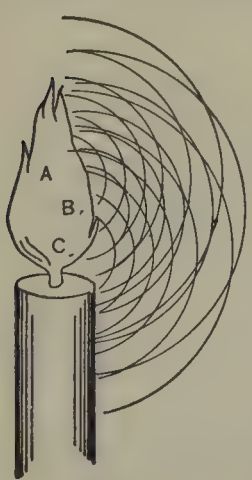

Fig. 4

But we must consider, in greater detail, the origin of these waves and the manner of their propagation from one point to another. And, first, it follows from what has already been said concerning the production of light that each point of a luminous body, such as the sun, a candle, or a piece of burning car-

bon, gives rise to its own waves, and is the centre of these waves. Thus if A, B, and C represent different points in a candle flame, concentric circles described about each of these points will represent the waves to which they give rise. And the same is true for all the points on the surface and within the flame. But since the disturbances at the centre of these waves do not follow each other in regular succession, we need not imagine the waves to follow one another at equal intervals; and if, in the figure, these waves are equally spaced, it is rather to indicate the progress which one and the same wave has made during equal intervals of time than to represent several waves having their origin at the same point.*

Nor does this enormous number of waves, crossing one another without confusion and without disturbing one another, appear unreasonable, for it is well known that one and the same particle of matter is able to transmit several waves coming from different, and even opposite, directions. And this is true not only in the case where the displacements follow one another in succession, but also where they are simultaneous. This is because the motion is propagated gradually. It is shown by the row of hard and equal spheres above mentioned. If we allow two equal spheres, A and D, to strike against the opposite sides of this row at the same instant, they will be observed to rebound each with the same speed that it had before collision, while all the other spheres remain at rest, although the motion has twice traversed the entire row. [*This evidently implies that the spheres A and D have equal speeds just before*

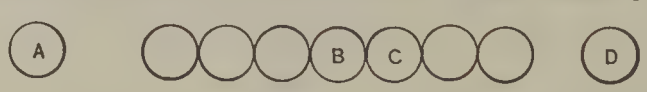

Fig. 5

collision.] If these two oppositely directed motions happen to meet at the middle sphere, B, or at any other sphere, say C, it will yield and spring back from both sides, thus transmitting both motions at the same instant.

* [*From this paragraph it would appear that Huygens had no conception of* trains *of light-waves. The experimental evidence for thinking that light-waves travel in trains seems first to have been furnished by Young. See pp.* 60, 61 *below. If, however, one prefers to interpret the colored rings of Newton in terms of the wave-theory, this experimental evidence may be ascribed to Newton.*]

But what is strangest and most astonishing of all is that waves produced by displacements and particles so minute should spread to distances so immense, as, for instance, from the sun or from the stars to the earth. For the intensity of these waves must diminish in proportion to their distance from the origin until finally each individual wave is of itself unable to produce the sensation of light. Our astonishment, however, diminishes when we consider that in the great distance which separates us from the luminous body there is an infinitude of waves which, although coming from different parts of the [*luminous*] body, are practically compounded into a single wave which thus acquires sufficient intensity to affect our senses. Thus the infinitely great number of waves which at any one instant leave a fixed star, as large possibly as our sun, unite to form what is equivalent to one single wave of intensity sufficient to affect the eye. Not only so, but each luminous point may send us thousands of waves in the shortest imaginable time, on account of the rapidity of the blows with which the particles of the luminous body strike the ether at these points. The effect of the waves would thus be rendered still more sensible.

In considering the propagation of waves, we must remember that each particle of the medium through which the wave spreads does not communicate its motion only to that neighbor which lies in the straight line drawn from the luminous point, but shares it with all the particles which touch it and resist its motion. Each particle is thus to be considered as the centre of a wave. Thus if DCF is a wave whose centre and origin is the luminous point A, a particle at B, inside the sphere DCF, will give rise to its own individual [*secondary*] wave, KCL, which will touch the wave DCF in the point C, at the same instant in which the principal wave, originating at A, reaches the position DCF. And it is clear that there will be only one point of the wave KCL which will touch the wave DCF, *viz.*, the point which lies in the straight line from A drawn through B. In like manner, each of the other particles, *bbbb*, etc., lying within the sphere DCF, gives rise to its

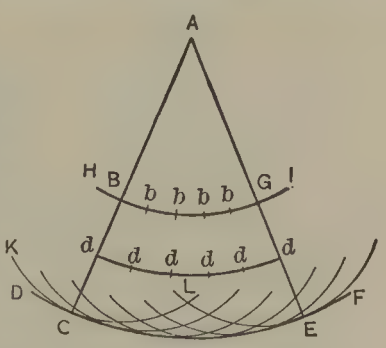

Fig. 6

own wave. The intensity of each of these waves may, however, be infinitesimal compared with that of DCF, which is the resultant of all those parts of the other waves which are at a maximum distance from the centre A.

We see, moreover, that the wave DCF is determined by the extreme limit to which the motion has travelled from the point A within a certain interval of time. For there is no motion beyond this wave, whatever may have been produced inside by those parts of the secondary waves which do not touch the sphere DCF. Let no one think this discussion mere hair-splitting. For, as the sequel will show, this principle, so far from being an ultra-refinement, is the chief element in the explanation of all the properties of light, including the phenomena of reflection and refraction. This is exactly the point which seems to have escaped the attention of those who first took up the study of light-waves, among whom are Mr. Hooke, in his *Micrographia*, and Father Pardies, who had undertaken to explain reflection and refraction on the wave-theory, as I know from his having shown me a part of a memoir which he was unable to finish before his death. But the most important fundamental idea, which consists in the principle I have just stated, is wanting in his demonstrations. On other points also his view is different from mine, as will some day appear in case his writings have been preserved.

Passing now to the properties of light, we observe first that each part of the wave is propagated in such a way that its extremities lie always between the same straight lines drawn from the luminous point.

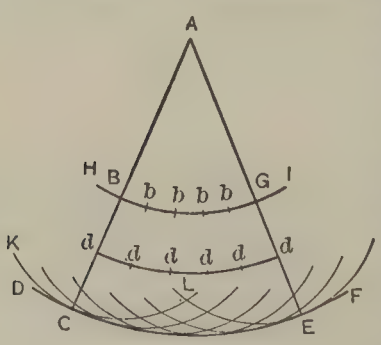

Fig. 6

For instance, that part of the wave BG, whose centre is the luminous point A, develops into the arc CE, limited by the straight lines, ABC and AGE. For although the secondary waves produced by the particles lying within the space CAE may spread to the region outside, nevertheless they do not combine at the same instant to produce one single wave limiting the motion and lying in the circumference CE which is their common tangent. This explains the fact that light, pro-

vided its rays are not reflected or refracted, always travels in straight lines, so that no body is illuminated by it unless the straight-line path from the source to this body is unobstructed.

Let us, for instance, consider the aperture BG as limited by the opaque bodies BH, GI; then, as we have just indicated, the light-waves will always be limited by the straight lines AC, AE. The secondary waves which spread into the region outside of ACE have not sufficient intensity to produce the sensation of light.

Now, however small we may make the opening BG, the circumstances which compel the light to travel in straight lines still remain the same; for this aperture is always sufficiently large to contain a great number of these exceedingly minute ether particles. It is thus evident that each particular part of any wave can advance only along the straight line drawn to it from the luminous point. And this justifies us in considering rays of light as straight lines.

From what has been said concerning the small intensity of the secondary waves, it would appear not to be necessary that all the ether particles be equal, although such an equality would favor the propagation of the motion. The effect of inequality would be to make a particle, in colliding with a larger one, use up a part of its momentum in an effort to recover. The secondary waves thus sent backward towards the luminous point would be unable to produce the sensation of light, and would not result in a primary wave similar to CE.

Another and more remarkable property of light is that when rays come from different, or even opposite, directions each produces its effect without disturbance from the other. Thus several observers are able, all at the same time, to look at different objects through one single opening; and two individuals can look into each other's eyes at the same instant.

If we now recall our explanation of the action of light and of waves crossing without destroying or interrupting each other, these effects which we have just described are readily understood, though they are not so easily explained from Descartes' point of view, *viz.*, that light consists in a continuous [*hydrostatic*] pressure which produces only a tendency to motion.

For such a pressure cannot, at the same instant, affect bodies from two opposite sides unless these bodies have some tendency to approach each other. It is, therefore, impossible to understand how two persons can look each other in the eye or how one torch can illuminate another.

CHAPTER II

ON REFLECTION

HAVING explained the effects produced by light-waves in a homogeneous medium, we shall next consider what happens when they impinge upon other bodies. First of all we shall see how reflection is explained by these same waves and how the equality of angles follows as a consequence. Let AB represent a plane polished surface of some metal, glass, or other substance, which, for the present, we shall consider as perfectly smooth (concerning irregularities which are unavoidable we shall have something to say at the close of this demonstration); and let the line AC, inclined to AB, represent a part of a light-wave whose centre is so far away that this part AC may be considered as a straight line. It may be mentioned here, once for all, that we shall limit our consideration to a single plane, *viz.*, the plane of the figure, which passes through the centre of the spherical wave and cuts the plane AB at right angles.

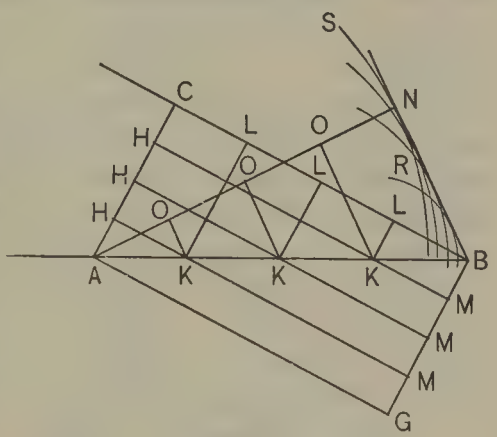

Fig. 7

The region immediately about C on the wave AC will, after a certain interval of time, reach the point B in the plane AB, travelling along the straight line CB, which we may think of as drawn from the source of light and hence drawn perpendicular to AC. Now in this same interval of time the region about A on the same wave is unable to transmit its entire motion beyond the plane AB; it must, therefore, continue its

motion on this side of the plane to a distance equal to CB, sending out a secondary spherical wave in the manner described above. This secondary wave is here represented by the circle SNR, drawn with its centre at A and with its radius AN equal to CB.

So, also, if we consider in turn the remaining parts H of the wave AC, it will be seen that they not only reach the surface AB along the straight lines HK parallel to CB, but they will produce, at the centres K, their own spherical waves in the transparent medium. These secondary waves are here represented by circles whose radii are equal to KM—that is, equal to the prolongations of HK to the straight line BG which is drawn parallel to AC. But, as is easily seen, all these circles have a common tangent in the straight line BN, *viz.*, the same line which passes through B and is tangent to the first circle having A as centre and AN, equal to BC, as radius.

This line BN (lying between B and the point N, the foot of the perpendicular let fall from A) is the envelope of all these circles, and marks the limit of the motion produced by the reflection of the wave AC. It is here that the motion is more intense than at any other point, because, as has been explained, BN is the new position which the wave AC has assumed at the instant when the point C has reached B. For there is no other line which, like BN, is a common tangent to these circles, unless it be BG, on the other side of the plane AB. And BG will represent the transmitted wave only in case the motion occurs in a medium which is homogeneous with that above the plane. If, however, one wishes to see just how the wave AC has gradually passed into the wave BN, he has only to use the same figure and draw the straight lines KO parallel to BN, and the straight lines KL parallel to AC. It is thus seen that the wave AC, from being a straight line, passes

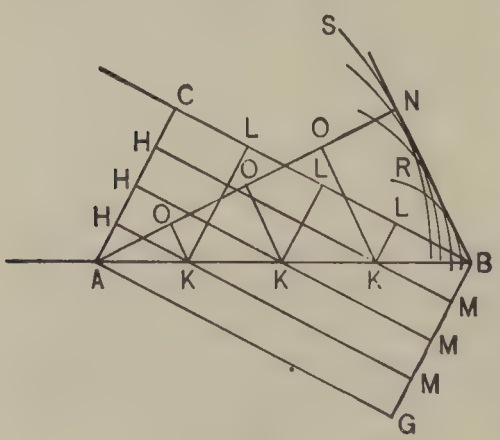

Fig. 7

successively into all the broken lines OKL, and reassumes the form of a single straight line NB.

It is now evident that the angle of reflection is equal to the angle of incidence. For the right-angled triangles ABC and BNA have the side AB in common, and the side CB equal to the side NA, whence it follows that the angles opposite these sides are equal, and hence also the angles CBA and NAB. But CB, perpendicular to CA, is the direction of the incident ray, while AN, perpendicular to the wave BN, has the direction of the reflected ray. These rays are, therefore, equally inclined to the plane AB.

Against this demonstration it may be urged that while BN is the common tangent of the circular waves in the plane of this figure, the fact is that these waves are really spherical and have an infinitely great number of similar tangents, *viz.*, all straight lines drawn through the point B and lying in the surface of a cone generated by the revolution of a straight line BN about BA as axis. It remains to be shown, therefore, that this objection presents no difficulty ; and, incidentally, we shall see that the incident and reflected rays each lie in one plane perpendicular to the reflecting plane.

I remark, then, that the wave AC, so long as it is considered merely a line, can produce no light. For a ray of light, however slender, must have a finite thickness in order to be visible. In order, therefore, to represent a wave whose path is along this ray, it is necessary to replace the straight line AC by a plane area, as, for instance, by the circle HC in the following figure, where the luminous point is supposed to be infinitely distant. From the preceding proof it is easily seen that each element of area on the wave HC, having reached the plane AB, will there give rise to its own secondary wave; and when C reaches the point B, these will all have a common tangent plane, *viz.*, the circle BN equal to CH. This circle will be cut through the centre and at right angles by the same plane which thus cuts the circle CH and the ellipse AB.

It is thus seen that the spherical secondary waves can have no common tangent plane other than BN. In this plane will be located more of the reflected motion than in any other, and it will therefore receive the light transmitted from the wave CH.

I have noted in the preceding explanation that the motion of the wave incident at A is not transmitted beyond the plane AB,

at least not entirely. And here it is necessary to remark that, although the motion of the ether may be partly communicated to the reflecting body, this cannot in the slightest alter the speed of the propagation of the waves, which determines the angle of reflection. For, in any one medium, a slight disturbance produces waves which travel with the same speed as those

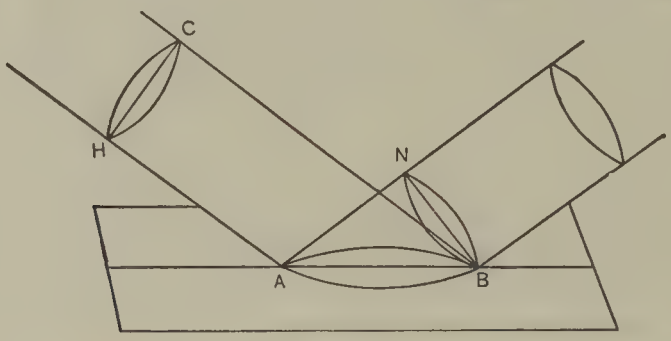

Fig. 8

due to a very great disturbance, a consequence of that property of elastic bodies concerning which we have spoken above, *viz.*, the time occupied in recovery is the same whether the compression be large or small. In every case of reflection of light from the surface of any substance whatever the angles of incidence and reflection are therefore equal, even though the body be of such a nature as to absorb a part of the motion delivered by the incident wave. And, indeed, experiment shows that among polished bodies there is no exception to this law of reflection.

We must emphasize the fact that in our demonstration there is no need that the reflecting surface be considered a perfectly smooth plane, as has been assumed by all those who have attempted to explain the phenomena of reflection. All that is called for is a degree of smoothness such as would be produced by the particles of the reflecting medium being placed one near another. These particles are much larger than those of the ether, as will be shown later when we come to treat of the transparency and opacity of bodies. Since, now, the surface consists thus of particles assembled together, the ether particles being above and smaller, it is evident that one cannot demonstrate the equality of the angles of incidence and reflection from the time-worn analogy with that which happens when

a ball is thrown against a wall. By our method, on the other hand, the fact is explained without difficulty.

Take particles of mercury, for instance, for they are so small that we must think of the least visible portion of surface as containing millions, arranged like the grains in a heap of sand which one has smoothed out as much as possible; this surface for our purpose is equal to polished glass. And, though such a surface may be always rough compared with ether particles, it is evident that the centres of all the secondary waves of reflection which we have described above lie practically in one plane. Accordingly, a single tangent comes as near touching them all as is necessary for the production of light. And this is all that is required in our demonstration to explain the equality of angles without allowing the rest of the motion, reflected in various directions, to produce any disturbing effect.

CHAPTER III

ON REFRACTION

In the same manner that reflection has been explained by light-waves reflected at the surface of polished bodies, we propose now to explain transparency and the phenomena of refraction by means of waves propagated into and through transparent bodies, whether solids, such as glass, or liquids, such as water and oils. But, lest the passage of waves into these bodies appear an unwarranted assumption, I will first show that this is possible in more ways than one.

Let us imagine that the ether does penetrate any transparent body, its particles will still be able to transmit the motion of the waves just as do those of the ether, supposing them each to be elastic. And this we can easily believe to be the case with water and other transparent liquids, since they are composed of discrete particles. But it may appear more difficult in the case of glass and other bodies that are transparent and hard, because their solidity would hardly allow that they should take up any motion except that of their mass as a whole. This, however, is not necessary, since this solidity is not what it appears to us to be, for it is more probable that these bodies are composed of particles which are placed near one another and bound together by an external pressure due to some other kind of matter and by irregularity of their own configurations. For their looseness of structure is seen in the facility with which they are penetrated by the medium of magnetic vortices and those which cause gravitation. One cannot go further than to say that these bodies have a structure similar to that of a sponge, or of light bread, because heat will melt them and change the relative positions of their particles. We infer, then, as has been indicated above, that they are assemblages of particles touching one another but not forming a continuous solid. This being the case, the motion which these particles receive

in the transmission of light is simply communicated from one to another, while the particles themselves remain tethered in their own positions and do not become disarranged among themselves. It is easily possible for this to occur without in any way affecting the solidity of the structure as seen by us.

By the external pressure of which I have spoken is not to be understood that of the air, which would be quite insufficient, but that of another and more subtle medium, whose pressure is exhibited by an experiment which I chanced upon a long while ago, namely, that water from which the air has been removed remains suspended in a glass tube open at the lower end, even though the air may have been exhausted from a vessel enclosing this tube.

We may thus explain transparency without assuming that bodies are penetrated by the luminiferous ether or that they contain pores through which the ether can pass. The fact, however, is not only that this medium penetrates ordinary bodies, but that it does so with the utmost ease, as indeed has already been shown by the experiment of Torricelli which we have cited above. When the mercury or the water leaves the upper part of the glass tube, the ether appears at once to take its place and transmit light. But following is still another argument for thinking that bodies, not only those which are transparent, but others also, are easily penetrable.

When light traverses a hollow glass sphere which is completely closed, it is evident that the sphere is filled with ether, just as is the space outside the sphere. And this ether, as we have shown above, consists of particles lying in close contact with each other. If, now, it were enclosed in the sphere in such a way that it could not escape through the pores of the glass, it would be compelled to partake of any motion which one might impress upon the sphere; consequently nearly the same force would be required to impress a given speed upon this sphere, lying upon a horizontal plane, as if it were filled with water, or possibly mercury. For the resistance which a body offers to any velocity one may wish to impart to it varies directly as the quantity of matter which the body contains and which is compelled to acquire velocity. But the fact is that the sphere resists the motion only in proportion to the amount of glass in it. Whence it follows that the ether within is not enclosed, but flows through the glass with perfect freedom.

Later we shall show, by this same process, that penetrability may be inferred for opaque bodies also.

A second and more probable explanation of transparency is to say that the light-waves continue on in the ether which always fills the interstices, or pores, of transparent bodies. For since it passes continuously and with ease, it follows that these pores are always full. Indeed, it may be shown that these interstices occupy more space than the particles which make up the body.

Now if it be true, as we have said, that the force required to impart a given horizontal velocity to a body is proportional to the mass of the body, and if this force be also proportional to the weight of the body, as we know by experiment that it is, then the mass of any body must be also proportional to its weight. Now we know that water weighs only $\frac{1}{14}$ part as much as an equal volume of mercury, therefore the substance of the water occupies only $\frac{1}{14}$ part of the space that encloses its mass. Indeed, it must occupy even a smaller fraction than this, because mercury is not so heavy as gold, and gold is a substance which is not very dense, since the medium of magnetic vortices and that which causes gravitation penetrate it with the utmost ease.

But it may be objected that if water be a substance of such small density, and if its particles occupy so small a portion of its apparent volume, it is very remarkable that it should offer such stubborn resistance to compression ; for it has not been condensed by any force hitherto employed, and remains perfectly liquid while under pressure.

This is, indeed, no small difficulty. But it may nevertheless be explained by supposing that the very violent and rapid motion of the subtle medium which keeps water liquid also sets in motion the particles of which it is composed, and maintains this liquid state in spite of any pressure which has hitherto been applied.

If, now, the structure of transparent bodies be as loose as we have indicated, we may easily imagine waves penetrating the ether which fills the interstices between the particles. Not only so, but we can easily believe that the speed of these waves inside the body must be a little less on account of the small détours necessitated by these same particles. I propose to show that in this varying velocity of light lies the cause of refraction.

I will first indicate a third and last method by which we may explain transparency, namely, by supposing that the motion of the light-waves is transmitted equally well by the ether particles which fill the interstices of the body, and by the particles which compose the body, the motion being handed on from one to the other. A little later we shall see how beautifully this hypothesis explains the double refraction of certain transparent substances. Should one object that the particles of ether are much smaller than those of the transparent body, since the former pass through the intervals between the latter, and that consequently they would be able to communicate only a small portion of their momentum, we may reply that the particles of the body are composed of other still smaller particles, and that it is these secondary particles that take up the momentum from those of the ether.

Moreover, if the particles of the transparent body are slightly less elastic than are the ether particles, which we may reasonably suppose, it would still follow that the speed of the light waves is less inside the body than outside in the ether.

We have here, what appears to me, the manner in which light-waves are probably transmitted by transparent bodies. But there still remains the consideration of opaque bodies and the difference between these and transparent bodies, a question all the more interesting in view of the ease with which ether penetrates all bodies, a fact to which attention has already been directed, and which might lead one to think that all bodies should be transparent. For considering the hollow sphere, by which I have shown the open structure of glass and the ease with which ether passes through it, one would naturally infer the same penetrability as a property of metals and all other substances. Imagine the sphere to be of silver; it would then certainly contain luminiferous ether, because this substance, as well as air, would be present in it when the opening in the sphere was closed up. But when closed and placed upon a horizontal plane it would resist motion only in proportion to the amount of silver in it, showing as above that the enclosed ether does not acquire the motion of the sphere. Silver, therefore, like glass, is easily penetrated by ether. In between the particles of silver and of all other opaque bodies this substance is distributed continuously and abundantly; and, since it can

transmit light, we are led to expect that these bodies should be as transparent as glass, which, however, is not the fact.

How, then, shall we explain their opacity? Are their constituent particles soft and built up of still smaller particles, and thus able to change shape when they are struck by ether particles? Do they thus damp out the motion and stop the propagation of the light-waves? This seems hardly possible; for if the particles of a metal were soft, how could polished silver and mercury reflect light so well? What seems to me more probable is that metallic bodies, which are almost the only ones that are really opaque, have interspersed among their hard particles some which are soft, the former producing reflection, the latter destroying transparency; while, on the other hand, transparent bodies are made up of only hard and elastic particles, which, together with the ether, propagate light-waves in the manner already indicated.

We pass now to the explanation of refraction, assuming, as above, that light-waves pass through transparent substances and in them undergo diminution of speed.

The fundamental phenomenon in refraction is the following, *viz.*, when any ray of light, AB, travelling in air, strikes obliquely upon the polished surface of a transparent body, FG, it undergoes a sudden change of direction at the point of incidence, B; and this change occurs in such a way that the angle CBE, which the ray makes with the normal to the surface, is less than the angle ABD, which the ray in air made with the same normal. To determine the numerical value of these angles, describe about the point B a circle cutting the rays AB, BC. Then the perpendiculars, AD, CE, let fall from these points of intersection upon the normal, DE, *viz.*, the sines of the angles ABD, CBE, bear to one another a certain ratio which, for any one transparent body, is constant for all directions of the incident ray. For glass this ratio is almost exactly $\frac{3}{2}$, while for water it is very nearly $\frac{4}{3}$, thus varying from one transparent body to another.

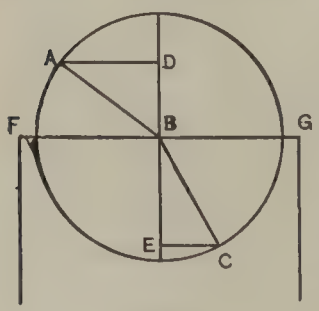

Fig. 9

Another property, not unlike the preceding, is that the refractions of rays entering and of rays emerging from a transpar-

ent body are reciprocal. That is to say, if an incident ray, AB, be refracted by a transparent body into the ray BC, so also will a ray, CB, in the interior of the body be refracted, on emergence, into the ray BA.

In order to explain these phenomena on our theory, let the straight line AB Fig. 10, represent the plane surface bounding a transparent body extending in a direction between C and N.

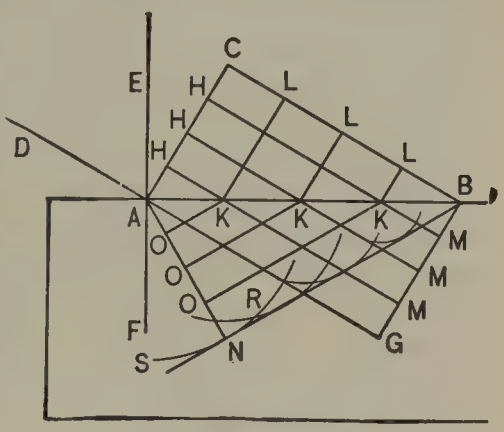

Fig. 10

By the use of the word plane we do not mean to imply a perfectly smooth surface, but merely such a one as was employed in treating of reflection, and for the same reason. Let the line AC represent a part of a light-wave whose source is so distant that this part may be treated as a straight line. The region C, on the wave AC, will, after a certain interval of time, arrive at the plane AB, along the straight line CB, which we must think of as drawn from the source of light, and which will, therefore, intersect AC at right angles. But during this same interval of time the region about A would have arrived at G, along the straight line AG, equal and parallel to CB, and, indeed, the whole of the wave AC would have reached the position GB, provided the transparent body were capable of transmitting waves as rapidly as the ether. But suppose that the rate of transmission is less rapid, say one-third less. Then the motion from the point A will extend into the transparent body to a distance which is only two-thirds of CB, while producing its secondary spherical wave as described above. This wave is represented by the circle SNR, whose centre is at A and whose radius is equal to $\frac{2}{3}$ CB. If we consider, in like manner, the other points H of the wave AC, it will be seen that, during the same time which C employs in going to B, these points will not only have reached the surface AB, along the straight lines HK, parallel to CB, but they will have started secondary waves into the transparent body from the points K as centres. These secondary waves are represented by cir-

cles whose radii are respectively equal to $\frac{2}{3}$ of the distances KM—that is, $\frac{2}{3}$ of the prolongations of HK to the straight line BG. If the two transparent media had the same ability to transmit light, these radii would equal the whole lengths of the various lines KM.

But all these circles have a common tangent in the line BN, *viz.*, the same line which we drew from the point B tangent to the circle SNR first considered. For it is easy to see that all the other circles from B up to the point of contact, N, touch, in the same manner, the line BN, where N is also the foot of the perpendicular let fall from A upon BN.

We may, therefore, say that BN is made up of small arcs of these circles, and that it marks the limits which the motion from the wave AC has reached in the transparent medium, and the region where this motion is much greater than anywhere else. And, furthermore, that this line, as already indicated, is the position assumed by the wave AC at the instant when the region C has reached the point B. For there is no other line below the plane AB, which, like BN, is a common tangent to all these secondary waves.

Accordingly, if one wishes to discover through what intermediate steps the wave AC reached the position BN, he has only to draw, in the same figure, the straight lines KO parallel to BN, and all the lines KL parallel to AC. He will thus see that the wave CA passes from a straight line into the successive broken lines LKO, reassuming the form of a straight line in the position BN. From what has preceded this will be so evident as to need no further explanation.

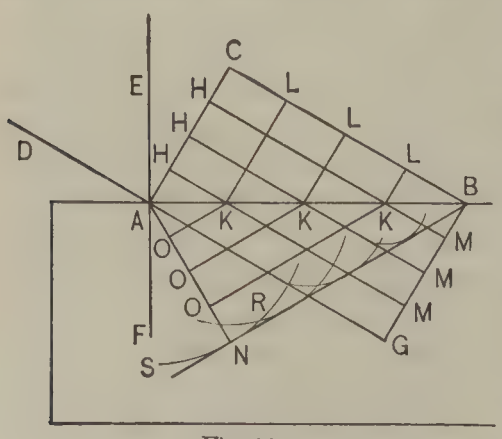

Fig. 10

If, now, using the same figure, we draw EAF normal to the plane AB at the point A, and draw DA at right angles to the wave AC, the incident ray of light will then be represented by DA; and AN, which is drawn perpendicular to BN, will be the refracted ray; for these rays are merely the straight lines along which the parts of the waves travel.

From the foregoing it is easy to deduce the principal law of refraction, *viz.*, that the sine of the angle DAE always bears a constant ratio of the sine of the angle NAF, whatever may be the direction of the incident ray, and that the ratio is the same as that which the speed of the waves in the medium on the side AE bears to their speed on the side AF.

For if we consider AB as the radius of a circle, the sine of the angle BAC is BC, and the sine of the angle ABN is AN. But the angles BAC and DAE are equal; for each is the complement of CAE. And the angle ABN is equal to NAF, since each is the complement of BAN. Hence the sine of the angle DAE is to the sine NAF as BC is to AN. But the ratio of BC to AN is the same as that of the speeds of light in the media on the side towards AE and the side towards AF, respectively; hence, also, the sine of the angle DAE bears to the sine of the angle NAF the same ratio as these two speeds of light.

In order to follow the refracted ray when the light-waves enter a body which transmits them more rapidly than the body from which they emerge (say in the ratio of 3 to 2), it is necessary only to repeat the same construction and the same demonstration which we have just been using, substituting, however, $\frac{3}{2}$ in place of $\frac{2}{3}$. And we find, by the same logical process, employing this other figure, that when the region C of the wave AC reaches the point B of the surface AB, the whole wave AC will have advanced to the position BN, such that the ratio of BC, perpendicular to AC, is to AN, perpendicular to BN, as 2 is to 3. The same ratio will also hold between the sine of the angle EAD and the sine of the angle FAN.

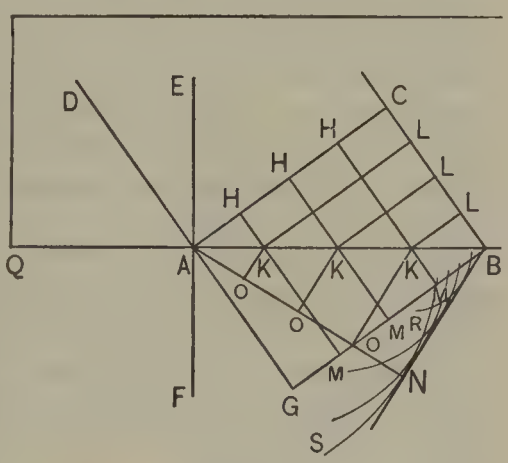

Fig. 11

The reciprocal relations between the refractions of a ray on entering and on emerging from one and the same medium is thus made evident. If the ray NA is incident upon the exterior surface AB, and is refracted into AD, then

the ray DA on emerging from the medium will be refracted into AN.

We are now able to explain a remarkable phenomenon which occurs in this refraction. When the incident ray DA exceeds a certain inclination it loses its ability to pass into the other medium. Because if the angle DAQ or CBA is such that, in the triangle ACB, CB is equal to or greater than $\frac{2}{3}$ of AB, then AN, being equal to or greater than AB, can no longer form one side of the triangle ANB. Therefore the wave BN does not exist, and consequently there can be no line AN drawn at right angles to it. And thus the incident ray DA cannot penetrate the surface AB.

When the wave-speeds are in the ratio of 2 to 3, as in the case of glass and air, which we have considered, the angle DAQ must exceed 48° 11′ if the ray DA is to emerge. And when the ratio of speeds is that of 3 to 4, as is almost exactly the case in water and air, this angle DAQ must be greater than 41° 24′. And this agrees perfectly with experiment.

But one may here ask why no light penetrates the surface, since the encounter of the wave AC against the surface AB must produce some motion in the medium on the other side. The answer is simple, if we recall what has already been said. For although an infinite number of secondary waves may be started into the medium on the other side of AB, these waves at no time have a common tangent line, either straight or curved. There is thus no line which marks the limit to which the wave AC has been transmitted beyond the plane AB, nor is there any line in which the motion has been sufficiently concentrated to produce light.

In the following manner one may easily recognize the fact that, when CB is greater than $\frac{2}{3}$ AB, the waves beyond the plane AB have no common tangent. About the centres K describe circles having radii respectively equal to $\frac{3}{2}$ LB. These circles will enclose one another and will each pass beyond the point B.

It is to be noted that just as soon as the angle DAQ becomes too small to allow the refracted ray DA to pass into the other medium, the internal reflection which occurs at the surface AB increases rapidly in brilliancy, as may be easily shown by means of a triangular prism. In terms of our theory, we may thus explain this phenomenon: While the angle DAQ is still large

enough for the ray DA to be transmitted, it is evident that the light from the wave-front AC will be concentrated into a much shorter line when it reaches the position BN. It will be seen also that the wave-front BN grows shorter in proportion as the angle CBA or DAQ becomes smaller, until finally, when the limit indicated above is reached, BN is reduced to a point. That is to say, when the region about C, on the wave AC, reaches B, the wave BN, which is the wave AC after transmission, is entirely compressed into this same point B ; and, in like manner, when the region about H has reached the point K the part AH is completely reduced to this same point K. It follows, therefore, that in proportion as the direction of propagation of the wave AC happens to coincide with the surface AB, so will be the quantity of motion along this surface.

Now this motion must necessarily spread into the transparent body and also greatly reinforce the secondary waves which produce internal reflection at the face AB, according to the laws of this reflection explained above.

And since a small diminution in the angle of incidence is sufficient to reduce the wave-front BN from a fairly large quantity to zero (for if this angle in the case of glass be 49° 11′, the angle BAN amounts to as much as 11° 21′; but if this same angle DAQ be diminished by one degree only, the angle BAN becomes zero and the wave-front BN is reduced to a point), it follows that the internal reflection occurs suddenly, passing from comparative darkness to brilliancy at the instant when the angle of incidence assumes a value which no longer permits refraction.

Now as to ordinary external reflection, *i. e.*, reflection which occurs when the angle of incidence DAQ is still large enough to allow the refracted ray to pass through the face AB, this reflection must be from the particles which bound the transparent body on the outside, apparently from particles of air and from others which are mixed with, but are larger than, the ether particles.

On the other hand, external reflection from bodies is produced by the particles which compose these bodies, and which are larger than those of the ether, since the ether flows through the interstices of the body.

It must be confessed that we here find difficulty in explaining the experimental fact that internal reflection occurs even

where the particles of air can cut no figure, as, for instance, in vessels and tubes from which the air has been exhausted.

Experiment shows further that these two reflections are of almost equal intensity, and that in various transparent bodies this intensity increases directly as the refractive index. Thus we see that reflection from glass is stronger than that from water, while in turn that from diamond is stronger than that from glass.

I shall conclude this theory of refraction by demonstrating a remarkable proposition depending upon it, namely, that when a ray of light passes from one point to another, the two points lying in different media, refraction at the bounding surface occurs in such a way as to make the time required the least possible ; and exactly the same thing occurs in reflection at a plane surface. M. Fermat discovered this property of refraction, believing with us and in opposition to M. Descartes that light travels more slowly through glass than through air. But, besides this, he assumed what we have just proved from the fact that the velocities in the two media are different, *viz.*, that the ratio of the sines is a constant ; or, what amounts to the same thing, he assumed, besides the different velocities, that the time employed was a minimum ; and from this he derived the constancy of the sine ratio.

His demonstration, which may be found in his works and in the correspondence of M. Descartes, is very long. It is for this reason that I here offer a simpler and easier one.

Let KF represent a plane surface ; imagine the point A in the medium through which the light travels more rapidly, say air ; the point C lies in another, say water, in which the speed of light is less. Let us suppose that a ray passes from A, through B, to C, suffering refraction at B, according to the law above demonstrated ; or, what is the same thing, having drawn PBQ perpendicular to the surface, the sine of the angle ABP is to the sine of the angle CBQ in the same ratio as the speed of light in the medium

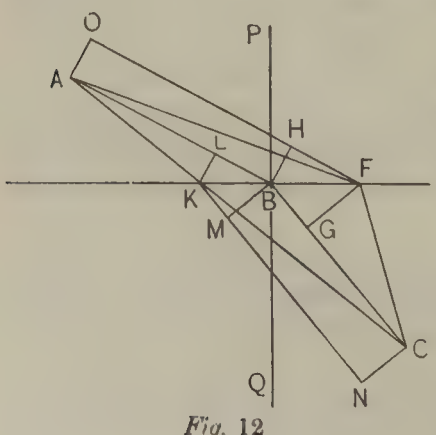

Fig. 12

containing A is to the speed in the medium containing C. It remains to show that the time required for the light to traverse AB and BC taken together is the least possible. Let us assume that the light takes some other path, say AF, FC, where F, the point at which refraction occurs, is more distant than B from A. Draw AO perpendicular to AB, and FO parallel to BA; BH perpendicular to FO, and FG perpendicular to BC. Since, now, the angle HBF is equal to PBA, and the angle BFG is equal to QBC, it follows that the sine of the angle HBF will bear to the sine of BFG the same ratio as the speed of light in the medium A bears to the speed in the medium C. But if we consider BF the radius of a circle, then sines are represented by the lines HF, BG. Accordingly, the lines HF, BG are in the ratio of these speeds. If, therefore, we imagine OF to be the incident ray, the time of passage from H to F will be the same as the time of passage from B to G in the medium C.

But the time from A to B is equal to the time from O to H. Hence the time from O to F is the same as the time from A to G, *via* B. Again, the time along FC is greater than the time along GC; and hence the time along the route OFC is greater than that along the path ABC. But AF is greater than OF; hence, *a fortiori*, the time along AFC is greater than that along ABC.

Let us now assume that the ray passes from A to C by the route AK, KC, the point of refraction, K, being nearer to A than is B. Draw CN perpendicular to BC; KN parallel to BC; BM perpendicular to KN; and KL perpendicular to BA.

Here BL and KM represent the sines of the angles BKL and KBM—that is, the angles PBA and QBC; and hence they are in the same ratio as the speeds of light in the media A and C respectively. The time, therefore, from L to B is equal to the time from K to M; and, since the time from B to C is equal to the time from M to N, the time by the path LBC is the same as the time *via* KMN. But the time from A to K is greater than the time from A to L, and, therefore, the time by the route AKN is greater than the route ABC.

Not only so, but since KC is greater than KN, the time by the path AKC will be so much the greater than by the path ABC. Hence follows that which was to be proved, namely, that the time along the path ABC is the least possible.

Biographical Sketch

While there are no sharp lines in nature, there is a very true sense in which the year 1642, marking the death of Galileo and the birth of Newton, serves as a line of demarcation between the foundation and the superstructure of modern physics.

Galileo, by his careful study of gravitation, by his clear grasp of force as determining acceleration, by his careful search after causes and their respective effects, by his profound faith in experiment, had more than cleared the ground for the builders of modern physics. The rapid rise of this structure at the hands of Newton and his brilliant contemporaries, Boyle, Leibnitz, Römer, Du Fay, Bradley, and Hooke, marks a distinctly modern era compared with that of Galileo.

The work of Christiaan Huygens, the "Dutch Archimedes," occupies, as regards both time and character, a position intermediate between these two periods. He was born at The Hague in 1629, and died there in 1695. A splendid ancestry, three years of university training at Leyden and Breda, much travel, and a rare group of associates, combined to give him an education which left little to be desired. Most of his life was spent in Holland, but for the fifteen years following 1666 he lived and worked in Paris, where he was the guest of Louis XIV. and the then recently founded French Academy of Sciences. This was for him a happy period of great activity, and it was only in anticipation of the revocation of the Edict of Nantes, in 1685, that he returned to his own country, where in private retirement and study he spent most of his remaining years.

His intellectual achievements fall into three not very distinct departments of science—namely, mathematics, physics, and physical astronomy. In mathematics, his chief accomplishments refer to—

(*a*) The quadrature of conics.
(*b*) The theory of probabilities.
(*c*) A discussion of the evolutes and involutes of curves and the introduction of the idea of the envelope of a moving straight line.

In physics he gave—

(*a*) A general solution of the problem of the Compound Pendulum, and in the demonstration enunciated the very general principle that in any mechanical system acting under gravity the centre of gravity can never rise to a point higher than that from which it fell—a principle which we now recognize as a special case of the law that the potential energy of any mechanical system tends to a minimum.

(*b*) The invention of the pendulum clock and its application to the measurement of gravity at various points on the earth's surface.

(*c*) An accurate description of the behavior of bodies in collision.

(*d*) The laws governing centrifugal forces.

(*e*) The undulatory theory of light and its application to the explanation of reflection, ordinary refraction, and double refraction.

Among his contributions to physical astronomy are—

(*a*) The construction of the first powerful telescope of the refracting kind.

(*b*) The discovery of the rings of Saturn and its sixth satellite.

(*c*) Improvements in the methods of grinding lenses and the addition of a tube to the object-glass and another to the eye-piece of the aërial telescope.

All his mechanical inventions are characterized by practicability, and all his intellectual work by clearness and elegance.

Those who wish a more detailed account of his activity will find it in the superb edition of his works* recently published by the Société Hollandaise des Sciences, while that delightful sketch of his life and work given by Dr. Bosscha† should be read by every one.

* *Œuvres Complètes de Christiaan Huygens* (La Haye: Martinus Nijhoff, 1888 to 19—).

† Bosscha: *Christiaan Huygens*, Rede am 200sten Gedächtnistage seines Lebensende. Übersetzt von Engelmann. (Engelmann: Leipzig, 1895), pp. 77.

ON THE THEORY OF LIGHT AND COLORS

From the *Philosophical Transactions* for 1802, p. 12.

AN ACCOUNT OF SOME CASES OF THE PRODUCTION OF COLORS NOT HITHERTO DESCRIBED

From the *Philosophical Transactions* for 1802, p. 387.

EXPERIMENTS AND CALCULATIONS RELATIVE TO PHYSICAL OPTICS

From the *Philosophical Transactions* for 1804.

BY

THOMAS YOUNG.

These three papers are reprinted in Young's *Miscellaneous Works*, vol. i., and also in his *Lectures on Natural Philosophy and Mechanical Arts.*

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ON THE THEORY OF LIGHT AND COLORS*

A BAKERIAN LECTURE

Read November 12, 1801.

ALTHOUGH the invention of plausible hypotheses, independent of any connection with experimental observations, can be of very little use in the promotion of natural knowledge, yet the discovery of simple and uniform principles, by which a great number of apparently heterogeneous phenomena are reduced to coherent and universal laws, must ever be allowed to be of considerable importance towards the improvement of the human intellect.

The object of the present dissertation is not so much to propose any opinions which are absolutely new, as to refer some theories, which have been already advanced, to their original inventors, to support them by additional evidence, and to apply them to a great number of diversified facts, which have hitherto been buried in obscurity. Nor is it absolutely necessary in this instance to produce a single new experiment; for of experiments there is already an ample store, which are so much the more unexceptionable as they must have been conducted without the least partiality for the system by which they will be explained; yet some facts, hitherto unobserved, will be brought forward, in order to show the perfect agreement of that system with the multifarious phenomena of nature.

The optical observations of Newton are yet unrivalled; and, excepting some casual inaccuracies, they only rise in our estimation as we compare them with later attempts to improve on

* From the *Philosophical Transactions* for 1802, p. 12

them. A further consideration of the colors of thin plates, as they are described in the second book of Newton's *Optics*, has converted that prepossession which I before entertained for the undulatory system of light into a very strong conviction of its truth and sufficiency, a conviction which has been since most strikingly confirmed by an analysis of the colors of striated substances. The phenomena of thin plates are indeed so singular that their general complexion is not without great difficulty reconcilable to any theory, however complicated, that has hitherto been applied to them; and some of the principal circumstances have never been explained by the most gratuitous assumptions; but it will appear that the minutest particulars of these phenomena are not only perfectly consistent with the theory which will now be detailed, but that they are all the necessary consequences of that theory, without any auxiliary suppositions; and this by inferences so simple that they become particular corollaries, which scarcely require a distinct enumeration.

A more extensive examination of Newton's various writings has shown me that he was in reality the first that suggested such a theory as I shall endeavor to maintain; that his own opinions varied less from this theory than is now almost universally supposed; and that a variety of arguments have been advanced, as if to confute him, which may be found nearly in a similar form in his own works; and this by no less a mathematician than Leonard Euler, whose system of light, as far as it is worthy of notice, either was, or might have been, wholly borrowed from Newton, Hooke, Huygens, and Malebranche.

Those who are attached, as they may be with the greatest justice, to every doctrine which is stamped with the Newtonian approbation, will probably be disposed to bestow on these considerations so much the more of their attention, as they appear to coincide more nearly with Newton's own opinions. For this reason, after having briefly stated each particular position of my theory, I shall collect, from Newton's various writings, such passages as seem to be the most favorable to its admission; and although I shall quote some papers which may be thought to have been partly retracted at the publication of the *Optics*, yet I shall borrow nothing from them that can be supposed to militate against his maturer judgment.

Hypothesis I

A luminiferous ether pervades the universe, rare and elastic in a high degree.

PASSAGES FROM NEWTON

"The hypothesis certainly has a much greater affinity with his own," that is, Dr. Hooke's, "hypothesis than he seems to be aware of; the vibrations of the ether being as useful and necessary in this as in his."—*Phil. Trans.*, vol. vii., p. 5087. Abr., vol. i., p. 145. Nov., 1672.

"To proceed to the hypothesis: first, it is to be supposed therein that there is an ethereal medium, much of the same constitution with air, but far rarer, subtler, and more strongly elastic. It is not to be supposed that this medium is one uniform matter, but compounded, partly of the main phlegmatic body of ether, partly of other various ethereal spirits, much after the manner that air is compounded of the phlegmatic body of air, intermixed with various vapors and exhalations: for the electric and magnetic effluvia and gravitating principle seem to argue such variety."—Birch, *Hist. of R. S.*, vol. iii., p. 249, Dec., 1675.

"Is not the heat (of the warm room) conveyed through the vacuum by the vibrations of a much subtler medium than air? And is not this medium the same with that medium by which light is refracted and reflected, and by whose vibrations light communicates heat to bodies, and is put into fits of easy reflection and easy transmission? And do not the vibrations of this medium in hot bodies contribute to the intenseness and duration of their heat? And do not hot bodies communicate their heat to contiguous cold ones by the vibrations of this medium propagated from them into the cold ones? And is not this medium exceedingly more rare and subtle than the air, and exceedingly more elastic and active? And doth it not readily pervade all bodies? And is it not, by its elastic force, expanded through all the heavens? May not planets and comets, and all the gross bodies, perform their motions in this ethereal medium? And may not its resistance be so small as to be inconsiderable? For instance, if this ether (for so I will call it) should be supposed 700,000 times more elastic than our air, and above 700,000 times more rare, its resistance would be

about 600,000,000 times less than that of water. And so small a resistance would scarce make any sensible alteration in the motions of the planets in ten thousand years. If any one would ask how a medium can be so rare, let him tell me how an electric body can by friction emit an exhalation so rare and subtle, and yet so potent? And how the effluvia of a magnet can pass through a plate of glass without resistance, and yet turn a magnetic needle beyond the glass?"—*Optics*, Qu. 18, 22.

Hypothesis II

Undulations are excited in this ether whenever a body becomes luminous.

Scholium. I use the word undulation in preference to vibration, because vibration is generally understood as implying a motion which is continued alternately backward and forward by a combination of the momentum of the body with an accelerating force, and which is naturally more or less permanent; but an undulation is supposed to consist in vibratory motion transmitted successively through different parts of a medium without any tendency in each particle to continue its motion, except in consequence of the transmission of succeeding undulations from a distinct vibrating body; as in the air the vibrations of a chord produce the undulations constituting sound.

PASSAGES FROM NEWTON

"Were I to assume an hypothesis, it should be this, if propounded more generally, so as not to determine what light is further than that it is something or other capable of exciting vibrations in the ether; for thus it will become so general and comprehensive of other hypotheses as to leave little room for new ones to be invented."—Birch, *Hist. of R. S.*, vol. iii., p. 249, Dec., 1675.

"In the second place, it is to be supposed that the ether is a vibrating medium like air, only the vibrations far more swift and minute; those of air, made by a man's ordinary voice, succeeding one another at more than half a foot or a foot distance, but those of ether at a less distance than the hundred-thousandth part of an inch. And as in air the vibrations are

some larger than others, but yet all equally swift (for in a ring of bells the sound of every tone is heard at two or three miles distance in the same order that the bells are struck), so, I suppose, the ethereal vibrations differ in bigness, but not in swiftness. Now, these vibrations, besides their use in reflection and refraction, may be supposed the chief means by which the parts of fermenting or putrefying substances, fluid liquors, or melted, burning, or other hot bodies, continue in motion."—BIRCH, *Hist. of R. S.*, vol. iii., p. 251, Dec., 1675.

"When a ray of light falls upon the surface of any pellucid body, and is there refracted or reflected, may not waves of vibrations, or tremors, be thereby excited in the refracting or reflecting medium? And are not these vibrations propagated from the point of incidence to great distances? And do they not overtake the rays of light, and by overtaking them successively, do not they put them into the fits of easy reflection and easy transmission described above?"—*Optics*, Qu. 17.

"Light is in fits of easy reflection and easy transmission before its incidence on transparent bodies. And probably it is put into such fits at its first emission from luminous bodies, and continues in them during all its progress."—*Optics*, Book ii., part iii., prop. 13.

HYPOTHESIS III

The sensation of different colors depends on the different frequency of vibrations excited by light in the retina.

PASSAGES FROM NEWTON

"The objector's hypothesis, as to the fundamental part of it, is not against me. That fundamental supposition is, that the parts of bodies, when briskly agitated, do excite vibrations in the ether, which are propagated every way from those bodies in straight lines, and cause a sensation of light by beating and dashing against the bottom of the eye, something after the manner that vibrations in the air cause a sensation of sound by beating against the organs of hearing. Now, the most free and natural application of this hypothesis to the solution of phenomena I take to be this—that the agitated parts of bodies, according to their several sizes, figures, and motions, do excite vibrations in the ether of various depths or bignesses, which,

being promiscuously propagated through that medium to our eyes, effect in us a sensation of light of a white color; but if by any means those of unequal bignesses be separated from one another, the largest beget a sensation of a red color; the least, or shortest, of a deep violet, and the intermediate ones of intermediate colors; much after the manner that bodies, according to their several sizes, shapes, and motions, excite vibrations in the air of various bignesses, which, according to those bignesses, make several tones in sound: that the largest vibrations are best able to overcome the resistance of a refracting superficies, and so break through it with least refraction; whence the vibrations of several bignesses—that is, the rays of several colors, which are blended together in light—must be parted from one another by refraction, and so cause the phenomena of prisms and other refracting substances; and that it depends on the thickness of a thin transparent plate or bubble whether a vibration shall be reflected at its further superficies or transmitted; so that, according to the number of vibrations interceding the two superficies, they may be reflected or transmitted for many successive thicknesses. And since the vibrations which make blue and violet are supposed shorter than those which make red and yellow, they must be reflected at a less thickness of the plate, which is sufficient to explicate all the ordinary phenomena of those plates or bubbles, and also of all natural bodies, whose parts are like so many fragments of such plates. These seem to be the most plain, genuine, and necessary conditions of this hypothesis; and they agree so justly with my theory that if the animadversor think fit to apply them, he need not, on that account, apprehend a divorce from it; but yet, how he will defend it from other difficulties I know not."—*Phil. Trans.*, vol. vii., p. 5088. Abr., vol. i., p. 145. Nov., 1672.

"To explain colors, I suppose that as bodies of various sizes, densities, or sensations do by percussion or other action excite sounds of various tones, and consequently vibrations in the air of different bigness, so the rays of light, by impinging on the stiff refracting superficies, excite vibrations in the ether of various bigness, the biggest, strongest, or most potent rays, the largest vibrations; and others shorter, according to their bigness, strength, or power: and therefore the ends of the capillamenta of the optic nerve, which pave or face the retina, being such refracting superficies, when the rays impinge

upon them, they must there excite these vibrations, which vibrations (like those of sound in a trunk or trumpet) will run along the aqueous pores or crystalline pith of the capillamenta, through the optic nerves, into the sensorium; and there, I suppose, affect the sense with various colors, according to their bigness and mixture; the biggest with the strongest colors, reds and yellows; the least with the weakest—blues and violets; the middle with green, and a confusion of all with white—much after the manner that, in the sense of hearing, nature makes use of aërial vibrations of several bignesses to generate sounds of divers tones, for the analogy of nature is to be observed."—BIRCH, *Hist. of R. S.*, vol. iii., p. 262, Dec., 1675.

"Considering the lastingness of the motions excited in the bottom of the eye by light, are they not of a vibrating nature? Do not the most refrangible rays excite the shortest vibrations, the least refrangible the largest? May not the harmony and discord of colors arise from the proportions of the vibrations propagated through the fibres of the optic nerve into the brain, as the harmony and discord of sounds arise from the proportions of the vibrations of the air?"—*Optics*, Qu. 16, 13, 14.

[*Scholium omitted.*]

HYPOTHESIS IV

All material bodies have an attraction for the ethereal medium, by means of which it is accumulated within their substance, and for a small distance around them, in a state of greater density but not of greater elasticity.

It has been shown that the three former hypotheses, which may be called essential, are literally parts of the more complicated Newtonian system. This fourth hypothesis differs perhaps, in some degree from any that have been proposed by former authors, and is diametrically opposite to that of Newton; but both being in themselves equally probable, the opposition is merely accidental, and it is only to be inquired which is the best capable of explaining the phenomena. Other suppositions might perhaps be substituted for this, and therefore I do not consider it as fundamental, yet it appears to be the simplest and best of any that have occurred to me.

Proposition I

All impulses are propagated in a homogeneous elastic medium with an equable velocity.

Every experiment relative to sound coincides with the observation already quoted from Newton, that all undulations are propagated through the air with equal velocity; and this is further confirmed by calculations. (Lagrange, *Misc. Taur.*, vol. i., p. 91. Also, much more concisely, in my syllabus of a course of lectures on Natural and Experimental Philosophy, about to be published. Art. 289.) If the impulse be so great as materially to disturb the density of the medium, it will be no longer homogeneous; but, as far as concerns our senses, the quantity of motion may be considered as infinitely small. It is surprising that Euler, although aware of the matter of fact, should still have maintained that the more frequent undulations are more rapidly propagated. (*Theor. mus.* and *Conject. phys.*) It is possible that the actual velocity of the particles of the luminiferous ether may bear a much less proportion to the velocity of the undulations than in sound, for light may be excited by the motion of a body moving at the rate of only one mile in the time that light moves a hundred millions.

Scholium 1. It has been demonstrated that in different mediums the velocity varies in the subduplicate ratio of the force directly and of the density inversely. (*Misc. Taur.*, vol. i., p. 91. Young's *Syllabus.* Art. 294.)

Scholium 2. It is obvious, from the phenomena of elastic bodies and of sounds, that the undulations may cross each other without interruption; but there is no necessity that the various colors of white light should intermix their undulations, for, supposing the vibrations of the retina to continue but a five-hundredth of a second after their excitement, a million undulations of each of a million colors may arrive in distinct succession within this interval of time, and produce the same sensible effect as if all the colors arrived precisely at the same instant.

Proposition II

An undulation conceived to originate from the vibration of a single particle must expand through a homogeneous medium

in a spherical form, but with different quantities of motion in different parts.

For, since every impulse, considered as positive or negative, is propagated with a constant velocity, each part of the undulation must in equal times have passed through equal distances from the vibrating-point. And, supposing the vibrating particle, in the course of its motion, to proceed forward to a small distance in a given direction, the principal strength of the undulation will naturally be straight before it ; behind it the motion will be equal in a contrary direction ; and at right angles to the line of vibration the undulation will be evanescent.

Now, in order that such an undulation may continue its progress to any considerable distance, there must be in each part of it a tendency to preserve its own motion in a right line from the centre ; for if the excess of force at any part were communicated to the neighboring particles, there can be no reason why it should not very soon be equalized throughout, or, in other words, become wholly extinct, since the motions in contrary directions would naturally destroy each other. The origin of sound from the vibration of a chord is evidently of this nature ; on the contrary, in a circular wave of water every part is at the same instant either elevated or depressed. It may be difficult to show mathematically the mode in which this inequality of force is preserved, but the inference from the matter of fact appears to be unavoidable ; and while the science of hydrodynamics is so imperfect that we cannot even solve the simple problem of the time required to empty a vessel by a given aperture, it cannot be expected that we should be able to account perfectly for so complicated a series of phenomena as those of elastic fluids. The theory of Huygens, indeed, explains the circumstance in a manner tolerably satisfactory. He supposes every particle of the medium to propagate a distinct undulation in all directions, and that the general effect is only perceptible where a portion of each undulation conspires in direction at the same instant ; and it is easy to show that such a general undulation would in all cases proceed rectilinearly, with proportionate force ; but, upon this supposition, it seems to follow that a greater quantity of force must be lost by the divergence of the partial undulations than appears to be consistent with the propagation of the effect to any considerable

distance; yet it is obvious that some such limitation of the motion must naturally be expected to take place, for if the intensity of the motion of any particular part, instead of continuing to be propagated straight forward, were supposed to affect the intensity of a neighboring part of the undulation, an impulse must then have travelled from an internal to an external circle in an oblique direction, in the same time as in the direction of the radius, and consequently with a greater velocity, against the first proposition. In the case of water the velocity is by no means so rigidly limited as in that of an elastic medium. Yet it is not necessary to suppose, nor is it indeed probable, that there is absolutely not the least lateral communication of the force of the undulation, but that, in highly elastic mediums, this communication is almost insensible. In the air, if a chord be perfectly insulated so as to propagate exactly such vibrations as have been described, they will, in fact, be much less forcible than if the chord be placed in the neighborhood of a sounding-board, and probably in some measure because of this lateral communication of motions of an opposite tendency. And the different intensity of different parts of the same circular undulation may be observed by holding a common tuning-fork at arm's-length, while sounding, and turning it, from a plane directed to the ear, into a position perpendicular to that plane.

Proposition III

A portion of a spherical undulation, admitted through an aperture into a quiescent medium, will proceed to be further propagated rectilinearly in concentric superficies, terminated laterally by weak and irregular portions of newly diverging undulations.

At the instant of admission the circumference of each of the undulations may be supposed to generate a partial undulation, filling up the nascent angle between the radii and the surface terminating the medium; but no sensible addition will be made to its strength by a divergence of motion from any other parts of the undulation, for want of a coincidence in time, as has already been explained with respect to the various force of a spherical undulation. If, indeed, the aperture bear but a small proportion to the breadth of an undulation, the newly gener-

ated undulation may nearly absorb the whole force of the portion admitted; and this is the case considered by Newton in the *Principia*. But no experiment can be made under these circumstances with light, on account of the minuteness of its undulations and the interference of inflection; and yet some faint radiations do actually diverge beyond any probable limits of inflection, rendering the margin of the aperture distinctly visible in all directions. These are attributed by Newton to some unknown cause, distinct from inflection (*Optics*, Book iii., obs. 5), and they fully answer the description of this proposition.

Let the concentric lines in Fig. 13 represent the contemporaneous situation of similar parts of a number of successive undulations diverging from the point A; they will also represent the successive situations of each individual undulation: let the force of each undulation be represented by the breadth of the line, and let the cone of light ABC be admitted through the aperture BC; then the principal undulations will proceed in a rectilinear direction towards GH, and the faint radiations on each side will diverge from B and C as centres, without receiving any additional force from any intermediate point D of the undulation, on account of the inequality of the lines DE and DF. But if we allow some little lateral divergence from the extremities of the undulations, it must diminish their force, without adding materially to that of the dissipated light; and their termination, instead of the right line BG, will assume the form CH, since the loss of force must be more considerable near to C than at greater distances. This line corresponds with the boundary of the shadow in Newton's first observation, Fig. 13; and it is much more probable that such a dissipation of light was the cause of the increase of the shadow in that observation than that it was owing to the action of the inflecting atmosphere, which must have extended a thirtieth of an inch each way in order to produce it; especially when it is considered that

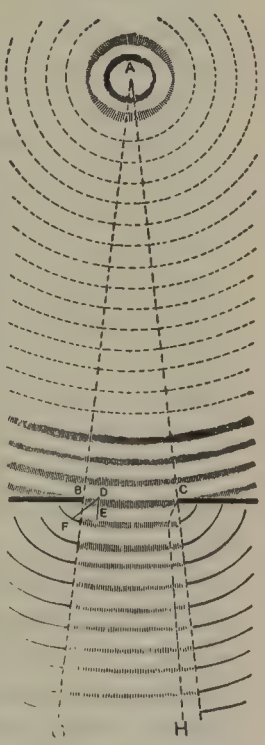

Fig. 13

the shadow was not diminished by surrounding the hair with a denser medium than air, which must in all probability have weakened and contracted its inflecting atmosphere. In other circumstances the lateral divergence might appear to increase, instead of diminishing, the breadth of the beam.

As the subject of this proposition has always been esteemed the most difficult part of the undulatory system, it will be proper to examine here the objections which Newton has grounded upon it.

"To me the fundamental supposition itself seems impossible—namely, that the waves or vibrations of any fluid can, like the rays of light, be propagated in straight lines, without a continual and very extravagant spreading and bending every way into the quiescent medium, where they are terminated by it. I mistake if there be not both experiment and demonstration to the contrary."—*Phil. Trans.*, vol. vii., p. 5089. Abr., vol. i., p. 146. Nov. 1672.

"Motus omnis per fluidum propagatus divergit a recto tramite in spatia immota."

"Quoniam medium ibi," in the middle of an undulation admitted, "densius est, quam in spatiis hinc inde, dilatabit sese tam versus spatia utrinque sita, quam versus pulsum rariora intervalla; eoque pacto—pulsus eadem *fere* celeritate sese in medii partes quiescentes hinc inde relaxare debent;—ideoque spatium totum occupabunt—Hoc experimur in sonis."—*Princip.*, lib. ii., prop. 42.

"Are not all hypotheses erroneous in which light is supposed to consist in pression or motion propagated through a fluid medium? If it consisted in pression or motion, propagated either in an instant, or in time, it would bend into the shadow. For pression or motion cannot be propagated in a fluid in right lines beyond an obstacle which stops part of the motion, but will bend and spread every way into the quiescent medium which lies beyond the obstacle. The waves on the surface of stagnating water passing by the sides of a broad obstacle which stops part of them, bend afterwards, and dilate themselves gradually into the quiet water behind the obstacle. The waves, pulses, or vibrations of the air, wherein sounds consist, bend manifestly, though not so much as the waves of water. For a bell or a cannon may be heard beyond a hill which intercepts the sight of the sounding body; and sounds

are propagated as readily through crooked pipes as straight ones. But light is never known to follow crooked passages nor to bend into the shadow. For the fixed stars, by the interposition of any of the planets, cease to be seen. And so do the parts of the sun by the interposition of the moon, Mercury, or Venus. The rays which pass very near to the edges of any body are bent a little by the action of the body; but this bending is not towards but from the shadow, and is performed only in the passage of the ray by the body, and at a very small distance from it. So soon as the ray is past the body it goes right on."—*Optics*, Qu. 28.

Now the proposition quoted from the *Principia* does not directly contradict this proposition; for it does not assert that such a motion must diverge equally in all directions; neither can it with truth be maintained that the parts of an elastic medium communicating any motion must propagate that motion equally in all directions. All that can be inferred by reasoning is that the marginal parts of the undulation must be somewhat weakened and that there must be a faint divergence in every direction; but whether either of these effects might be of sufficient magnitude to be sensible could not have been inferred from argument, if the affirmative had not been rendered probable by experiment.

As to the analogy with other fluids, the most natural inference from it is this: "The waves of the air, wherein sounds consist, bend manifestly, though not so much as the waves of water"; water being an inelastic and air a moderately elastic medium; but ether being most highly elastic, its waves bend very far less than those of the air, and therefore almost imperceptibly. Sounds are propagated through crooked passages, because their sides are capable of reflecting sound, just as light would be propagated through a bent tube, if perfectly polished within.

The light of a star is by far too weak to produce, by its faint divergence, any visible illumination of the margin of a planet eclipsing it; and the interception of the sun's light by the moon is as foreign to the question as the statement of inflection is inaccurate.

To the argument adduced by Huygens in favor of the rectilinear propagation of undulations Newton has made no reply; perhaps because of his own misconception of the nature of the

motions of elastic mediums, as dependent on a peculiar law of vibration, which has been corrected by later mathematicians. On the whole, it is presumed that this proposition may be safely admitted as perfectly consistent with analogy and with experiment.

Proposition IV

When an undulation arrives at a surface which is the limit of mediums of different densities, a partial reflection takes place proportionate in force to the difference of the densities.

This may be illustrated, if not demonstrated, by the analogy of elastic bodies of different sizes. "If a smaller elastic body strikes against a larger one, it is well known that the smaller is reflected more or less powerfully, according to the difference of their magnitudes: thus, there is always a reflection when the rays of light pass from a rarer to a denser stratum of ether; and frequently an echo when a sound strikes against a cloud. A greater body striking a smaller one propels it, without losing all its motion: thus, the particles of a denser stratum of ether do not impart the whole of their motion to a rarer, but, in their effort to proceed, they are recalled by the attraction of the refracting substance with equal force; and thus a reflection is always secondarily produced when the rays of light pass from a denser to a rarer stratum." But it is not absolutely necessary to suppose an attraction in the latter case, since the effort to proceed would be propagated backward without it, and the undulation would be reversed, a rarefaction returning in place of a condensation; and this will perhaps be found most consistent with the phenomena.

[*Propositions V., VI., and VII. omitted.*]

Proposition VIII

When two undulations, from different origins, coincide either perfectly or very nearly in direction, their joint effect is a combination of the motions belonging to each.

Since every particle of the medium is affected by each undulation, wherever the directions coincide, the undulations can proceed no otherwise than by uniting their motions, so that the joint motion may be the sum or difference of the separate

motions, accordingly as similar or dissimilar parts of the undulations are coincident.

I have, on a former occasion, insisted at large on the application of this principle to harmonics; and it will appear to be of still more extensive utility in explaining the phenomena of colors. The undulations which are now to be compared are those of equal frequency. When the two series coincide exactly in point of time, it is obvious that the united velocity of the particular motions must be greatest, and, in effect at least, double the separate velocities; and also that it must be smallest, and, if the undulations are of equal strength, totally destroyed when the time of the greatest direct motion belonging to one undulation coincides with that of the greatest retrograde motion of the other. In intermediate states the joint undulation will be of intermediate strength; but by what laws this intermediate strength must vary cannot be determined without further data. It is well known that a similar cause produces in sound that effect which is called a beat; two series of undulations of nearly equal magnitude co-operating and destroying each other alternately, as they coincide more or less perfectly in the times of performing their respective motions.

[*Proposition IX. and five corollaries to Proposition VIII. are here omitted.*]

AN ACCOUNT OF SOME CASES OF THE PRODUCTION OF COLORS NOT HITHERTO DESCRIBED*

READ JULY 1, 1802

WHATEVER opinion may be entertained of the theory of light and colors which I have lately had the honor of submitting to the Royal Society, it must at any rate be allowed that it has given birth to the discovery of a simple and general law capable of explaining a number of the phenomena of colored light, which, without this law, would remain insulated and unintelligible. The law is, that "wherever two portions of the same light arrive at the eye by different routes, either exactly or very nearly in the same direction, the light becomes most intense when the difference of the routes is any multiple of a certain length, and least intense in the intermediate state of the interfering portions; and this length is different for light of different colors."

I have already shown in detail the sufficiency of this law for explaining all the phenomena described in the second and third books of Newton's *Optics*, as well as some others not mentioned by Newton. But it is still more satisfactory to observe its conformity to other facts, which constitute new and distinct classes of phenomena, and which could scarcely have agreed so well with any anterior law, if that law had been erroneous or imaginary: these are the colors of fibres and the colors of mixed plates.

As I was observing the appearance of the fine parallel lines of light which are seen upon the margin of an object held near

*From the *Philosophical Transactions* for 1802, p. 387.

the eye, so as to intercept the greater part of the light of a distant luminous object, and which are produced by the fringes caused by the inflection of light already known, I observed that they were sometimes accompanied by colored fringes, much broader and more distinct; and I soon found that these broader fringes were occasioned by the accidental interposition of a hair. In order to make them more distinct, I employed a horse-hair, but they were then no longer visible. With a fibre of wool, on the contrary, they became very large and conspicuous; and, with a single silk-worm's thread, their magnitude was so much increased that two or three of them seemed to occupy the whole field of view. They appeared to extend on each side of the candle, in the same order as the colors of thin plates seen by transmitted light. It occurred to me that their cause must be sought in the interference of two portions of light, one reflected from the fibre, the other bending round its opposite side, and at last coinciding nearly in direction with the former portion; that, accordingly, as both portions deviated more from a rectilinear direction, the difference of the length of their paths would become gradually greater and greater, and would consequently produce the appearances of color usual in such cases; that supposing them to be inflected at right angles, the difference would amount nearly to the diameter of the fibre, and that this difference must consequently be smaller as the fibre became smaller; and, the number of fringes in a right angle becoming smaller, that their angular distances would consequently become greater, and the whole appearance would be dilated. It was easy to calculate that for the light least inflected the difference of the paths would be to the diameter of the fibre very nearly as the deviation of the ray at any point from the rectilinear direction to its distance from the fibre.

I therefore made a rectangular hole in a card, and bent its ends so as to support a hair parallel to the sides of the hole; then, upon applying the eye near the hole, the hair, of course, appeared dilated by indistinct vision into a surface, of which the breadth was determined by the distance of the hair and the magnitude of the hole, independently of the temporary aperture of the pupil. When the hair approached so near to the direction of the margin of a candle that the inflected light was sufficiently copious to produce a sensible effect, the fringes

began to appear; and it was easy to estimate the proportion of their breadth to the apparent breadth of the hair across the image of which they extended. I found that six of the brightest red fringes, nearly at equal distances, occupied the whole of that image. The breadth of the aperture was $\frac{66}{1000}$, and its distance from the hair $\frac{8}{10}$ of an inch; the diameter of the hair was less than $\frac{1}{500}$ of an inch; as nearly as I could ascertain it was $\frac{1}{600}$. Hence, we have $\frac{11}{1000}$ for the deviation of the first red fringe at the distance $\frac{8}{10}$; and as $\frac{8}{10} : \frac{11}{1000} :: \frac{1}{600} : \frac{11}{480000}$, or $\frac{1}{43636}$ for the difference of the routes of the red light where it was most intense. The measure deduced from Newton's experiments is $\frac{1}{39200}$. I thought this coincidence, with only an error of one-ninth of so minute a quantity, sufficiently perfect to warrant completely the explanation of the phenomenon, and even to render a repetition of the experiment unnecessary; for there are several circumstances which make it difficult to calculate much more precisely what ought to be the result of the measurement.

When a number of fibres of the same kind—for instance, a uniform lock of wool—are held near to the eye, we see an appearance of halos surrounding a distant candle; but their brilliancy, and even their existence, depends on the uniformity of the dimensions of the fibres; and they are larger as the fibres are smaller. It is obvious that they are the immediate consequences of the coincidence of a number of fringes of the same size, which, as the fibres are arranged in all imaginable directions, must necessarily surround the luminous object at equal distances on all sides, and constitute circular fringes.

There can be little doubt that the colored atmospherical halos are of the same kind; their appearance must depend on the existence of a number of particles of water of equal dimensions, and in a proper position with respect to the luminary and to the eye. As there is no natural limit to the magnitude of the spherules of water, we may expect these halos to vary without limit in their diameters, and accordingly Mr. Jordan has observed that their dimensions are exceedingly various, and has remarked that they frequently change during the time of observation.

I first noticed the colors of mixed plates in looking at a candle through two pieces of plate-glass with a little moisture between them. I observed an appearance of fringes resembling

the common colors of thin plates; and, upon looking for the fringes by reflection, I found that these new fringes were always in the same direction as the other fringes, but many times larger. By examining the glasses with a magnifier, I perceived that wherever these fringes were visible the moisture was intermixed with portions of air, producing an appearance similar to dew. I then supposed that the origin of the colors was the same as that of the colors of halos; but, on a more minute examination, I found that the magnitude of the portions of air and water was by no means uniform, and that the explanation was, therefore, inadmissible. It was, however, easy to find two portions of light sufficient for the production of these fringes; for the light transmitted through the water, moving in it with a velocity different from that of the light passing through the interstices filled only with air, the two portions would interfere with each other and produce effects of color according to the general law. The ratio of the velocities in water and in air is that of 3 to 4; the fringes ought, therefore, to appear where the thickness is six times as great as that which corresponds to the same color in the common case of thin plates; and, upon making the experiment with a plane glass and a lens slightly convex, I found the sixth dark circle actually of the same diameter as the first in the new fringes. The colors are also very easily produced when butter or tallow is substituted for water; and the rings then become smaller, on account of the greater refractive density of the oils; but when water is added, so as to fill up the interstices of the oil, the rings are very much enlarged; for here the difference only of the velocities in water and in oil is to be considered, and this is much smaller than the difference between air and water. All these circumstances are sufficient to satisfy us with respect to the truth of the explanation; and it is still more confirmed by the effect of inclining the plates to the direction of the light; for then, instead of dilating, like the colors of thin plates, these rings contract: and this is the obvious consequence of an increase of the length of the paths of light, which now traverse both mediums obliquely; and the effect is everywhere the same as that of a thicker plate.

It must, however, be observed that the colors are not produced in the whole light that is transmitted through the mediums: a small portion only of each pencil, passing through

the water contiguous to the edges of the particle, is sufficiently coincident with the light transmitted by the neighboring portions of air to produce the necessary interference; and it is easy to show that, on account of the natural concavity of the surface of each portion of the fluid adhering to the two pieces of glass, a considerable portion of the light which is beginning to pass through the water will be dissipated laterally by reflection at its entrance, and that much of the light passing through the air will be scattered by refraction at the second surface. For these reasons the fringes are seen when the plates are not directly interposed between the eye and the luminous object; and on account of the absence of foreign light, even more distinctly than when they are in the same right line with that object. And if we remove the plates to a considerable distance out of this line, the rings are still visible and become larger than before; for here the actual route of the light passing through the air is longer than that of the light passing more obliquely through the water, and the difference in the times of passage is lessened. It is, however, impossible to be quite confident with respect to the causes of these minute variations, without some means of ascertaining accurately the forms of the dissipating surfaces.

In applying the general law of interference to these colors, as well as to those of thin plates already known, I must confess that it is impossible to avoid another supposition, which is a part of the undulatory theory—that is, that the velocity of light is the greater the rarer the medium; and that there is also a condition annexed to the explanation of the colors of thin plates which involves another part of the same theory—that is, that where one of the portions of light has been reflected at the surface of a rarer medium, it must be supposed to be retarded one-half of the appropriate interval—for instance, in the central black spot of a soap-bubble, where the actual lengths of the paths very nearly coincide, but the effect is the same as if one of the portions had been so retarded as to destroy the other. From considering the nature of this circumstance, I ventured to predict that if the two reflections were of the same kind, made at the surfaces of a thin plate of a density intermediate between the densities of the mediums containing it, the effect would be reversed, and the central spot, instead of black, would become white; and I have now

the pleasure of stating that I have fully verified this prediction by interposing a drop of oil of sassafras between a prism of flint-glass and a lens of crown-glass; the central spot seen by reflected light was white and surrounded by a dark ring. It was, however, necessary to use some force in order to produce a contact sufficiently intimate; and the white spot differed, even at last, in the same degree from perfect whiteness as the black spot usually does from perfect blackness.

[*Three pages of speculation concerning dispersion are here omitted.*]

EXPERIMENTS AND CALCULATIONS RELATIVE TO PHYSICAL OPTICS*

A BAKERIAN LECTURE

Read November 24, 1803

I. EXPERIMENTAL DEMONSTRATION OF THE GENERAL LAW OF THE INTERFERENCE OF LIGHT.

In making some experiments on the fringes of colors accompanying shadows, I have found so simple and so demonstrative a proof of the general law of the interference of two portions of light, which I have already endeavored to establish, that I think it right to lay before the Royal Society a short statement of the facts which appear to me so decisive. The proposition on which I mean to insist at present is simply this—that fringes of colors are produced by the interference of two portions of light; and I think it will not be denied by the most prejudiced that the assertion is proved by the experiments I am about to relate, which may be repeated with great ease whenever the sun shines, and without any other apparatus than is at hand to every one.

Experiment 1. I made a small hole in a window-shutter, and covered it with a piece of thick paper, which I perforated with a fine needle. For greater convenience of observation I placed a small looking-glass without the window-shutter, in such a position as to reflect the sun's light in a direction nearly horizontal upon the opposite wall, and to cause the cone of diverging light to pass over a table on which were several little screens of card-paper. I brought into the sunbeam a slip of

* From the *Philosophical Transactions* for 1804.

card about one-thirtieth of an inch in breadth, and observed its shadow, either on the wall or on other cards held at different distances. Besides the fringes of color on each side of the shadow, the shadow itself was divided by similar parallel fringes of smaller dimensions, differing in number according to the distance at which the shadow was observed, but leaving the middle of the shadow always white. Now these fringes were the joint effects of the portions of light passing on each side of the slip of card, and inflected, or rather diffracted, into the shadow; for a little screen being placed a few inches from the card so as to receive either edge of the shadow on its margin, all the fringes which had before been observed in the shadow on the wall immediately disappeared, although the light inflected on the other side was allowed to retain its course, and although this light must have undergone any modification that the proximity of the other edge of the slip of card might have been capable of occasioning. When the interposed screen was more remote from the narrow card, it was necessary to plunge it more deeply into the shadow, in order to extinguish the parallel lines; for here the light diffracted from the edge of the object had entered farther into the shadow in its way towards the fringes. Nor was it for want of a sufficient intensity of light that one of the two portions was incapable of producing the fringes alone; for when they were both uninterrupted, the lines appeared, even if the intensity was reduced to one-tenth or one-twentieth.

Experiment 2. The crested fringes described by the ingenious and accurate Grimaldi afford an elegant variation of the preceding experiment and an interesting example of a calculation grounded on it. When a shadow is formed by an object which has a rectangular termination besides the usual external fringes there are two or three alternations of colors, beginning from the line which bisects the angle, disposed on each side of it in curves, which are convex towards the bisecting line, and which converge in some degree towards it as they become more remote from the angular point. These fringes are also the joint effect of the light which is inflected directly towards the shadow from each of the two outlines of the object; for if a screen be placed within a few inches of the object, so as to receive only one of the edges of the shadow, the whole of the fringes disappear; if, on the contrary, the rectangular point

of the screen be opposed to the point of the shadow so as barely to receive the angle of the shadow on its extremity, the fringes will remain undisturbed.

II. COMPARISON OF MEASURES DEDUCED FROM VARIOUS EXPERIMENTS.

If we now proceed to examine the dimensions of the fringes under different circumstances, we may calculate the differences of the lengths of the paths described by the portions of light which have thus been proved to be concerned in producing those fringes; and we shall find that where the lengths are equal the light always remains white; but that where either the brightest light or the light of any given color disappears and reappears a first, a second, or a third time, the differences of the lengths of the paths of the two portions are in arithmetical progression, as nearly as we can expect experiments of this kind to agree with each other. I shall compare, in this point of view, the measures deduced from several experiments of Newton and from some of my own.

In the eighth and ninth observations of the third book of Newton's *Optics* some experiments are related which, together with the third observation, will furnish us with the data necessary for the calculation. Two knives were placed, with their edges meeting at a very acute angle, in a beam of the sun's light, admitted through a small aperture, and the point of concourse of the two first dark lines bordering the shadows of the respective knives was observed at various distances. The results of six observations are expressed in the first three lines of the first table. On the supposition that the dark line is produced by the first interference of the light reflected from the edges of the knives, with the light passing in a straight line between them, we may assign, by calculating the difference of the two paths, the interval for the first disappearance of the brightest light, as it is expressed in the fourth line. The second table contains the results of a similar calculation from Newton's observations on the shadow of a hair; and the third, from some experiments of my own of the same nature; the second bright line being supposed to correspond to a double interval, the second dark line to a triple interval, and the succeeding lines to depend on a continuation of the progression. The unit of all the tables is an inch.

TABLE I.—*Observation* 9. N.

Distance of the knives from the aperture						101
Distance of the paper from the knives	$1\frac{1}{8}$	$3\frac{1}{8}$	$8\frac{3}{5}$	32	96	131
Distance between the edges of the knives opposite to the point of concourse	.012	.020	.034	.057	.081	.087
Interval of disappearance	.0000122	.0000155	.0000182	.0000167	.0000166	.0000166

TABLE II.—*Observation* 3. N.

Breadth of the hair		$\frac{1}{280}$
Distance of the hair from the aperture		144
Distances of the scale from the aperture	150	252
(Breadths of the shadow	$\frac{1}{54}$	$\frac{1}{9}$)
Breadth between the second pair of bright lines	$\frac{2}{47}$	$\frac{4}{17}$
Interval of disappearance, or half the difference of the paths	.0000151	.0000173
Breadth between the third pair of bright lines	$\frac{4}{73}$	$\frac{3}{10}$
Interval of disappearance, one-fourth of the difference	.0000130	.0000143

TABLE III.—*Experiment* 3.

Breadth of the object	.434
Distance of the object from the aperture	125
Distance of the wall from the aperture	250
Distance of the second pair of dark lines from each other	1.167
Interval of disappearance, one-third of the difference	.0000149

Experiment 4.

Breadth of the wire	.083
Distance of the wire from the aperture	32
Distance of the wall from the aperture	250
(Breadth of the shadow, by three measurements	.815, .826, or .827; mean, .823)
Distance of the first pair of dark lines	1.165, 1.170, or 1.160; mean, 1.165
Interval of disappearance	0000194
Distance of the second pair of dark lines	1.402, 1.395, or 1.400; mean, 1.399
Interval of disappearance	.0000137
Distance of the third pair of dark lines	1.594, 1.580, or 1.585; mean, 1.586
Interval of disappearance	.0000128

It appears, from five of the six observations of the first table, in which the distance of the shadow was varied from about 3 inches to 11 feet, and the breadth of the fringes was increased in the ratio of 7 to 1, that the difference of the routes constituting the interval of disappearance varied but one-eleventh at most; and that in three out of the five it agreed with the mean, either exactly or within $\frac{1}{160}$ part. Hence we are warranted in inferring that the interval appropriate to the extinction of the brightest light is either accurately or very nearly constant.

But it may be inferred from a comparison of all the other observations that when the obliquity of the reflection is very great some circumstance takes place which causes the interval thus calculated to be somewhat greater; thus, in the eleventh line of the third table it comes out one-sixth greater than the mean of the five already mentioned. On the other hand, the mean of two of Newton's experiments and one of mine is a result about one-fourth less than the former. With respect to the nature of this circumstance I cannot at present form a decided opinion; but I conjecture that it is a deviation of some of the light concerned, from the rectilinear direction assigned to it, arising either from its natural diffraction, by which the magnitude of the shadow is also enlarged, or from some other unknown cause. If we imagined the shadow of the wire and the fringes nearest it to be so contracted that the motion of the light bounding the shadow might be rectilinear, we should thus make a sufficient compensation for this deviation; but it is difficult to point out what precise track of the light would cause it to require this correction.

The mean of the three experiments which appear to have been least affected by this unknown deviation gives .0000127 for the interval appropriate to the disappearance of the brightest light; and it may be inferred that if they had been wholly exempted from its effects the measure would have been somewhat smaller. Now the analogous interval, deduced from the experiments of Newton on this plate, is .0000112, which is about one-eighth less than the former result; and this appears to be a coincidence fully sufficient to authorize us to attribute these two classes of phenomena to the same cause. It is very easily shown, with respect to the colors of thin plates, that each kind of light disappears and reappears where the differ-

ences of the routes of two of its portions are in arithmetical progression; and we have seen that the same law may be in general inferred from the phenomena of diffracted light, even independently of the analogy.

The distribution of the colors is also so similar in both cases as to point immediately to a similarity in the causes. In the thirteenth observation of the second part of the first book Newton relates that the interval of the glasses where the rings appeared in red light was to the interval where they appeared in violet light as 14 to 9; and, in the eleventh observation of the third book, that the distances between the fringes, under the same circumstances, were the twenty-second and the twenty-seventh of an inch. Hence, deducting the breadth of the hair and taking the squares, in order to find the relation of the difference of the routes, we have the proportion of 14 to $9\frac{1}{4}$, which scarcely differs from the proportion observed in the colors of the thin plate.

We may readily determine from this general principle the form of the crested fringes of Grimaldi, already described; for it will appear that, under the circumstances of the experiment related, the points in which the differences of the lengths of the paths described by the two portions of light are equal to a constant quantity, and in which, therefore, the same kinds of light ought to appear or disappear, are always found in equilateral hyperbolas, of which the axes coincide with the outlines of the shadow, and the asymptotes nearly with the diagonal line. Such, therefore, must be the direction of the fringes; and this conclusion agrees perfectly with the observation. But it must be remarked that the parts near the outlines of the shadow are so much shaded off as to render the character of the curve somewhat less decidedly marked where it approaches to its axis. These fringes have a slight resemblance to the hyperbolic fringes observed by Newton; but the analogy is only distant.

[*III. Application to the Supernumerary Rainbows, omitted.*]

IV. ARGUMENTATIVE INFERENCE RESPECTING THE NATURE OF LIGHT.

The experiment of Grimaldi on the crested fringes within the shadow, together with several others of his observations

equally important, has been left unnoticed by Newton. Those who are attached to the Newtonian theory of light, or to the hypothesis of modern opticians founded on views still less enlarged, would do well to endeavor to imagine anything like an explanation of these experiments derived from their own doctrines; and if they fail in the attempt, to refrain at least from idle declamation against a system which is founded on the accuracy of its application to all these facts, and to a thousand others of a similar nature.

From the experiments and calculation which have been premised, we may be allowed to infer that homogeneous light at certain equal distances in the direction of its motion is possessed of opposite qualities capable of neutralizing or destroying each other, and of extinguishing the light where they happen to be united; that these qualities succeed each other alternately in successive concentric superficies, at distances which are constant for the same light passing through the same medium. From the agreement of the measures, and from the similarity of the phenomena, we may conclude that these intervals are the same as are concerned in the production of the colors of thin plates; but these are shown, by the experiments of Newton, to be the smaller the denser the medium; and since it may be presumed that their number must necessarily remain unaltered in a given quantity of light, it follows, of course, that light moves more slowly in a denser than in a rarer medium; and this being granted, it must be allowed that refraction is not the effect of an attractive force directed to a denser medium. The advocates for the projectile hypothesis of light must consider which link in this chain of reasoning they may judge to be the most feeble, for hitherto I have advanced in this paper no general hypothesis whatever. But since we know that sound diverges in concentric superficies, and that musical sounds consist of opposite qualities, capable of neutralizing each other, and succeeding at certain equal intervals, which are different according to the difference of the note, we are fully authorized to conclude that there must be some strong resemblance between the nature of sound and that of light.

I have not, in the course of these investigations, found any reason to suppose the presence of such an inflecting medium in the neighborhood of dense substances as I was formerly

inclined to attribute to them; and, upon considering the phenomena of the aberration of the stars, I am disposed to believe that the luminiferous ether pervades the substance of all material bodies, with little or no resistance, as freely, perhaps, as the wind passes through a grove of trees.

The observations on the effects of diffraction and interference may, perhaps, sometimes be applied to a practical purpose in making us cautious in our conclusions respecting the appearances of minute bodies viewed in a microscope. The shadow of a fibre, however opaque, placed in a pencil of light admitted through a small aperture, is always somewhat less dark in the middle of its breadth than in the parts on each side. A similar effect may also take place, in some degree, with respect to the image on the retina, and impress the sense with an idea of a transparency which has no real existence; and if a small portion of light be really transmitted through the substance, this may again be destroyed by its interference with the diffracted light, and produce an appearance of partial opacity, instead of uniform semi-transparency. Thus a central dark spot and a light spot, surrounded by a darker circle, may respectively be produced in the images of a semi-transparent and an opaque corpuscle, and impress us with an idea of a complication of structure which does not exist. In order to detect the fallacy, we make two or three fibres cross each other, and view a number of globules contiguous to each other; or we may obtain a still more effectual remedy by changing the magnifying power; and then, if the appearance remain constant in kind and in degree, we may be assured that it truly represents the nature of the substance to be examined. It is natural to inquire whether or not the figures of the globules of blood delineated by Mr. Hewson in the *Phil. Trans.*, vol. lxiii., for 1773, might not in some measure have been influenced by a deception of this kind; but, as far as I have hitherto been able to examine the globules with a lens of one-fiftieth of an inch focus, I have found them nearly such as Mr. Hewson has described them.

[*V. Remarks on the Colors of Natural Bodies, omitted.*]

VI. EXPERIMENT ON THE DARK RAYS OF RITTER

Experiment 6. The existence of solar rays accompanying light, more refrangible than the violet rays and cognizable by

their chemical effects, was first ascertained by Mr. Ritter; but Dr. Wollaston made the same experiments a very short time afterwards without having been informed of what had been done on the Continent. These rays appear to extend beyond the violet rays of the prismatic spectrum, through a space nearly equal to that which is occupied by the violet. In order to complete the comparison of their properties with those of visible light, I was desirous of examining the effect of their reflection from a thin plate of air, capable of producing the well-known rings of colors. For this purpose I formed an image of the rings, by means of the solar microscope, with the apparatus which I have described in the Journals of the Royal Institution, and I threw this image on paper dipped in a solution of nitrate of silver, placed at the distance of about nine inches from the microscope. In the course of an hour portions of three dark rings were very distinctly visible, much smaller than the brightest rings of the colored image, and coinciding very nearly in their dimensions with the rings of violet light that appeared upon the interposition of violet glass. I thought the dark rings were a little smaller than the violet rings, but the difference was not sufficiently great to be accurately ascertained; it might be as much as $\frac{1}{30}$ or $\frac{1}{40}$ of the diameters, but not greater. It is the less surprising that the difference should be so small, as the dimensions of the colored rings do not by any means vary at the violet end of the spectrum so rapidly as at the red end. For performing this experiment with very great accuracy a heliostat would be necessary, since the motion of the sun causes a slight change in the place of the image; and leather impregnated with the muriate of silver would indicate the effect with greater delicacy. The experiment, however, in its present state, is sufficient to complete the analogy of the invisible with the visible rays, and to show that they are equally liable to the general law which is the principal subject of this paper. If we had thermometers sufficiently delicate, it is probable that we might acquire, by similar means, information still more interesting with respect to the rays of invisible heat discovered by Dr. Herschel; but at present there is great reason to doubt of the practicability of such an experiment.

THE WAVE-THEORY OF LIGHT

Biographical Sketch

Thomas Young was born at Milverton, England, in 1773, and died at London in 1829. His education, in respect to the amount of ground it covered, is quite as remarkable as his later scientific work. As a lad he showed marked proficiency in linguistic studies, acquired great mechanical skill, distinguished himself in drawing, music, and athletics. As a young man he pursued his university studies at London, Edinburgh, Göttingen, and Cambridge.

The following programme of his daily work at Göttingen in the autumn of 1795 characterizes at once the lad, the youth, and the mature man:

"At 8, I attend Spittler's course on the History of the Principal States of Europe, exclusive of Germany.

"At 9, Arnemann on Materia Medica.

"At 10, Richter on Acute Diseases.

"At 11, twice a week, private lessons from Blessman, the academical dancing-master.

"At 12, I dine at Ruhlander's table d'hôte.

"At 1, twice a week, lessons on the clavichord from Forkel; and twice a week at home, from Fiorillo on Drawing.

"At 2, Lichtenberg on Physics.

"At 3, I ride in the academical manége, under the instruction of Ayrer, four times a week.

"At 4, Stromeyer on Diseases.

"At 5, Blumenbach on Natural History.

"At 6, twice Blessman with other pupils, and twice Forkel."

He was born of a well-to-do Quaker family; he inherited ample money; he had all that travel, leisure, and good society could do for a man. Only in one particular does his education appear to have been defective—*viz.*, in the absence of any training in advanced dynamics or in higher mathematical analysis.

In 1800 he completed his medical studies at Cambridge, and settled as a practising physician in London. In the year following he was appointed to the professorship of natural philosophy in the then newly founded Royal Institution, a position from which he resigned at the end of two years in order to devote himself more completely to the practice of medicine. It was during his occupancy of this chair that he published the

three papers reprinted in this volume, the first of which is possibly the most important of his contributions to physics. It was during this period also that he wrote his *Lectures on Natural Philosophy*, which must always be reckoned as a potent factor in the spread of sound physical science in the nineteenth century, while its bibliography of more than four hundred quarto pages is to-day valuable as well as classic.

But nothing short of a catalogue of his papers can give one an adequate idea of the varied activity of this man during the remaining quarter-century of his life. His contributions cover fields as diverse as the physiology of the human eye, hydrodynamics, music, paleography, atmospheric refraction, theory of tides, tables of mortality, theory of structures. His explanation of color-vision as due to the presence of three sets of nerve fibres in the retina, which, when excited, give respectively sensations of red, green, and violet, has been adopted and modified by Helmholtz, and is to-day perhaps the most widely accepted of the various theories on this subject.

After all, it must be confessed, even by his most ardent admirers, that Young's style is, in general, far from clear. Whether this is in any way connected with his lack of mathematical training, or whether it is due to the fact that his own clear intuitions bridged most of the gaps in his written work, it is difficult to say; but in any event many of his papers are obscure, and few of them are read. The reader who desires a full biography will find it in Dr. Peacock's *Life of Young* (London, 1855). This biographer also edited his *Miscellaneous Works*, 3 vols. (London, 1855). All his papers, however, which are of especial interest to the student of physics are contained in the lectures on *Natural Philosophy* (London, 1807).

MEMOIR ON THE DIFFRACTION OF LIGHT

"CROWNED" BY THE FRENCH ACADEMY OF SCIENCES IN 1819

BY

A. FRESNEL

Natura simplex et fecunda.

MEMOIR ON THE ACTION OF RAYS OF POLARIZED LIGHT UPON EACH OTHER

BY

MESSRS. ARAGO AND FRESNEL

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FRESNEL'S PRIZE MEMOIR ON THE DIFFRACTION OF LIGHT

[*The Introduction, covering fourteen pages and describing in the most general way the defects of the emission-theory and some of the merits of the wave-theory, is omitted.*]

SECTION I

11. It might appear that on the emission-theory nothing would be simpler than the phenomena of shadows, especially when the source of light is merely a point; but, on the contrary, nothing is more complicated. If we suppose the surface of the body producing the shadow to be endowed with a repulsive property capable of changing the direction of rays of light passing very near it, we should then expect only to see the shadows increase in size and, towards their edges, to blend a little with the illuminated area; while, as a matter of fact, they are bordered with three very distinct colored fringes when one employs white light, and with a still greater number of bright and dark bands when one uses light which is practically homogeneous. These fringes we shall call *exterior*, and those which are observed in the midst of very narrow shadows we shall call *interior* fringes.

If one adopts the Newtonian theory, he is tempted at first to explain the exterior fringes as produced by a force which is alternately attractive and repulsive, and which has its source in the surface of the body producing the shadows. I shall now consider the consequences of this theory and show that its results are not justified by experiment; but, first of all, I must explain the experimental method which I have employed.

12. We know that the effect of a magnifying-glass placed in front of the eye is to reproduce accurately upon the retina any object or image which is located at its conjugate focus; at least

this is the case whenever all the rays which go to make up the image are incident upon the surface of the glass. In place, then, of projecting the fringes upon a white card or a ground glass, one may observe them directly with a magnifying-glass, and he then sees them as they are at its focus. One has then only to look towards the luminous point and place the glass between his eye and the opaque body in such a way that the point where the refracted rays cross each other falls in the middle of the pupil ; this position is recognized by the fact that the entire surface of the magnifying-glass appears to be filled with light. This method is much preferable to the other two in that it enables us to study conveniently phenomena of diffraction in a weak light, and has, at the same time, the further advantage of allowing us to follow the exterior fringes right up to their source. Using a lens of 2 mm. focus and light which is practically homogeneous, I have been able to follow these fringes very close to their origin and yet observe the dark band of the fifth order. The interval which separates this band from the edge of the shadow I have measured on the micrometer and find it to be less than 0.015 mm., while the first three fringes are comprised within a space not exceed-0.01 mm. ; by using a lens of shorter focus, one would doubtless still further diminish this distance. We may thus regard the dark and bright bands as beginning at the very edge of the opaque body, so long as we do not push the accuracy of our measures beyond the hundredth part of a millimeter—an accuracy which proves to be sufficient, and which it is difficult to exceed except when the fringes are somewhat larger, as is the case with those most frequently observed.

13. This point established, suppose that we measure the exterior fringes at any given distance from the [*opaque*] screen and then allow the luminous point to approach ; the fringes are observed to grow much larger. Meanwhile the angle which the incident ray passing through the origin of the fringes makes with the tangent drawn from the luminous point to the edge of the screen will be almost zero. And since these fringes take their rise at a distance less than 0.01 mm. from the edge, the variation of this angle would not be able to sensibly affect the size of the fringes. To explain this enlargement, we must therefore assume that the repulsive force increases in proportion as the opaque body approaches the luminous point. But this is

impossible, for the intensity of this force can evidently depend only upon the distance at which the light corpuscle passes the opaque body, upon the size and form of the surface of this body, upon its density, mass, or nature; and by hypothesis these all remain constant.

But even if we suppose the origin of the dark and bright bands to lie at a greater distance from the edge of the screen, a supposition which would explain the fact that they grow larger in proportion as one approaches the luminous point, it is still impossible to make the results of experiment agree with the formula deduced from the [*Newtonian*] hypothesis which we are here discussing.

14. The following table gives the distance between the darkest point in the dark band of the fourth order and the edge of the geometrical shadow* for different distances of the opaque body from the luminous point. These measures have been taken with a micrometer eye-piece which carries in its focal plane a silk fibre, the whole being moved by a micrometer screw. By the aid of a head divided into one hundred parts, passing an index, fixed with reference to the screw, one is able to read the displacement of the silk thread to within about 0.01 mm.

No. of Observation	Distance of luminous point from opaque screen	Distance of opaque body from micrometer	Distance from edge of geometrical shadow to the middle of the dark band of the fourth order
	m.	m.	mm.
1	0.100	0.7985	5.96
2	0.510	1.005	3.84
3	1.011	0.996	3.12
4	2.008	0.999	2.71
5	3.018	1.003	2.56
6	4.507	1.018	2.49
7	6.007	0.999	2.40

These experiments were made with practically homogeneous red light, which was obtained by means of a colored glass trans-

* I define *geometrical shadow* as the space included between the straight lines drawn through the luminous point and tangent to the edges of the screen; this is the shadow which the light would project if it were not diffracted.

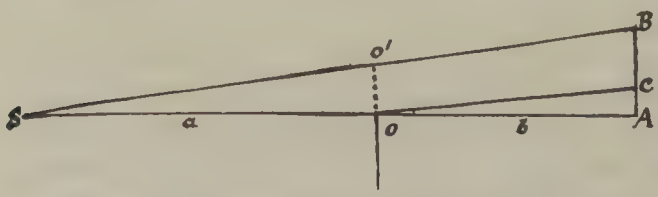
mitting only the red rays and a small portion of the orange rays. One might obtain more homogeneous light by use of a prism, but he would not be so certain as to its identity in the various observations—a condition which it is very necessary to satisfy.

15. Let us represent by a and b the respective distances of the opaque body from the luminous point and from the micrometer; let d be the distance from the edge of the body to the origin of the dark band of the fourth order, and r the tangent of the small angle of inflection resulting from the action of the repulsive forces. We then have the following expression* for the distance between the edge of the geometrical shadow and the darkest point in the dark band:

$$br + \frac{d\,(a+b)}{a}.$$

Now since r and d remain constant whatever be the distances of the luminous point from the opaque body and from the micrometer respectively, two observations suffice to determine their value. Combining the first and the last observations, we find $d = 0.5019$ mm. and $r = 1.8164$. We are thus compelled to suppose that at its origin the dark band of the fourth order is distant one-half a millimeter from the edge of the opaque body. If, now, we substitute these values in the formula and apply it to the intermediate observations, we obtain the following values, several of which evidently differ widely from the results of experiment.

* [*In diagram* S *is luminous point,* O *is edge of opaque body,* A *is edge of geometrical shadow,* O′ *is origin of dark band of fourth* (*or any*) *order. Hence*



AB *is* $\frac{d\,(a+b)}{a}$; *and the centre of the dark band is a distance* br *farther, where* AC $= br$. *The unit which Fresnel here employs for* r *is evidently one hundred times smaller than that in which we ordinarily express natural tangents.*]

No. of Observation	Distance of luminous point from opaque body	Distance of opaque body from micrometer	Distance between the edge of geometrical shadow and darkest point of fourth band		Differences
			Observed	Computed from formula $br+\frac{d(a+b)}{a}$	
	m.	m.	m.	mm.	mm.
1	0.1000	0.7985	5 96	——	——
2	0.510	1.005	3.84	3.32	−0.52
3	1.011	0.996	3.12	2 81	−0.31
4	2.008	0.999	2.71	2.57	−0.14
5	3.018	1.003	2.56	2.49	−0.07
6	4.507	1.018	2.49	2 46	−0.03
7	6.007	0.999	2.40	——	——

16. In attributing the production of fringes to the alternate expansion and contraction of rays which pass very near the opaque body, we are led to still another inference which is contradicted by experiment—*viz.*, that the centres of the dark and bright bands ought to lie along straight lines which would be the axes of the expanded or condensed pencils of rays. But experiment shows that in the case of exterior fringes their trajectories are hyperbolas, of which the curvature is quite sensible whenever the body which produces the shadow is sufficiently distant from the luminous point.

The screen being placed at a distance of 3.018 m. from the luminous point, I measured in succession the deviation of the darkest point of the dark band of the third order, first at 0.0017 m. from the screen, then at 1.003 m., and lastly at 3.995 m. from the screen; and I found for its distance from the edge of the geometrical shadow first 0.08 mm., secondly 2.20 mm., and thirdly 5.83 mm. If, now, we join the two extreme points by a straight line, we find for the ordinate corresponding to the intermediate point 1.52 mm. in place of 2.20 mm., the difference being 0.68 mm.—that is to say, about one and one-half times the interval between the middle of the third and the middle of the second bands. For this interval at a distance of 1.003 m. from the opaque body was only 0.42 mm., from which it is evident that the difference of 0.68 mm. cannot be attributed to an error resulting from lack of definition in the fringes observed. Nor is one able to explain this discrepancy by supposing an error in the observation made at 3.995 m. from the opaque body. From the fact that the

fringes are larger, the measures should be less accurate; but in repeating them several times I find variations which at most amount to three or four hundredths of a millimeter. Even supposing that there were an error of one-half a millimeter in this measure, it would produce only a difference of 0.13 mm. at a distance of 1.003 m.; so that experiment shows conclusively that the exterior fringes lie on curved lines with their convex side outwards.

The following table gives these trajectories, referred to their chords, for different series of observations, in each of which the distance of the opaque body from the luminous point remains constant. In the fourth series I suppose first that the chord joins the two extreme readings, and next I suppose it to be drawn from the edge of the opaque body itself where the deviation of the fringes from their origin is, as we have already seen, very small. In the other series the chord joins the edge of the opaque body and the point most distant from it.

Distance from luminous point to opaque screen, or the value of a	Distance from opaque body to micrometer, or the value of b	Ordinates of Trajectories of dark bands referred to their chords				
		1st order	2d order	3d order	4th order	5th order
		1st Series				
m. 0.510	0	0 mm.	0 mm.	0 mm.	0 mm.	0 mm.
	m. 0.110	0.19	0.29	0.35	0.40	0.44
	0.501	0.14	0.21	0.25	0.30	0.34
	1.005	0	0	0	0	0
		2d Series				
m. 1.011	0	0 mm.	0 mm.	0 mm.	0 mm.	0 mm.
	m. 0.116	0.23	0.35	0.42	0.49	0 55
	0.502	0.27	0.40	0.51	0.57	0.63
	0.996	0.21	0.30	0.38	0.42	0.49
	2.010	0	0	0	0	0
		3d Series				
m. 2.008	0	0 mm.	0 mm.	0 mm.	0 mm.	0 mm.
	m. 0.118	0.26	0.38	0.47	0.54	0.60
	0.999	0.34	0.48	0.60	0.68	0.76
	2.998	0	0	0	0	0

4TH SERIES referred to the chord joining the extreme readings

	m.					
	0.0017	0	0	0	0	0
		mm.	mm.	mm.		
	0.253	0.30	0.45	0 56	—	—
m.	0.500	0.38	0.53	0.65	—	—
3.018	1.003	0.38	0.56	0.68	—	—
	1.998	0.31	0.45	0.54	—	—
	3.002	0.17	0.23	0.28	—	—
	3.995	0	0	0	0	0

4TH SERIES referred to the chord drawn from the edge of the opaque body

	0	0	0	0	0	0
	m.	mm.	mm.	mm.		
	0.0017	0.04	0.06	0.08	—	—
					mm.	mm.
	0.253	0.34	0.50	0.63	0.73	0.83
m.	0.500	0.41	0.58	0 72	0.85	0.94
3.018	1.003	0 41	0.60	0.74	0.87	0.97
	1.998	0.32	0.48	0 57	0 67	0.75
	3.002	0.18	0.25	0.30	0.38	0.39
	3 995	0	0	0	0	0

5TH SERIES

	0	0	0	0	0	0
	m.	mm.	mm.	mm.	mm.	mm.
m.	0.131	0.27	0.40	0.50	0.58	0.66
4.507	1.018	0 32	0.48	0.59	0.71	0.81
	2.506	0	0	0	0	0

6TH SERIES

	0	0	0	0	0	0
	m.	mm.	mm.	mm.	mm.	mm.
m.	0 117	0.23	0 33	0.42	0.49	0.53
6 007	0.999	0	0	0	0	0

It is thus evident that the hypothesis of contraction and expansion produced by the action of the body upon rays of light is insufficient to explain the phenomena of diffraction. Introducing the principle of interference, however, we are able to predict not only the variation in size of the exterior fringes when the screen is made to approach or recede from the luminous point, but also the curved path of the bright and dark bands. The law of interference, or the mutual influence of rays of light, is an immediate consequence of the wave-theory; not only so, but it is proved or confirmed by so many different experiments that it is really one of the best-established principles of optics.

17. Grimaldi was the first to observe the effect which rays

of light produce upon one another. Recently the distinguished Dr. Thomas Young has shown by a simple and ingenious experiment that the interior fringes are produced by the meeting of rays inflected at each side of the opaque body. This he proved by using a screen to intercept one of the two pencils of light; and in this way he was able to make the interior fringes completely vanish, whatever might be the form, mass, or nature of the screen, and whether he intercepted the luminous pencil before or after its passage into the shadow.

18. Brighter and sharper fringes may be produced by cutting two parallel slits close together in a piece of cardboard or a sheet of metal, and placing the screen thus prepared in front of the luminous point. We may then observe, by use of a magnifying-glass between the opaque body and the eye, that the shadow is filled with a large number of very sharp-colored fringes so long as the light shines through both openings at the same time, but these disappear whenever the light is cut off from one of the slits.

19. If we allow two pencils of light, each coming from the same source and regularly reflected by two metallic mirrors, to meet under a very small angle, we obtain similar fringes, the colors of which are even purer and more brilliant than before. To obtain these bands, it is necessary to be very careful that in the region where the two mirrors come into contact, or at least throughout a portion of their line of contact, the surface of the one is not shifted sensibly past that of the other. This is necessary in order that the difference of path traversed by two reflected rays meeting in the area common to the two luminous* fields may be very small. I may remark in passing that the theory of interference alone will

* In the case of white light, or even in light as homogeneous as possible, the number of fringes which one can see is always limited, because even when the light has reached a degree of simplicity as great as possible without too far diminishing its intensity, it is still composed of rays which are heterogeneous; and since the bright and dark bands thus produced do not all have the same size, they encroach the one upon the other in proportion as their order increases, and finally they completely destroy each other; and this is why one does not see any fringes when the difference of paths becomes slightly sensible. Concerning the details of this experiment and its explanation on the principle of interference, see the article upon *Light* in the French translation of Thomson's Chemistry, already cited.

explain this experiment, and that the experiment calls for manipulation so delicate and effort so continued that it is almost impossible that one should strike upon it by accident.

If we raise one of the mirrors or intercept the light which it reflects either before or after reflection, the fringes disappear as in the preceding case. This furnishes still further evidence that the fringes are produced, not by the action of the edges of the mirrors, but by the meeting of two pencils of light. For these fringes are always at right angles to the line which joins the two images of the luminous point, whatever be its inclination with respect to these edges, at least throughout the extent of the area which is common to the two regularly reflected pencils.*

20. Since the fringes which one sees in the interior of the shadow of a very narrow body and those which one obtains by the use of two mirrors result evidently from the mutual influence of rays of light, analogy would indicate that the same thing ought to be true for the exterior fringes of the shadows of bodies illuminated by a point source. The first explanation which occurs to one is that these fringes are produced by the interference of direct rays with those which are reflected at the edge of the opaque body, while the interior fringes result from the combined action of rays inflected into the shadow from the two sides of the opaque body, these inflected rays having their origin either at the surface or at points indefinitely near it. This appears to be the opinion of Mr. Young, and it was at first my own opinion ; but a closer examination of the phenomena convinced me of its falsity. Nevertheless, I propose to follow it to its logical conclusion and to state the formula which I have derived in order to facilitate comparison of this theory with that which I offer as a substitute.

Let R, Fig. 14, be the radiant point, AA′ the opaque body, and FT′ either a white screen upon which the shadow of this body falls or the focal plane of a magnifying-glass with which the

* When the fringes extend outside, all their exterior portions resulting from the meeting of rays regularly reflected by one of the mirrors and rays inflected near the edge of the other should have different directions. If one observes this phenomenon carefully, he will see that the form and position of the fringes are in each case in accord with the theory of interference.

fringes are observed. RT and RT′ are rays tangent at the edge of the opaque body, T and T being the limits of the geometrical shadow. Let us indicate by a the distance RB from the luminous point to the opaque body, by b the distance BC of the body from the white screen, and by c its diameter, AA′, which we shall consider very small compared with the distances a and b. This assumption is made in order that we may measure the size of the fringes either in a plane perpendicular to RT or perpendicular to the line RC, which passes through the middle of the shadow.

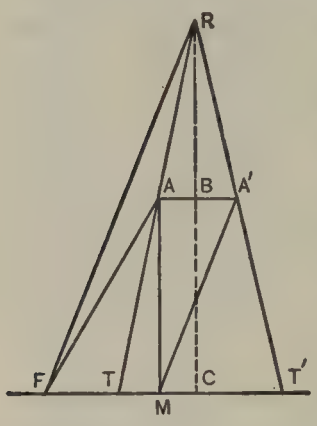


Fig. 14

With these conventions we shall consider, first, the exterior fringes. Let F be any point on the receiving screen outside the shadow. The difference of path traversed by the direct ray, and by the ray reflected at the edge of the opaque body, and meeting the direct ray at this point, is RA + AF − RF. Let us represent FT by x, and express in series the values of RF, AR, and AF. Then, if we neglect all terms involving any power of x or of c higher than the second, since they are very small compared with distances a and b, the terms which contain c will disappear and we shall have for the difference of path traversed

$$d=\frac{a}{2b\,(a+b)}x^2;$$

whence follows

$$x=\sqrt{\frac{2db(a+b)}{a}}.$$

21. If we call λ the length of a light-wave, that is to say, the distance between two points in the ether where vibrations of the same kind are occurring at the same time and in the same sense, then λ/2 will be the distance between two ether particles whose velocities of vibration are at any one instant equal but oppositely directed. Thus two trains of waves separated by an interval equal to λ are in perfect accord as to their vibrations; but when the distance between corresponding points is λ/2, then their vibrations are directly opposed. Accordingly

the above formula gives for the value of x, corresponding to the centre of the dark band of the first order, the following value: $\sqrt{\frac{\lambda b\,(a+b)}{a}}$; while observation shows that, as a matter of fact, this is the brightest part of the first fringe. On the same theory, the edge of the geometrical shadow, where the difference of path vanishes, ought to be brighter than the rest of the fringe, while, as a matter of fact, this is precisely the darkest region outside the geometrical shadow. In general, the position of the dark and bright bands deduced from this formula is almost exactly the inverse of that determined by experiment. This is the first difficulty presented by this theory. To avoid it, we must suppose that the rays reflected at the edge of the screen suffer the loss of half a wave-length; adding $\lambda/2$ to the difference of path, d, the general expression becomes

$$x=\sqrt{\frac{(2d+\lambda)\,b\,(a+b)}{a}}.$$

Replacing d in this formula by $\lambda/2$, $3\,\lambda/2$, $5\,\lambda/2$, $7\,\lambda/2$, etc., we have for the values of x corresponding to dark bands of the first, second, third, fourth, etc., orders:

$$\sqrt{\frac{2\lambda b\,(a+b)}{a}},\ \sqrt{\frac{4\lambda b\,(a+b)}{a}},\ \sqrt{\frac{6\lambda b\,(a+b)}{a}},\ \sqrt{\frac{8\lambda b\,(a+b)}{a}},\ \text{etc.}$$

These formulæ appear to agree fairly well with the observations; however, closer measurements show that the ratios between the sizes of the fringes derived from these expressions are not exactly correct, as we shall see later.

22. I pass now to the consideration of interior fringes produced in the shadow by the meeting of two pencils of light inflected at A and A′. Let M, Fig. 14, be any point located in the interior of the shadow; the intensity of the light at this point depends upon the amount of disagreement between the vibrations of the rays AM and A′M, which meet at this point, or upon the difference of path A′M−AM. I shall denote by x the distance MC of the point M from the middle of the shadow, and by d the difference of paths, and hence

$$d=\sqrt{b^2+(\tfrac{1}{2}c+x)^2}-\sqrt{b^2+(\tfrac{1}{2}c-x)^2}.$$

Expanding the radicals and neglecting the higher powers of x, since this quantity is very small compared with b, we have

$$d=cx/b,$$

or $$x = bd/c.$$

If in place of d in this expression we substitute successively $\lambda/2$, $3\lambda/2$, $5\lambda/2$, $7\lambda/2$, etc., we obtain the values of x corresponding to dark bands of the first, second, third, fourth, etc., order, namely,

$$\frac{b\lambda}{2c}, \quad \frac{3b\lambda}{2c}, \quad \frac{5b\lambda}{2c}, \quad \frac{7b\lambda}{2c}, \text{ etc.},$$

and consequently for the distance between the middle points of two consecutive dark bands, $b\lambda/c$.

The general expression for n such intervals is, therefore, $nb\lambda/c$.

23. So long as the extreme fringes are sufficiently distant from the edges of the shadow, this formula agrees fairly well with experiment; but when they approach very near or pass beyond the edges, one detects a slight difference between their actual position and that deduced from the formula. In general, the calculated values are always a little larger than the observed. The reason for this I shall show when we come to the true theory of diffraction. It also follows from this formula that the size of the interior fringes ought to be entirely independent of the distance, a, of the luminous point from the opaque body; this prediction, however, is not completely verified by experiment, especially when the fringes completely fill the shadow; their position then varies distinctly with the distance a.

24. According to the formula

$$\sqrt{\frac{2n\lambda b\,(a+b)}{a}},$$

which we have just derived for the exterior fringes, their position depends upon a as well as upon b. Experiment shows that, in fact, their size increases or diminishes according as the opaque body approaches or recedes from the luminous point, and that the ratios between the different sizes of one and the same fringe deduced from the formula are precisely those given by observation. But the most remarkable inference from this formula is that, when a remains constant, the distance of any dark or bright band from the edge of the geometrical shadow is not directly proportional to b as in the case of interior fringes, but varies in such a way that this band traces out, not a straight line, but a hyperbola of sensible curvature. This is

also confirmed by experiment, as may be seen from the observations given above.

Considering the striking agreement of these formulæ with experiment, it is natural to suppose that they are accurate expressions of fact, and therefore natural to attribute any small differences between calculated and observed values to the errors which are unavoidable in such delicate measurements.*

But a closer examination of the hypotheses from which they are derived, and of the inferences derivable from them, shows that they do not agree with the facts of nature.

25. If the fringes at the edge of a shadow are really due to the meeting of the direct rays with those reflected at the edge of the screen, their intensity ought to depend upon the area and the curvature of its surface, and the fringes produced by the back of a razor, for instance, ought to be much more vis-

* It might appear at first sight that one would be able to adapt this theory to the ideas of Newton by introducing the principle of interference, as I have indicated above; but besides the complication of fundamental hypotheses and the small probability of any of them, this principle, it appears to me, would lead to consequences which contradict the emission-theory.

M. Arago has remarked that the interposition of a thin transparent plate at the edge of an opaque body sufficiently narrow to produce interior fringes in its shadow displaces these fringes and shifts them towards the side [*see paper by Arago* (*Ann. Chim. et Phys.*, i., p. 199, 1816)] where is placed the transparent plate. This being so, it follows from the principle of interference that the rays which have traversed the plate have been retarded in their path, because the same fringes in each case must correspond to equal intervals between the times of arrival of rays. This inference at once confirms the wave-theory and manifestly contradicts the emission-theory, in which one is compelled to assume that light travels more rapidly in dense than in rare media.

This objection can be avoided only by substituting for difference of path difference of "fit"; but we lose all that was gained by the principle of interference in thus replacing a sharp idea by a hazy one, a satisfactory explanation by one which does not aid our understanding of the phenomena; for one can readily see how two light particles striking the retina at the same point may produce sensations more or less intense, according as the interval of time which separates two consecutive impacts is sufficient to produce unison or dissonance between the vibrations at the optic nerve; while it is by no means so easy to see how this effect could be produced by a difference of "fit" between two light particles, or how by simultaneous impact on the optic nerve they would produce no effect at all when they were in opposite "fits," even though their mechanical impacts were in perfect unison.

ible than those produced by the edge; but, using a magnifying-glass at a distance of some centimeters, one detects practically no difference in intensity in these two cases. This test is more easily made by using a steel plate one edge of which is round throughout a part of its length and sharp throughout the remainder of its length, these two edges lying in the same straight line. One is thus easily convinced that the fringes have the same intensity throughout their entire length.

26. We know that under large angles of incidence dull surfaces reflect light almost as well as polished mirrors. This is easily explained either on the emission-theory or on the wave-theory. But although one can understand how difference of polish cuts a small figure when the angle of incidence is large, it is not easy to see how the intensity of the reflected light can be independent of the curvature of the reflecting surface; indeed, it is clear that as the radius of curvature diminishes the reflected rays will diverge more and more, whatever be their angle of incidence.

27. Not only so, but I have convinced myself by another simple experiment of the incorrectness of the hypothesis which I had first adopted, and which I am now opposing. I cut a sheet of copper into the shape represented in Fig. 15, and placed it in a dark room about four meters in front of a luminous point, and examined its shadow with a magnifying-glass. What I observed, on slowly receding, was as follows: When the large fringes produced by each of the very narrow openings CEE′C′ and DFF′D′ had spread out into the geometrical shadow of CDFE, which received practically only white light from each separate slit, the interior fringes produced by the meeting of these two pencils of light showed colors much sharper and purer than the interior fringes of the shadow of ABDC, and were, at the same time, much brighter. On receding still farther, I noticed that the light diminished throughout the whole of the shadow of ABFE, but much more rapidly back of EFDC than in the upper part of the shadow, so that there was one particular instant when the intensity of the light ap-

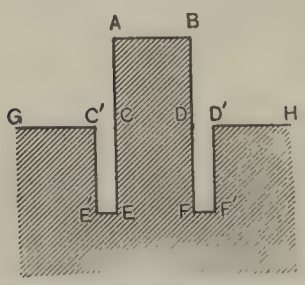


Fig 15.

peared to be the same above and below, after which the fringes remained less intense in the lower* part, although their colors were always much purer.

If, now, the only inflected light were that which grazed the edges of the opaque bodies, the fringes of the upper part ought to be sharper and ought to show purer colors than those of the lower part; for the first are produced by the meeting of two systems of waves which have their centres upon the edges AC and BD, while the others are formed by the meeting of four systems of waves having their origin at the edges C′E′, CE, DF, DF′; and this would necessarily diminish the difference of intensity between the dark and bright bands, in the case of homogeneous light, or the purity of the colors, in the case of white light, because the fringes produced by the rays reflected and inflected at C′E′ and DF would not exactly coincide with those produced by the meeting of rays coming from CE and D′F′. Now experiment shows, as I have just said, that exactly the reverse of this is true. One might explain on this same hypothesis how it happens that the shadow of ECDF is much brighter than that of ABDC arising from the double source of light presented by the two edges of each slit; but from this it would follow that the lower part ought always to be brighter, and we have just seen that this is not the fact.

28. From the experiments which I have just described it is evident that we cannot attribute the phenomena of diffraction solely to rays which graze the edge of the body; but we must, on the contrary, admit that there is an infinitude of other rays sensibly distant from the body and yet deviated from their original direction, so as to meet and form these fringes.

29. The spreading out of a pencil of light in passing through a very narrow opening shows in an even better manner that the inflection of light occurs at a sensible distance from the edges

* In order that this difference of intensity between the two parts of the shadow shall be as marked as possible, it is necessary that the slits CE and DF be very narrow as compared with the distance which separates them, and that the sheet of copper should be as far away as possible from the luminous point.

[*In repeating this experiment, it will be found very convenient to use, instead of sheet copper, an unfixed photographic plate: lantern slide is best. The two slits can be cut either with a pocket-knife or, better still, by means of a dividing engine.*]

of the diaphragm. It was in the consideration of this phenomenon that I discovered the error into which I had previously fallen. When one brings the edges of two opaque screens very close together in front of a luminous point in a dark room, he observes that the region illuminated by the aperture greatly increases. Such screens were Newton's two knife-edges. I shall suppose, as in his experiment, that the edges of the aperture are thin and perfectly sharp; not that this has any effect upon the phenomena, but simply for making clearer the conclusion which is to be drawn. The small number of rays which graze these sharp edges, being spread out over a rather large area, could produce only an insensible amount of illumination, or, at most, an exceedingly feeble light, and in the midst of it one ought to see a bright band traced out by the pencil of direct rays. This, however, is not the fact; for white light of almost uniform intensity fills a space much larger than the projection of the aperture,* and gradually grows weaker, shading into the dark bands of the first order. It was doubtless in order to account for the large amount of light inflected that Newton supposed the action of the body upon rays of light to extend to sensible distances, but this hypothesis will not bear careful scrutiny.

30. If the expansion of a pencil of light which passes through a narrow opening were brought about by attractive and repulsive forces having their origin at the edges of the aperture, the intensity of these forces, and consequently their effect upon the light, would *necessarily* vary with the nature, the mass, and the surface of the edges of the screen. All forces produced by a body acting at a sensible distance and taking their rise in any considerable extent of its mass or its surface would depend upon the relative positions and upon the number of particles contained within this sphere of activity, or, what is the same thing, upon the shape of the surface. If, then, the phenomena in question are due to the action of such forces, one would expect that, on placing a sharp body opposite a round body, the rays of light would be inflected more to the one side than the

* The illuminated space increases so rapidly in comparison with the width of the conical projection of the aperture as the receiving screen recedes from the aperture, and likewise when the aperture itself is further withdrawn from the luminous point, that by making these two distances sufficiently great one can obtain the same effect with an opening of any size.

other ; but, as I have shown by a very simple experiment, this is not the fact. I passed a pencil of rays between two steel plates whose vertical edges were brought very close together and were carefully straightened throughout their entire length. A part of each edge was sharp, the rest of it round, and these edges were arranged so that the round portion of one plate corresponded to the sharp one of the other. Thus, if a sharp edge were located on the right in the upper part of the opening, another was located on the left in the lower part, so that if there had been any difference in the action of the two edges upon the rays, I should have noticed it in the relative positions of the upper and lower parts of the bright interval at the middle, and especially in the fringes in that neighborhood, as they would be interrupted at the point of passage from the sharp to the round edge ; but, on observing them closely, I noted that they were perfectly straight throughout their entire length, even at the bright interval in the middle, exactly in the same way as when two edges of the same kind are opposed one to the other. The experiment may be varied by using plates made of two different substances, but the result* obtained will certainly remain the same.

31. All the experiments which I have tried so far have shown that the nature of the body interposed has in other respects no more influence upon the inflection of light than is exerted by the mass or the shape of the two edges. I shall cite only one experiment, in which I have taken every precaution necessary to determine the correctness of this principle, which, indeed, is already well established by the preceding experiment.

I covered an unsilvered mirror with a layer of India ink spread over a thin layer of paper, forming together a thickness of one-tenth of a millimeter. With a sharp point I traced two parallel lines, and then carefully removed from between these two lines the paper and the India ink which adhered to the surface of the glass. This aperture, as measured by the microm-

* Messrs. Berthollet and Malus found a long while ago that the nature of the body had no effect upon the diffraction of light. For screens they employed plates composed of different substances, with edges made up, for instance, of very dense metal and a piece of ivory ; but they had no means of observation so convenient and accurate as mine, and consequently one might suspect that some small difference might have escaped them.

eter, was 1.17 mm. I then placed opposite each other two copper cylinders, each having a diameter of 14.5 mm., and by means of a graduated wedge I made the interval between these cylinders also 1.17 mm. The cylinders, placed alongside the blackened glass, were at a distance of 4.015 m. from the luminous point, and at a distance of 1.663 m. from the micrometer. I then measured the size of the fringes produced by these two openings, and found that they were absolutely the same. The following are the results of the two observations made with white light:

		mm.
The distance between the darkest point of the two dark bands of the first order at the point of separation of the brownish red from the violet .	First reading,	1.49
	Second reading,	1.49
The interval between the two fringes of the second order at the point of separation of red and green . . .	First reading,	3.22
	Second reading,	3.22

It is hardly possible that two sets of circumstances should differ more than these as regards the mass and the nature of the edges of the aperture. In the one case there is a single layer of India ink producing the fringes, for the glass to which it adheres completely fills the aperture; in the other case we have two massive cylinders of copper, 14.5 mm. in diameter, giving us an aperture whose edges have very considerable masses and areas, but we observe no difference in the expansion of the pencil of light.

32. It is therefore certain that the phenomena of diffraction do not at all depend upon the nature, the mass, or the shape of the body which intercepts the light,* but only upon the size of the intercepting body or upon the size of the aperture through which it passes. We must, therefore, reject any hypothesis which assigns these phenomena to attractive and repulsive

* This is so, at least provided one does not consider the shadow too close up to the edge of the screen, or provided the surface grazed by the rays of light has not too large an area compared with this distance; for in this case it may happen that the reflected rays sensibly affect the phenomenon —as, for instance, occurs when the surface grazed by the rays is a plane mirror of one or two decimeters in size and when one observes the fringes at a short distance. Besides, there would then be *successive* diffractions over an area too considerable for one to neglect.

forces whose action extends to a distance from the body as great as that at which rays are inflected. We are equally unable to admit that diffraction is caused by a shallow atmosphere which has the same thickness as the sphere of activity of these forces, and whose refractive index differs from that of the neighboring medium; for this second hypothesis, like the first, would lead us to think that the inflection of light ought to vary with the form and the nature of the edge of the screen, and ought not to be the same, for instance, at the edge and at the back of a razor. Now, on the emission-theory it is impossible to explain in any other manner the expansion of a beam of light passing through a narrow opening, and this expansion is a well-established fact.* Consequently, *the phenomena of diffraction cannot be explained on the emission-theory.*

SECTION II

33. In the first section of this memoir I have shown that the corpuscular theory, and even the principle of interference when applied only to direct rays and to rays *reflected or inflected at the very edge of the opaque screen,* is incompetent to explain the phenomena of diffraction. I now propose to show that we may find a satisfactory explanation and a general theory in terms of waves, without recourse to any auxiliary hypothesis, by basing everything upon the principle of Huygens and upon that of interference, both of which are inferences from the fundamental hypothesis.

Admitting that light consists in vibrations of the ether similar to sound-waves, we can easily account for the inflection of rays of light at sensible distances from the diffracting body. For when any small portion of an elastic fluid under-

* The rise of a liquid in a capillary tube occurs between two surfaces separated by a finite distance, although the attraction which these surfaces exert upon the liquid extends only to an infinitely small distance. The reason of this is, that the molecules of the liquid, attracted by the surface of the tube, also in their turn attract other molecules of the liquid situated within their sphere of action, and so on, step by step; but in the emission-theory an analogous explanation is not admissible, for the fundamental hypothesis is that the luminous particles never exert any sensible effect upon the path of neighboring particles. No interdependence of motion is here admissible, for such an assumption would be the assumption of a fluid medium.

goes condensation, for instance, it tends to expand in all directions; and if throughout the entire wave the particles are displaced only along the normal, the result would be that all points of the wave lying upon the same spherical surface would simultaneously suffer the same condensation or expansion, thus leaving the transverse pressures in equilibrium; but when a portion of the wave-front is intercepted or retarded in its path by interposing an opaque or transparent screen, it is easily seen that this transverse equilibrium is destroyed and that various points of the wave may now send out rays along new directions.

To follow by analytical mechanics all the various changes which a wave-front undergoes from the instant at which a part of it is intercepted by a screen would be an exceedingly difficult task, and we do not propose to derive the laws of diffraction in this manner, nor do we propose to inquire what happens in the immediate neighborhood of the opaque body, where the laws are doubtless very complicated and where the form of the edge of the screen must have a perceptible effect upon the position and the intensity of the fringes. We propose rather to compute the relative intensities at different points of the wave-front only after it has gone a large number of wave-lengths beyond the screen. Thus the positions at which we study the waves are always to be regarded as separated from the screen by a distance which is very considerable compared with the length of a light-wave.

34. We shall not take up the problem of vibrations in an elastic fluid from the point of view which the mathematicians have ordinarily employed—that is, considering only a single disturbance. Single vibrations are never met with in nature. Disturbances occur in groups, as is seen in the pendulum and in sounding bodies. We shall assume that vibrations of luminous particles occur in the same manner—that is, one after another and series after series. This hypothesis follows not only from analogy, but as an inference from the nature of the forces which hold the particles of a body in equilibrium. To understand how a single luminous particle may perform a large series of oscillations all of which are nearly equal, we have only to imagine that its density is much greater than that of the fluid in which it vibrates—and, indeed, this is only what has already been inferred from the uniformity of the motions of

the planets through this same fluid which fills planetary space. It is not improbable also that the optic nerve yields the sensation of sight only after having received a considerable number of successive stimuli.

However extended one may consider systems of wave-fronts to be, it is clear that they have limits, and that in considering interference we cannot predicate of their extreme portions that which is true for the region in which they are superposed. Thus, for instance, two systems of equal wave-length and of equal intensity, differing in path by half a wave, interfere destructively only at those points in the ether where they meet, and the two extreme half wave-lengths escape interference.

Nevertheless, we shall assume that the various systems of waves undergo the same change throughout their entire extent, the error introduced by this assumption being inappreciable; or, what amounts to the same thing, we shall assume in our discussion of interference that these series of light-waves represent general vibrations of the ether, and are undefined as to their limits.

The Problem of Interference

35. *Given the intensities and relative positions of any number of trains of light-waves of the same length* and travelling in the same direction, to determine the intensity of the vibrations produced by the meeting of these different trains of waves, that is, the oscillatory velocity of the ether particles.*†

* We shall not here consider light-waves of different lengths which, in general, come from different sources and which cannot, therefore, give rise to simultaneous disturbances and cannot by their interaction produce any phenomena which are uniform; and even if they were uniform, the rise and fall of intensity produced by the interference of two different kinds of waves, after the manner of beats in sound, would be far too rapid to be detected, and would produce only a sensation of constant intensity.

† It was Mr. Thomas Young who first introduced the principle of interference into optics, where he showed much ingenuity in applying it to special cases; but in the problems which he has thus solved he has considered, I think, only the limiting cases, where the difference in phase between the two trains of waves is either a maximum or a minimum, and has not computed the intensity of the light for any intermediate cases or for any number whatever of trains of waves, as I here propose to do.

Employing the general principle of the superposition of small motions, the total velocity impressed upon any particle of a fluid is equal to the sum of the velocities impressed by each train of waves acting by itself. When these waves do not coincide, these different velocities depend not only upon the intensity of each wave, but also upon its phase at the instant under consideration. We must, therefore, know the law according to which the velocity of vibration varies in any one wave, and for this purpose we must trace the wave back to the origin whence it derives all its characteristics.

36. It is natural to suppose that the particles whose vibrations produce light perform their oscillations like those of sounding bodies—that is, according to the laws which hold for the pendulum; or, what is the same thing, to suppose that the acceleration tending to make a particle return to its position of equilibrium is directly proportional to the displacement. Let us denote this displacement by x. A suitable function of this displacement can then be represented by the expression $Ax+Bx^2+Cx^3+$etc., since this will vanish when $x=0$. If, now, we suppose the excursion of the particle to be very small when compared with the radius of the sphere throughout which the forces of attraction and repulsion act, we can neglect in comparison with Ax all other terms of the series and consider the acceleration as practically proportional to the distance x. This hypothesis, to which we are led by analogy, and which is the simplest that one can make concerning the vibrations of light particles, ought to lead to accurate results, since the laws of optics remain the same for all intensities of light.

Let us represent by v the velocity of vibration of a light particle at the end of a time t. We shall then have $dv=-Axdt$; but $v=dx/dt$, or $dt=dx/v$. Substituting in the first equation, we have $vdv=-Axdx$. Integrating, we have $v^2=C-Ax^2$; and hence

$$x=-\sqrt{\frac{C-v^2}{A}}.$$

Substituting this value of x in the first equation, we have

$$dt=\frac{dv}{\sqrt{A(C-v^2}},$$

which, on integration, gives

$$t=C'+\frac{1}{\sqrt{A}}\sin^{-1}\frac{v}{\sqrt{C}}.$$

If we measure time from the instant at which the velocity is zero, the constant C' becomes zero, and we have

$$t=\frac{1}{\sqrt{A}}\sin^{-1}\frac{v}{\sqrt{C}},\text{ or }v=\sqrt{C}\sin t\sqrt{A}.$$

If we employ as unit of time the interval occupied by the particle in one complete vibration, we have $v=\sqrt{C}\sin(2\pi t)$. Thus, in isochronous vibrations, the velocities for equal values of t are always proportional to the constant $\sqrt{C}$, which, therefore, measures the intensity of the vibration.

37. Let us now consider the wave produced in the ether by the vibrations of this particle. The energy of motion in the ether at any point on the wave depends upon the velocity of the point-source at the instant when it started a disturbance which has just reached this point. The velocity of the ether particles at any point in space after an interval of time t is proportional to that of the point-source at the instant $t-x/\lambda$, x being the distance of this point from the source of motion and λ the length of a light-wave. Let us denote by u the velocity of the ether particles. We then have

$$u=a\sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right].$$

We know that the intensity a of vibration* [*oscillatory velocity*] in a fluid is in inverse ratio to the distance of the wave from the centre of disturbance; but, considering how minute these waves are when compared with the distance which separates them from the luminous point, we may neglect the variation of a and consider it as constant throughout the extent of one or even of several waves.

38. By the aid of this expression one can compute the intensity of vibration produced by the meeting of any number of pencils of light whenever he knows the intensity of the different trains of waves and their respective positions.

Let us first determine the velocity of a luminous particle in a vibration which results from the interference of two trains

* [*See last sentence of section 57 below.*]

of waves displaced, one with respect to the other, by a quarter of a wave-length [*i.e., differing in phase by* 90°], and having intensities which we shall denote by a and a'. We shall count time, t, from the moment at which the vibrations of the first train begin. Let u and u' be the velocities which the first and second trains of waves would impress upon a light particle whose distance from the source of motion is x. We then have

$$u=a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right] \text{ and } u'=a' \sin\left[2\pi\left(\frac{t-x+\frac{\lambda}{4}}{\lambda}\right)\right],$$

or

$$u'=-a' \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right].$$

Hence, the resultant velocity U will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-a' \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right].$$

Putting $a=A \cos i$ and $a'=A \sin i$, this expression may always be placed in the following form:

$$A \cos i \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-A \sin i \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

or

$$A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right].$$

Thus the wave produced by the meeting of two others will be of the same nature, but will have a different position [*phase*] and a different intensity. From the equations $A \cos i=a$ and $A \sin i=a'$, we have for the value of A (that is, for the intensity of the resultant wave) $\sqrt{a^2+a'^2}$; but this is exactly the value of the resultant of two mutually rectangular forces, a and a'.

From the same equations it is easily seen also that the new wave exactly corresponds in angular position [*phase*] to the resultant of the two mutually rectangular forces a and a'; for the equation

$$U=A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right]$$

shows that the linear displacement of this wave with respect to the first is $\frac{i\lambda}{2\pi}$; but i is also the angle which the force a

makes with the resultant A, because $A \cos \iota = a$. Thus we have complete analogy between the resultant of two mutually rectangular forces and the resultant of two trains of waves differing in phase by a quarter of a wave-length.

39. The solution of this particular case for waves differing by a quarter of a wave-length suffices to solve all other cases. In fact, whatever be the number of the trains of waves, and whatever be the intervals which separate them, we can always substitute for each of them its components referred to two reference points which are common to each train of waves and which are distant from each other by a quarter of a wave-length; then adding or subtracting, according to sign, the intensities of the components referred to the same point, we may reduce the whole motion to that of two trains of waves separated by the distance of a quarter of a wave-length; and the square root of the sum of the squares of their intensities will be the intensity of their resultant; but this is exactly the method employed in statics to find the resultant of any number of forces; here the wave-length corresponds to one circumference in the statical problem, and the interval of a quarter of a wave-length between the trains of waves to an angular displacement of 90° between the components.

40. It very often happens in optics that the intensities of light or the particular tint which one wishes to compute is produced by the meeting of only two trains of waves, as in the case of [*Newton's*] colored rings and the ordinary phenomena of color presented by crystalline plates. It is, therefore, well to know the general expression for the resultant of two trains of waves differing in phase by any amount whatever. The result is easily predicted from the general method which I have explained, but I think it will be wise to emphasize somewhat the theory of vibrations, and to show directly that the wave resulting from two others, separated by any interval whatever, corresponds exactly in intensity and position to the resultant of two forces whose intensities are equal to those of the two pencils of light, making an angle with each other which bears to one complete circumference the same ratio that the interval between the two trains of waves bears to one wave-length.

Let x be the distance from the origin of the first train of waves to the light particle under consideration, and t the

instant for which we wish to compute its velocity. The speed impressed by the first train of waves will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

where a represents the intensity of this ray of light.

Let us call a' the intensity of the second pencil, and let us denote by c the distance between corresponding points on the two trains of waves; the [*oscillatory*] velocity due to the second train will then be

$$a' \sin\left[2\pi\left(t-\frac{x+c}{\lambda}\right)\right],$$

and hence the total velocity impressed upon the particle will be

$$a \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]+a' \sin\left[2\pi\left(t-\frac{x+c}{\lambda}\right)\right],$$

or

$$\left[a+a' \cos\left(2\pi\frac{c}{\lambda}\right)\right]\sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-a' \sin\left(2\pi\frac{c}{\lambda}\right)\cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

an expression to which may always be given the following form:

$$A \cos i \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)\right]-A \sin i \cos\left[2\pi\left(t-\frac{x}{\lambda}\right)\right],$$

or

$$A \sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right],$$

where

$$a+a' \cos\left(2\pi\frac{c}{\lambda}\right)=A \cos i,$$

and

$$a' \sin\left(2\pi\frac{c}{\lambda}\right)=A \sin i.$$

Squaring and adding, we have

$$A^2=a^2+a'^2+2aa' \cos\left(2\pi\frac{c}{\lambda}\right).$$

Hence,

$$A=\pm\sqrt{a^2+a'^2+2aa' \cos\left(2\pi\frac{c}{\lambda}\right)}.$$

But this is precisely the value of the resultant of two forces, a and a', inclined to each other at an angle $2\pi\frac{c}{\lambda}$.

41. From this general expression it is seen that the resultant intensity of the light vibrations is equal to the sum of intensities of the two constituent pencils when they are in perfect agreement and to their difference when they are in exactly opposite phases, and, lastly, to the square root of the sum of their squares when their phase difference is a quarter of a wave-length, as we have already shown.

It thus follows that the phase of the wave corresponds exactly to the angular position of the resultant of two forces, a and a'. The distance from the first wave to the second is c, to the resultant wave $\frac{i\lambda}{2\pi}$, and from the resultant wave to the second is $c-\frac{i\lambda}{2\pi}$; accordingly, the corresponding angles are $2\pi.\frac{c}{\lambda}$, i, and $2\pi.\frac{c}{\lambda}-i$. Let us multiply the equation

$$a+a'\cos\left(2\pi\frac{c}{\lambda}\right)=A\cos i$$

by sin i, and the following equation

$$a'\sin\left(2\pi\frac{c}{\lambda}\right)=A\sin i$$

by cos i. Subtracting one from the other, we have

$$a\sin i=a'\sin\left(2\pi\frac{c}{\lambda}-i\right),$$

which, together with

$$a'\sin\left(2\pi\frac{c}{\lambda}\right)=A\sin i,$$

gives the following proportion:

$$\sin\left(2\pi\frac{c}{\lambda}-i\right):\sin i:\sin 2\pi\frac{c}{\lambda}::\quad :a':A.$$

42. The general expression, $A\sin\left[2\pi\left(t-\frac{x}{\lambda}\right)-i\right]$, for the velocity of the particles in a wave produced by the meeting of two others shows that this wave has the same length as its components and that the velocities at corresponding points are proportional, so that the resultant wave is always of the same nature as its components and differs only in intensity—that is to say, in the constant by which we must multiply the velocities in either of the components in order to obtain the correspond-

ing velocities in the resultant. In combining this resultant with still another new wave, one again arrives at an expression of the same form—a remarkable property of a function of this kind. Thus in the resultant of any number of trains of waves of the same length the light particles are always urged by velocities proportional to those of the components at points located at the same distance from the end of each wave. [*This is seen by multiplying each of the last three terms in the preceding proportion by* sin ωt. *For then,*

$$a \sin \omega t : a' \sin \omega t : A \sin \omega t :: a : a' : A'$$
$$:: \textit{constant ratio.}]$$

Applications of Huygens's Principle to the Phenomena of Diffraction

43. Having determined the resultant of any number of trains of light-waves, I shall now show how by the aid of these interference formulæ and by the principle of Huygens alone it is possible to explain, and even to compute, all the phenomena of diffraction. This principle, which I consider as a rigorous deduction from the basal hypothesis, may be expressed thus: *The vibrations at each point in the wave-front may be considered as the sum of the elementary motions which at any one instant are sent to that point from all parts of this same wave in any one of its previous* positions, each of these parts acting independently the one of the other.* It follows from the principle of the superposition of small motions that the vibrations produced at any point in an elastic fluid by several disturbances are equal to the resultant of all the disturbances reaching this point at the same instant from different centres of vibration, whatever be their number, their respective positions, their nature, or the epoch of the different disturbances. This general principle must apply to all particular cases. I shall suppose that all of these disturbances, infinite in number, are of the same kind, that they take place simultaneously, that they

* I am here discussing only an infinite train of waves, or the most general vibration of a fluid. It is only in this sense that one can speak of two light waves annulling one another when they are half a wave-length apart. The formulæ of interference just given do not apply to the case of a single wave, not to mention the fact that such waves do not occur in nature.

are contiguous and occur in the single plane or on a single spherical surface. I shall make still another hypothesis with reference to the nature of these disturbances, *viz.*, I shall suppose that the velocities impressed upon the particles are all directed in the same sense, perpendicular to the surface of the sphere,* and, besides, that they are proportional to the compression, and in such a way that the particles have no retrograde motion. I have thus reconstructed a primary wave out of partial [*secondary*] disturbances. We may, therefore, say that the vibrations at each point in the wave-front can be looked upon as the resultant of all the secondary displacements which reach it at the same instant from all parts of this same wave in some previous position, each of these parts acting independently one of the other.

44. If the intensity of the primary wave is uniform, it follows from theoretical as well as from all other considerations that this uniformity will be maintained throughout its path, provided only that no part of the wave is intercepted or retarded with respect to its neighboring parts, because the resultant of the secondary displacements mentioned above will be the same at every point. But if a portion of the wave be stopped by the interposition of an opaque body, then the intensity of each point varies with its distance from the edge of the shadow, and these variations will be especially marked near the edge of the geometrical shadow.

Let C be the luminous point, AG the screen, AME a wave which has just reached A and is partly intercepted by the opaque body. Imagine it to be divided into an infinite number of small arcs—Am', $m'm$, mM, Mn, nn', $n'n''$, etc. In order to determine the intensity at any point P in any of the later positions of the wave BPD, it is necessary to find the resultant of

*It is possible for light-waves to occur in which the direction of the absolute velocity impressed upon the particles is not perpendicular to the wave surface. In studying the laws of interference of polarized light, I have become convinced since the writing of this memoir that light vibrations are at right angles to the rays or parallel to the wave surface. The arguments and computations contained in this memoir harmonize quite as well with this new hypothesis as with the preceding, because they are quite independent of the actual direction of the vibrations and pre-suppose only that the direction of these vibrations is the same for all rays belonging to any system of waves producing fringes.

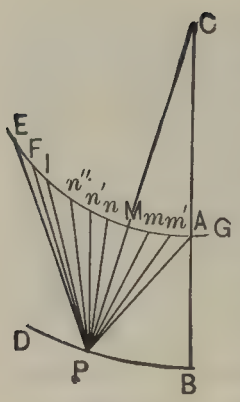


Fig. 16

all the secondary waves which each of these portions of the primitive wave would send to the point P, provided they were acting independently one of the other.

Since the impulse communicated to every part of the primitive wave was directed along the normal, the motion which each [*part of the wave*] tends to impress upon the ether ought to be more intense in this direction than in any other; and the rays which would emanate from it, if acting alone, would be less and less intense as they deviated more and more from this direction.

45. The investigation of the law according to which their intensity varies about each centre of disturbance is doubtless a very difficult matter;* but, fortunately, we have no need of knowing it, for it is easily seen that the effects produced by these rays are mutually destructive when their directions are sensibly inclined towards the normal. Consequently, the rays which produce any appreciable effect upon the quantity of light received at any point P may be regarded as of equal intensity.†

Let us now consider the rays EP, FP, and IP, which are sen-

* [*This is the problem solved by Stokes*; *Math. and Phys. Papers*, vol. ii., p. 243.]

† When the centre of disturbance has been compressed, the force of expansion tends to thrust the particles in all directions; and if they have no backward motion, the reason is simply that their initial velocities forward destroy those which expansion tends to impress upon them towards the rear; but it does not follow that the disturbance can be transmitted only along the direction of the initial velocities, for the force of expansion in a perpendicular direction, for instance, combines with a primitive impulse without having its effect diminished. It is clear that the intensity of the wave thus produced must vary greatly at different points of its circumference, not only on account of the initial impulse, but also because the compressions do not obey the same law around the centre of disturbance; but the variations of intensity in the resultant wave must follow the law of continuity, and may, therefore, be considered as vanishing throughout a small angle, especially along the normal to the primitive wave. For the initial velocities of the particles in any direction whatever are proportional to the cosine of the angle which this direction makes with that of the normal, so that these components vary much less rapidly than the angle so long as the angle is small.

sibly inclined and which meet at P, a point whose distance from the wave EA I shall suppose to include a large number of wave-lengths. Take the two arcs EF and FI of such a length that the differences EP—FP and FP—IP shall be equal to a half wave-length. Since these rays are quite oblique, and since a half wave-length is very small compared with their length, these two arcs will be very nearly equal, and the rays which they send to the point P will be practically parallel; and since corresponding rays on the two arcs differ by half a wave-length, the two are mutually destructive.

We may then suppose that all the rays which various parts of the primary wave AE send to the point P are of equal intensity, since the only rays for which this assumption is not accurate produce no sensible effect upon the quantity of light which it receives. In the same manner, for the sake of simplifying the calculation of the resultant of all the elementary waves, we may consider their vibrations as taking place in the same direction, since the angles which these rays make with each other are very small; so that the problem reduces itself to the one which we have already solved—namely, *to find the resultant of any number of parallel trains of light-waves of the same length, the intensities and relative positions being given.* The intensities are here proportional to the lengths of the illuminating arcs, and the relative positions of the wave trains are given by the differences of path traversed.

46. Properly speaking, we have considered up to this point only the section of the wave made by a plane perpendicular to the edge of the screen projected at A. We shall now consider it in its entirety, and shall think of it as divided by equidistant meridians perpendicular to the plane of the figure into infinitely thin spindles. We shall then be able to employ the same process of reasoning which we have just used for a section of the wave, and thus show that the rays which are quite oblique are mutually destructive.

In the case we are now considering these spindles are indefinitely extended in a direction parallel to the edge of the screen, for the wave is intercepted only on one side. Accordingly the intensity of the resultant of all the vibrations which they send to the point P would be the same for each of them; for, owing to the extremely small difference of path, the rays which emanate from these spindles must be considered as of equal in-

tensity, at least throughout that region of the primitive wave which produces a sensible effect upon the light sent to P. Further, it is evident that each elementary resultant will differ in phase by the same quantity with respect to the ray coming from that point of the spindle nearest P, that is to say, from the point at which the spindle cuts the plane of the figure. The intervals between these elementary resultants will then be equal to the difference of path traversed by the rays AP, m'P, mP, etc., all lying in the plane of the figure; and their intensities will be proportional to the arcs Am', $m'm$, mM, etc. In order now to obtain the intensity of the total resultant, we have to make the same calculation which we have already made, considering only the section of the wave by a plane perpendicular to the edge of the screen.*

47. Before deriving the analytical expression for this resultant I propose to draw from the principle of Huygens some of the inferences which follow from simple geometrical considerations.

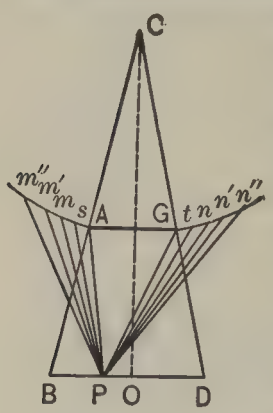


Fig. 17

Let AG represent an opaque body sufficiently narrow for one to distinguish fringes in its shadow at the distance AB. Let C be the luminous point and BD be either the focal plane of the magnifying-glass with which one observes these fringes or a white card upon which the fringes are projected.

Let us now imagine the original wave divided into small arcs—Am, mm', $m'm''$, etc., Gn, nn', $n'n''$, $n''n'''$, etc.—in such a way that the rays drawn from the point P in the shadow to two consecutive points of division will differ by half a wave-length. All of the secondary waves sent to the point P by the elements of each of these arcs will completely interfere with those which emanate

* So long as the edge of the screen is rectilinear we can determine the position of the dark and bright bands and their relative intensities by considering only the section of the wave made by a plane which is perpendicular to the edge of the screen. But when the edge of the screen is curved or composed of straight edges inclined at an angle it is then necessary to integrate along two directions at right angles to each other, or to integrate around the point under consideration. In some particular cases this latter method is simpler, as, for instance, when we have to calculate the intensity of the light in the centre of the shadow produced by a screen or in the projection of a circular aperture.

from the corresponding parts of the two arcs immediately adjoining it; so that, if all these arcs were equal, the rays which they would send to the point P would be mutually destructive, with the exception of the extreme arc *m*A. Half of the intensity of this arc would be left, for half the light sent by the arc *mm'* (with which *m*A is in complete discordance) would be destroyed by half of the preceding arc *m''m'*. As soon as the rays meeting at P are considerably inclined with respect to the normal, these arcs are practically equal. The resultant wave, therefore, corresponds in phase almost exactly to the middle of *m*A, the only arc which produces any sensible effect. It is thus seen that it differs in phase by one-quarter of a wave-length from the element at the edge A of the opaque screen. Since the same thing takes place in the other part of the incident wave G*n*, the interference between these two vibrations occurring at the point P is determined by the difference of length between the two rays *s*P and *t*P, which take their rise at the middle of the arcs A*m* and G*n*, or, what amounts to the same thing, by the difference between the two rays AP and GP coming from the very edge of the opaque body. It thus happens that when the interior fringes under consideration are rather distant from the edges of the geometrical shadow, we are able to apply practically without error the formula based upon the hypothesis that the inflected waves have their origin at the very edges of the opaque body; but in proportion as the point P approaches B the arc A*m* becomes greater in comparison with the arc *mm'*, the arc *mm'* with respect to the arc *m'm''*, etc.; and likewise in the arc *m*A the elements in the immediate vicinity of the point A become sensibly greater than the elements which are situated near the point *m*, and which correspond to equal differences of path. It happens, therefore, that the *effective** ray, *s*P, will not be the mean between the outside rays, *m*P and AP, but will more nearly approach the length of the latter. On the other side of the opaque body we have slightly different circumstances. The difference between the ray GP and the effective ray *t*P approximates more and more nearly a quarter of a wave-length

* I have given this name to the distance of the resultant wave from the original wave because the positions of the dark and bright bands are the same as they would be if these effective rays alone produced them.

as the point P moves farther and farther away from D, so that the difference of path traversed varies more rapidly between the effective rays sP and tP than between the rays AP and GP; consequently, the fringes in the neighborhood of the point B ought to be a little farther from the centre of the shadow than would be indicated by the formula based upon the first hypothesis.

48. Having considered the case of fringes produced by a narrow body, I pass to the consideration of those which are caused by a small aperture.

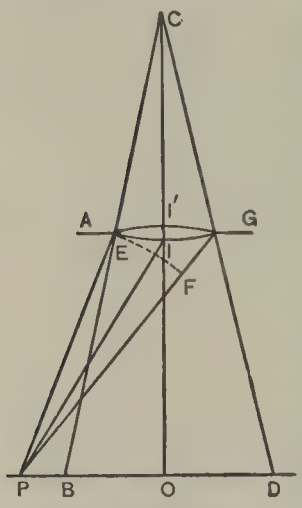


Fig. 18

Let AG be the aperture through which the light passes. I shall at first suppose that it is sufficiently narrow for the dark bands of the first order to fall inside the geometrical shadow of the screen, and at the same time to be fairly distant from the edges B and D. Let P be the darkest point in one of these two bands; it is then easily seen that this must correspond to a difference of one whole wave-length between the two extreme rays AP and GP. Let us now imagine another ray, PI, drawn in such a way that its length shall be a mean between the other two. Then, on account of its marked inclination to the arc AIG, the point I will fall almost exactly in the middle. We now have the arc divided into two parts, whose corresponding elements are almost exactly equal, and send to the point P vibrations in exactly opposite phases, so that these must annul each other.

By the same reasoning it is easily seen that the darkest points in the other dark bands also correspond to differences of an even number of half wave-lengths between the two rays which come from the edges of the aperture; and, in like manner, the brightest points of the bright bands correspond to differences of an uneven number of half wave-lengths—that is to say, their positions are exactly reversed as compared with those which are deduced from the interference of the limiting rays on the hypothesis that these alone are concerned in the production of fringes. This is true with the exception of the point at the middle, which, on either hypothesis, must be

bright. The inferences deduced from the theory that the fringes result from the superposition of all of the disturbances from all parts of the arc AG are verified by experiments, which at the same time disprove the theory which looks upon these bands as produced only by rays inflected and reflected at the edges of the diaphragm. These are precisely the phenomena which first led me to recognize the insufficiency of this hypothesis, and suggested the fundamental principle of the theory which I have just explained—namely, the principle of Huygens combined with the principle of interference.

49. In the case which we have just considered, where, by virtue of a very small aperture, the dark bands of the first order fall at some distance from the edges of the geometrical shadow, it follows from theory, as well as from experiment, that the distance comprised between the darkest points is almost exactly double that of the other intervals between the middle points of two consecutive dark bands, and this is all the more nearly true in proportion as the aperture becomes smaller or more distant from the luminous point and from the focus of the magnifying-glass with which one observes the fringes; for, by sufficiently increasing these distances one may produce the same effects with an aperture of any size whatever.

But when these distances are not very great, and when their aperture is too large for the rays producing the fringes to be very much inclined to the wave-front, AG, it follows that corresponding elements of the arcs into which we have supposed a wave to be divided can no longer be considered as each equal to the other, for they are sensibly larger on the side next the band under consideration. Under these conditions we can rigorously deduce the positions of maximum and minimum intensity only by computing the resultant of all the small secondary waves which are sent out by the incident wave.

50. But there is one very remarkable case where a knowledge of this integral is not needed for the determination of the law of the fringes by an aperture of very considerable size. This is the case where a lens is placed in front of the diaphragm, and brings the refracted rays to focus upon the plane in which the fringes are observed. The problem is now greatly simplified by the fact that the centre of curvature of the

emergent wave now lies in this plane instead of at the luminous point.

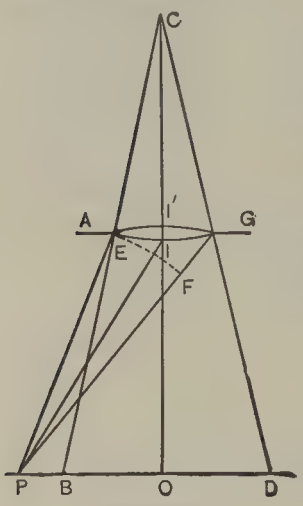


Fig. 18

Let O be the projection of the middle point of the aperture upon this plane. From the point O as centre, and with a radius equal to AQ, let us now describe the arc AI'G, which will now represent the incident wave as modified by the interposition of the lens. If, now, from the point P as centre, and with a radius AP, we describe the arc AEF, those portions of the luminous rays meeting at the point P which are comprised between the arc AI'G and the arc AEF will be the differences of path traversed by the secondary waves; and, since these two arcs have equal curvatures and are convex towards the same side, it follows that equal differences of path will correspond to equal intervals upon the wave-front AI'G. Let us suppose this wave divided in such a manner that any two consecutive rays drawn through the points of division shall differ by one-half a wave-length. If, then, the point P be located in such a way that the total number of these arcs is even, it will no longer receive any light. For these arcs, taken two and two, are mutually destructive, since the vibrations due to corresponding elements are at the same time of equal intensity and opposite phase. The light reaching any point P will be a maximum when the total number of arcs is uneven. The brightest points of the bright bands, therefore, correspond to a difference of an uneven number of half wave-lengths between the two rays coming from the edges of the diaphragm, and the darkest points on the dark bands to a difference of an even number of half wave-lengths. Consequently, all the dark bands will be equally spaced among themselves, with the exception of the first two, where the interval is exactly double that which separates the others. This result, which had already been suggested by theory, I found to be thoroughly confirmed by experiment. I shall cite only one experiment of this kind made in homogeneous red light. In order to bring the centre of the incident wave to the plane of the micrometer wire, I used, instead of an ordinary lens, a

glass cylinder, which, in order to get the full length of the fringes, I placed with its generating line parallel to the edges of the aperture in the diaphragm.

	mm.
Size of the aperture	2.00
	m.
Distance from the luminous point to the diaphragm, or *a*	2 507
Distance from the diaphragm to the micrometer, or *b*	1.140
	mm.
Interval between the middle points of the two dark bands of the first order	0.72
Interval between the band of the first order and the third	0.73
Interval between the band of the third order and the fifth	0.72

It will be observed that the first interval is double that of the others.

I have observed that the same law holds, even at distances which are not very great, for apertures which are much wider, a centimeter or even a centimeter and a half ; but if we further increase the aperture of the diaphragm, the fringes become confused, however much care be taken to place the micrometer in the focus of the cylindrical lens; which goes to show that the rays refracted by this glass vibrate in unison [*in the same phase*] only within rather narrow limits, just as happens with ordinary lenses.

51. When the aperture of the diaphragm thus backed with a cylindrical lens is not too great, the dark and bright bands produced are as sharp as the fringes which result from the union of rays reflected from two mirrors. But, in the latter case, the intensity of the light is the same for all fringes, or, at least, whatever differences there are appear to arise merely from the fact that the light employed is not perfectly homogeneous; and if it happen that the bright bands diminish in brilliancy, the dark bands become less dark, so that the sum of the light in one entire fringe remains practically constant. But in the other phenomenon, as one recedes from the centre he observes a rapid diminution of the light, which is easily accounted for by the theory we have just explained. For, indeed, all the rays which leave the wave-front AI′G and meet at the centre of the bright band of the first order have traversed equal paths ; so that all the small secondary waves which they bring to this point coincide [*in phase*] and strengthen each other.

But this is not the case with the other bright bands. The brightest band of the second order, for instance, corresponds to a division of the wave AI'G into three arcs, the extreme rays of which differ by one-half a wave-length; the effects produced by two of these arcs annul each other. Consequently, this band receives light from only one-third of the incident wave-front, while even the effect produced by this third is somewhat diminished by the fact that there is a difference of one-half a wave-length between the rays from its edges. A similar process of reasoning shows that the middle of the bright band of the third order is illuminated by only one-fifth of the wave-front AI'G, the light of this one-fifth being still further diminished by opposition of phase in its extreme rays.

[*Here are omitted six pages, including a geometrical discussion of the general relations between size of aperture* (*or obstacle*), *distance of screen, distance of luminous point, etc.*]

56. I have just explained the general relations between the size of any particular fringe and the respective distances of the obstacle from the luminous point and from the micrometer. As we have seen, these laws may be derived from theory quite independently of any knowledge of the integral which at each point represents the resultant of all the secondary waves; but in order to find the absolute size of these fringes, it is essential that we compute this resultant, for the positions of maxima and minima of intensity can be determined only by a comparison of the different values of this resultant, or at least by knowing the function which represents it.

In order to do this, we propose to apply to the principle of Huygens the method which we have already explained for computing the resultant of any number of trains of waves when their intensities and relative positions [*phases*] are given.

Application of Theory of Interference to Huygens's Principle

57. Let the waves from any luminous point C be partly intercepted by an opaque body AG. To begin with, we shall suppose that this screen is so large that no light comes around the edge G, so that we need consider only that part of the wave which lies to the left of the point A. Let DB represent the

plane upon which are received the shadow and its fringes. The problem then is to find the intensity of the light at any point P in this plane.

If from C as centre and with a radius CA we describe the circle AMI, it will represent the light-wave at the instant it is partly intercepted by the opaque body. It is from this position of the wave that I have computed the resultant of the secondary waves sent to the point P. If we consider the wave in an earlier position, say A′M′I′, it then becomes necessary to calculate the effect of the obstacle on each of the secondary waves arising from the arc A′M′I′; and if we consider the wave in a later position, say A″M″I″, it becomes necessary to first determine the intensities of its various points, for they are no longer equal, having been changed by the interposition of the screen. In this case the computation is vastly more complicated, possibly quite impracticable. If, however, we consider the wave at the instant it reaches A, the process is simple; for then all parts of the wave have the same intensity. Not only so, but none of the secondary waves are now affected by the opaque screen. However numerous the subdivisions into which we may consider these elementary waves divided, it is evident that the number will be the same for each, since they are transmitted freely in all directions. And, therefore, we need only consider the axes of these pencils of split rays—*i. e.*, the straight lines drawn from the various points on the wave AMI to the point P. The differences of length in these direct rays are the differences of path traversed by the elementary or partial resultants meeting at P.*

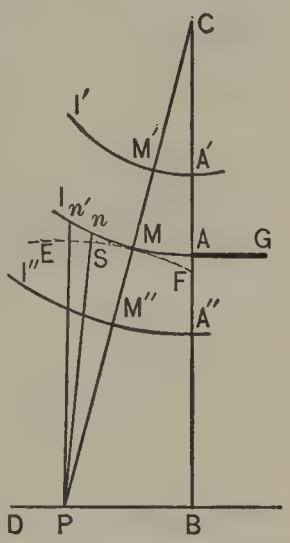


Fig. 19

In order to compute the total effect, I refer these partial resultants to the wave emitted by the point M on the straight line CP, and to another wave displaced a quarter of a wave-length with reference to the preceding. This is the process already employed (p. 101) in the general solution of the interference

* [*A foot-note is here omitted.*]

problem. We shall consider only a section of the wave made by a plane perpendicular to the edge of the screen, and shall indicate by dz an element, nn', of the primary wave, and by z its distance from the point M. These, as I have shown, suffice to determine the position and the *relative* intensities of the bright and dark bands. The distance nS included between the wave AMI and the tangential arc, EMF, described about the point P as centre is $\frac{1}{2}\frac{z^2(a+b)}{ab}$, where a and b are, as before, the distances CA and AB. If we denote the wave-length by λ, we have for the component in question, referred to the wave leaving the point M, the following expression

$$dz \cos\left(\pi \frac{z^2(a+b)}{ab\lambda}\right);$$

while for the other component,* referred to a wave displaced a quarter of a wave-length from the first, we have

$$dz \sin\left(\pi \frac{z^2(a+b)}{ab\lambda}\right).$$

If, now, we take the sum of all similar components of all the other elements, we shall have

$$\int dz \cos\left(\pi \frac{z^2(a+b)}{ab\lambda}\right) \text{ and } \int dz \sin\left(\pi \frac{z^2(a+b)}{ab\lambda}\right).$$

Hence the intensity of the vibration at P resulting from all these small disturbances is

$$\sqrt{\left[\int dz \cos\left(\pi \frac{z^2(a+b)}{ab\lambda}\right)\right]^2 + \left[\int dz \sin\left(\pi \frac{z^2(a+b)}{ab\lambda}\right)\right]^2}.$$

The intensity of the sensation, being proportional to the square of the speeds of the particles, is

$$\left[\int dz \cos\left(\pi \frac{z^2(a+b)}{ab\lambda}\right)\right]^2 + \left[\int dz \sin\left(\pi \frac{z^2(a+b)}{ab\lambda}\right)\right]^2.$$

This is what I have called the *intensity of the light* in order to conform to ordinary usage, while reserving the expression *intensity of vibration* to designate the speed of an ether particle during its oscillation.

* [*These expressions for amplitude follow directly from sec.* 40, *when in the general expression for velocity we make* $a=o$, $a'=dz$, *and* $c=\frac{1}{2}\frac{z^2(a+b)}{ab}$.]

58. In the case we are now considering, where the body, AG, is so large that we can neglect any light coming around the edge G, the integration extends from A to infinity on the side towards I. This integral naturally divides into two parts, one extending from A to M, the other from M to infinity. This latter integral remains constant, while the former varies with the position of the point P. This variation, indeed, is the determining factor in the size and relative intensity of the bright and dark bands.

The integrals

$$\int dz \cos\left(\pi \frac{z^2(a+b)}{ab\lambda}\right) \text{ and } \int dz \sin\left(\pi \frac{z^2(a+b)}{ab\lambda}\right)$$

may be evaluated in finite terms when the limits of z are taken at zero and infinity; but between any other limits their values can be expressed only in terms of a series or by means of partial integration.

The latter method seems to me more convenient, and I have, therefore, employed it in the computation of the following table, where the limits of integration are taken so close together that we can neglect the square of half the arc included between them.*

* Let i and $i+t$ be the narrow limits between which it is proposed to integrate $dv \cos qv^2$ and $dv \sin qv^2$. Neglecting the square of $\frac{t}{2}$, we then find the following approximate values for these integrals:

$$\int dv \cos qv^2 = \frac{1}{2q\left(i+\frac{t}{2}\right)}\left[\sin q\left(i+\frac{t}{2}\right)\left(i+\frac{3t}{2}\right) - \sin q\left(i+\frac{t}{2}\right)\left(i-\frac{t}{2}\right)\right]$$

$$\int dv \sin qv^2 = \frac{1}{2q\left(i+\frac{t}{2}\right)}\left[-\cos q\left(i+\frac{t}{2}\right)\left(i+\frac{3t}{2}\right) + \cos q\left(i+\frac{t}{2}\right)\left(i-\frac{t}{2}\right)\right]$$

These are the formulæ which I have used in the computation of the table. When the limits are sufficiently narrow for us to neglect t^2 instead of $\left(\frac{t}{2}\right)^2$, the following still simpler formulæ may be employed:

$$\int_{i+t}^{i} dv \cos qv^2 = \frac{1}{2iq}\left[\sin qi\,(i+2t) - \sin qi^2\right]$$

$$\int_{i+t}^{i} dv \sin qv^2 = \frac{1}{2iq}\left[-\cos qi\,(i+2t) + \cos qi^2\right]$$

This arc here amounts to $\frac{1}{10}$ of a quadrant, since this furnishes results of an accuracy greater than is attainable in the observations. In place of the integrals mentioned above, I have substituted $\int dv \sin qv^2$ and $\int dv \cos qv^2$, where q stands for quadrant or $\frac{\pi}{2}$.

To pass from one of these forms to the other is a simple matter.

[*Following is a derivation of these formulæ which Verdet found in one of Fresnel's journals.*

Let the limits of integration be denoted by a *and* $a+2p$.

Put $v=a+p+u$. *Then* $du=dv$; *and when* $v=a$, $u=-p$; *but when* $v=a+2p$, $u=+p$.

Substituting for v,

$$\int dv \cos qv^2 = \int du \cos q\left[u^2+2(a+p)u+(a+p)^2\right].$$

If the limiting values of u *are* $+p$ *and* $-p$, *we may take* p *so small, say* $\frac{1}{10}$, *that we may neglect its square,* u^2. *We then have*

$$\int_{a+2p}^{a} dv \cos qv^2 = \int_{-p}^{+p} du \cos q\left[2u(a+p)+(a+p)^2\right]$$

$$= \left|\frac{1}{2q(a+p)} \sin q\left[2u(a+p)+(a+p)^2\right]\right|_{-p}^{+p}$$

$$= \frac{1}{2q(a+p)}\left\{\sin q(a+p)(a+3p) - \sin q(a+p)(a-p)\right\}$$

TABLE OF THE NUMERICAL VALUES OF THE INTEGRALS

$\int dv \cos qv^2$ and $\int dv \sin qv^2$.*

Limits of Integrals	$\int dv \cos qv^2$	$\int dv \sin qv^2$	Limits of Integrals	$\int dv \cos qv^2$	$\int dv \sin qv^2$
From $v=0^q$			From $v=0^q$		
to $v=0.10^q$	0.0999	0.0006	to $v=2.90^q$	0.5627	0.4098
to $v=0.20$	0.1999	0.0042	to 3.00	0.6061	0.4959
0.30	0.2993	0.0140	3.10	0.5621	0.5815
0.40	0.3974	0.0332	3.20	0.4668	0.5931
0.50	0.4923	0.0644	3.30	0.4061	0.5191
0.60	0.5811	0.1101	3.40	0.4388	0.4294
0.70	0.6597	0.1716	3.50	0.5328	0.4149
0.80	0.7230	0.2487	3.60	0.5883	0.4919
0.90	0.7651	0.3391	3.70	0.5424	0.5746
1.00	0.7803	0.4376	3.80	0.4485	0.5654
1.10	0.7643	0.5359	3.90	0.4226	0.4750
1.20	0.7161	0.6229	4.00	0.4986	0.4202
1.30	0.6393	0.6859	4.10	0.5739	0.4754
1.40	0.5439	0.7132	4.20	0.5420	0.5628
1.50	0.4461	0.6973	4.30	0.4497	0.5537
1.60	0.3662	0.6388	4.40	0.4385	0.4620
1.70	0.3245	0.5492	4.50	0.5261	0.4339
1.80	0.3342	0.4509	4.60	0.5674	0.5158
1.90	0.3949	0.3732	4.70	0.4917	0.5668
2.00	0.4886	0.3432	4.80	0.4340	0.4965
2.10	0.5819	0.3739	4.90	0.5003	0.4347
2.20	0.6367	0.4553	5.00	0.5638	0.4987
2.30	0.6271	0.5528	5.10	0.5000	0.5620
2.40	0.5556	0.6194	5.20	0.4390	0.4966
2.50	0.4581	0.6190	5.30	0.5078	0.4401
2.60	0.3895	0.5499	5.40	0.5573	0.5136
2.70	0.3929	0.4528	5.50	0.4785	0.5533
2.80	0.4678	0.3913			

* From the text, and also from the first column of this table, one would be led to think that the second and third columns in the table give the values of the integrals $\int_0^v dv \cos \frac{\pi}{2} v^2$ and $\int_0^v dv \sin \frac{\pi}{2} v^2$ for the following values of v:

$$v = \tfrac{1}{10} \tfrac{\pi}{2},$$

$$v = \tfrac{2}{10} \tfrac{\pi}{2},$$

$$v = \tfrac{3}{10} \tfrac{\pi}{2},$$

etc.

That this is not the case, however, may be shown by using the approximation formulæ of Fresnel to compute any pair of consecutive values of

Either of the integrals $\int dv \cos qv^2$ and $\int dv \sin qv^2$ taken from zero to infinity have the value $\frac{1}{2}$. We may thus by the aid of the above table find the intensity of light corresponding to any given position of the point P, or, what is the same thing, corresponding to any definite value of v, where v is one limit of integration and infinity the other. We have only to take from the table the values of $\int dv \cos qv^2$ and $\int dv \sin qv^2$, using the value of v as an argument, then add to each $\frac{1}{2}$, and finally take the sum of their squares.

59. Simple inspection of this table shows a periodic change in the intensity of light as one leaves the geometrical shadow. To obtain the values of v corresponding to *maxima* and *minima*, *i. e.*, the brightest and darkest points in the respective bright and dark bands, I take from the table the numbers which most nearly correspond to them and then compute the corresponding intensities. Finally, by means of these data and a simple formula of approximation, I determine with sufficient accuracy the values of v which give *maxima* and *minima*.

Let us represent by i the approximate value of v taken directly from the table, by I and Y the corresponding values of $\frac{1}{2}+\int dv \cos qv^2$ and $\frac{1}{2}+\int dv \sin qv^2$, and by t the small arc by which v must be increased in order to give the *maximum* or *minimum* of light. Neglecting the square of t, we find that the following formula gives the value of t which yields a *maximum* or a *minimum*.

$$\operatorname{Sin}\left[q\left(i^2+2it\right)\right]=\frac{2qiI-\sin qi^2}{\sqrt{(qiI-\sin qi^2)^2+(2qiY+\cos qi^2)^2}}.$$

[*A foot-note containing the derivation of this expression is here omitted.*]

If in this formula we substitute the numbers taken from the table, we obtain the following results :

either integral. The successive values of v employed in the first column are

$v=0.1,$
$v=0.2,$
$v=0.3,$
etc.

The same remark applies to the following tables. [*E. Verdet.*]

TABLE OF MAXIMA AND MINIMA FOR EXTERIOR FRINGES AND OF THE CORRESPONDING INTENSITIES

	Values of V	Intensities of Light
Maximum of 1st order.....	1.2172	2.7413
Minimum of 1st order.....	1.8726	1.5570
Maximum of 2d order.....	2.3449	2.3990
Minimum of 2d order.....	2.7392	1.6867
Maximum of 3d order.....	3.0820	2.3022
Minimum of 3d order.....	3.3913	1.7440
Maximum of 4th order.....	3.6742	2.2523
Minimum of 4th order.....	3.9372	1.7783
Maximum of 5th order.....	4.1832	2.2206
Minimum of 5th order.....	4.4160	1 8014
Maximum of 6th order.....	4.6369	2.1985
Minimum of 6th order.....	4.8479	1.8185
Maximum of 7th order.....	5.0500	2.1818
Minimum of 7th order.....	5.2442	1.8317

It is to be observed that here none of the *minima* become zero, as in the case of Newton's rings, or in fringes produced by the meeting of two beams of light of equal intensities; here the difference between *maxima* and *minima* diminishes as one goes farther away from the edge of the opaque screen.

This explains why the fringes which border shadows are not so bright or so numerous as the colored rings, or as the bands produced by the reflection of a luminous point in two slightly inclined mirrors.

60. To employ the above table in computing the size of the exterior fringes, we must first recall the substitution of the integrals $\int dv \cos qv^2$ and $\int dv \sin qv^2$ for the integrals* in question,

$$\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right) \text{ and } \int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right);$$

whence

$$2q\frac{z^2(a+b)}{ab\lambda}=qv^2,$$

and

$$z=v\sqrt{\frac{ab\lambda}{2(a+b)}}.$$

* [*In what follows Fresnel replaces* $\frac{\pi}{2}$ *by q, the initial letter of "quadrant."*]

Therefore

$$\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right)=\sqrt{\frac{ab\lambda}{2(a+b)}}\int dv \cos qv^2,$$

and

$$\int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right)=\sqrt{\frac{ab\lambda}{2(a+b)}}\int dv \sin qv^2.$$

Also

$$\left[\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right)\right]^2+\left[\int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right)\right]^2$$
$$=\frac{ab\lambda}{2(a+b)}\left[\left(\int dv \cos qv^2\right)^2+\left(\int dv \sin qv^2\right)\right]^2.$$

Now the factor $\frac{ab\lambda}{2(a+b)}$ is constant; whence we infer that these two quantities,

$$\left[\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right)\right]^2+\left[\int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right)\right]^2$$

and

$$\left(\int dv \cos qv^2\right)^2+\left(\int dv \sin qv^2\right)^2,$$

will each reach their maximum or minimum values at the same time. Let us now denote by n the value of v which yields a maximum or minimum value for these integrals; the corresponding value of z will then be

$$z=n\sqrt{\frac{ab\lambda}{2(a+b)}}.$$

The size of the fringe, x, [*that is, its distance from the edge of the opaque screen*] then follows from the proportion

$$a:z::a+b:x;$$

whence

$$x=\frac{z(a+b)}{a};$$

or, substituting for z,

$$x=n\sqrt{\frac{\frac{1}{2}(a+b)b\lambda}{a}}.$$

This radical, it may be remarked, is exactly the distance between the edge of the geometrical shadow and that point

which corresponds to a difference* of a quarter of a wave-length between the direct ray and the ray coming *via* the edge of the opaque screen. But this is precisely what might have been predicted, inasmuch as the corresponding value of v [*viz.*, *the quadrant*] has been taken as unity in the table of numerical values of $\int dv \cos qv^2$ and $\int dv \sin qv^2$.

If in the formula

$$x = n\sqrt{\frac{(a+b)b\lambda}{2a}}$$

we substitute for n the value corresponding to a minimum of the first order—*i. e.*, to the darkest part of the first dark band, we have

$$x = 1.873\sqrt{\frac{(a+b)b\lambda}{2a}}.$$

61. If, however, we assume that the fringes are produced by the meeting of the direct rays with those reflected at the edge of the opaque screen, and if we suppose further that the reflected rays lose half a wave-length, we have [*section* 20] for the same band

$$x = \sqrt{\frac{2(a+b)b\lambda}{a}} \text{ or } x = 2\sqrt{\frac{(a+b)b\lambda}{2a}}.$$

Accordingly, these two quantities are in the ratio of 2 to 1.873. The second is measurably smaller than the first, differing as they do by nearly a fifteenth; so that by accurate observations on homogeneous light of well-determined wave-length one might distinguish between these two theories by means of experiment.

62. The method which I at first thought best adapted to the determination of the wave-length was to measure the size of the fringes produced by two mirrors slightly inclined to one another, and also the distance between the two images of the luminous point; but the slightest curvature in the mirrors diminishes the accuracy, and so I preferred to use the bands

* [*The general expression for this difference of path, d, has been given above—section* 20.

$$d = \frac{ax^2}{2b(a+b)}.$$

Put $d = \frac{\lambda}{4}$, *and we have the value of* x *in question.*]

produced by a narrow slit combined with the cylindrical lens of which I have already spoken. We have already found that the distance between any two consecutive dark bands, either to the right or the left of the aperture, is $\frac{b\lambda}{c}$, where λ is the wave-length, c the width of the aperture, and b its distance from the micrometer. The distance between the two dark bands of the first order is just twice this amount. With these data, it is an easy matter to determine λ from measurements on the fringes.

The following table gives the results of five observations of this kind, together with the wave-lengths computed from them. In order to describe all the conditions of the experiment, I include in the table the various values of a, the distance from the luminous point to the screen, even though this quantity is not employed in the calculation. These measures have been made with practically homogeneous red light, obtained by use of the same colored glass which, for the purpose of getting results that are comparable, I have used in all my observations. Each measure recorded in the table is the mean of four observations.

Distance from luminous point to diaphragm a	Distance from diaphragm to micrometer b	Size of aperture	Number of bands $\frac{b\lambda}{c}$ included in each measure	Mean of micrometer measures	Wave-lengths computed from these measures
m.	m.	mm.		mm.	mm.
2.507	1.140	2.00	6	2.185	0.000639
2.010	1.302	4.00	10	2.075	0.000637
2.010	1.302	3.00	8	2.222	0.000640
1.304	2.046	3.00	8	3.466	0.000635
1.304	2.046	2.00	6	3.922	0.000639
				Sum of these results=	0.003190
				Fifth of the sum or mean=	0.000638

The agreement of these results with each other is very satisfactory, differing, as they do, among themselves by less than one per cent. Accordingly I have adopted the value 0.000638 mm., and have employed it in all my comparisons of theory and experiment.

[*Four pages devoted to verifications of the results in this table are here omitted.*]

65. Having thus, by means of simple and well-known methods, verified the wave-length determination made with the single slit and cylindrical lens, I have used this same value to compute the exterior fringes by use of the formula

$$x = n\sqrt{\frac{(a+b)b\lambda}{2a}},$$

substituting for n those values of v which, according to the table, give *maxima* and *minima*.

The table on page 130 summarizes the results of calculation and observation. In my experiments I have measured the positions of the *minima* only, because I considered this a sufficient test of the theory, and because my eye can determine the darkest point of a dark band with greater accuracy than it can set upon the brightest point of a bright band.

[*Only every fifth observation in the table is reproduced.*]

More striking agreement between theory and experiment could scarcely be expected. When these small differences are compared with the quantities measured, and when the great variations in the quantities a and b are noted, one can no longer doubt that the integrals which led to these results accurately describe the law governing the phenomena. The probability in favor of the new theory is still further increased by the fact that the wave-length here employed has been deduced from different and simpler phenomena.

[*Four pages, devoted to a description of some experimental precautions and to a computation of exterior fringes on Young's hypothesis of reflection from the edge of the opaque body, are here omitted.*]

68. We have just seen that both the formation and the position of the exterior fringes can be explained in a satisfactory manner by considering them as produced by the meeting of an infinitely great number of secondary waves which originate on that part of the primary wave which is not intercepted by the opaque screen. From this view it follows that the light which is inflected into the shadow ought not to produce any bright or dark band, but ought to diminish gradually in intensity, provided the screen is sufficiently large to allow no light to go around the other side; and this is true, even though this inflected light, like that which gives rise to the exterior fringes, is the resultant of an infinitude of secondary waves. This will

TABLE COMPARING THE RESULTS OF EXPERIMENT WITH THOSE OF THEORY

EXTERIOR FRINGES IN RED LIGHT OF WAVE-LENGTH 0.000638 MM.

Number of observation	Distance from luminous point to opaque screen a	Distance from opaque body to micrometer b	Order of dark band	Distance from darkest point in each band to edge of geometrical shadow		Difference
				Observed	Computed	
	m.	m.		mm.	mm.	
1	0.1000	0.7985	1	2.84	2 83	−1
			2	4 14	4.14	0
			3	5.14	5.13	−1
			4	5.96	5.96	0
			5	6.68	6.68	0
5	0.510	0.501	1	1.05	1.05	0
			2	1.54	1.54	0
			3	1.90	1.91	+1
			4	2.21	2.22	+1
			5	2.49	2.49	0
10	1.011	2.010	1	2.59	2 59	0
			2	3.79	3.79	0
			3	4.68	4.69	+1
			4	5.45	5.45	0
			5	6.10	6.11	+1
15	3.018	0.253	1	0.54	0.55	+1
			2	0.80	0.81	+1
			3	1.00	1.00	0
			4	1.16	1.16	0
			5	1.31	1.31	0
20	3.018	3.995	1	3.19	3.22	+3
			2	4.70	4.71	+1
			3	5.83	5.84	+1
			4	6.73	6.78	+5
			5	7.58	7.60	+2
25	6.007	0.999	1	1.13	1.14	+1
			2	1.67	1.67	0
			3	2.06	2.07	+1
			4	2.40	2.49	0
			5	2.69	2.69	0

be easily seen by looking at the table below, which gives the intensity of light in the shadow for rays inflected at various angles. These intensities have been computed by means of the table giving the numerical values of the integrals

$$\int dv \cos qv^2 \text{ and } \int dv \sin qv^2,$$

by taking the sums of the squares of the corresponding numbers and subtracting $\frac{1}{2}$. In spite of the inaccuracy introduced by the method of partial integration employed in the first table, it is seen that the intensity of light diminishes rapidly as v increases, presenting none of the *maxima* and *minima* observed outside of the shadow.

INTENSITIES OF LIGHT DIFFRACTED INTO THE SHADOW UNDER DIFFERENT ANGLES

Values of v	Corresponding intensities	Values of v	Corresponding intensities
q		q	
0.10	0.4095	2.90	0.0121
0.20	0.3359	3.00	0.0113
0.30	0.2765	3.10	0.0105
0.40	0.2284	3.20	0.0098
0.50	0.1898	3.30	0.0092
0.60	0.1586	3.40	0.0087
0.70	0.1334	3.50	0.0083
0.80	0.1129	3.60	0.0079
0.90	0.0962	3.70	0.0074
1.00	0.0825	3.80	0.0069
1.10	0.0711	3.90	0.0066
1.20	0.0618	4.00	0.0064
1.30	0.0540	4.10	0.0061
1.40	0.0474	4.20	0.0057
1.50	0.0418	4.30	0.0054
1.60	0.0372	4.40	0.0052
1.70	0.0332	4.50	0.0051
1.80	0.0299	4.60	0 0048
1.90	0.0271	4.70	0.0045
2.00	0.0247	4.80	0.0044
2.10	0.0226	4.90	0.0043
2.20	0.0207	5.00	0.0041
2.30	0.0189	5.10	0 0038
2.40	0.0173	5.20	0 0037
2.50	0 0159	5.30	0.0036
2.60	0.0147	5.40	0.0035
2.70	0.0137	5.50	0.0033
2.80	0.0129		

As usual, a and b represent the distances of the screen from the luminous point and from the plane in which the shadow lies, while x is the distance from the edge of the geometrical shadow to the point in this plane under consideration, so that we have

$$x = v\sqrt{\frac{(a+b)b\lambda}{2a}},$$

and therefore

$$\frac{x}{b}=v\sqrt{\frac{(a+b)\lambda}{2ab}}.$$

69. By the aid of these formulæ we can find the value of the distance x or the angle x/b of the inflected ray corresponding to the various values of b; and *vice versa*, if x or the slant x/b be given, we can find v, and thus determine the intensity of the inflected light. One striking inference from this formula, $x=v\sqrt{\frac{(a+b)b\lambda}{2a}}$, is that the values of x are not directly proportional to those of b, but are related to them as the ordinates of a hyperbola are to its abscissas. It thus follows that points of equal intensity along the edge of the geometrical shadow do not lie upon a straight line as we vary b, but upon a hyperbola of appreciable curvature, like the corresponding loci in exterior fringes.

70. I have not yet succeeded in verifying by direct experiment the ratios of intensity in the inflected light as predicted by the theory of interference applied to the principle of Huygens. A measurement of this kind is very difficult [*foot-note omitted*], and I hardly think that one would be able to reach the same accuracy as in the determination of the darkest and brightest points in fringes. The results already obtained for fringes, however, appear to me as verifications—indirect, it must be confessed—of these very ratios of intensity; for whenever the positions of *maxima* and *minima* have been deduced from the general expression for the intensity of light and have been found to coincide accurately with experiment, it becomes more and more probable that this integral correctly represents all the variations of intensity in the inflected light.

71. In the case of exterior fringes one may, as we have seen, use the table of *maxima* and *minima* to compute the positions of the darkest and brightest points in the dark and bright bands for all values of a and b. This, however, is not the case with regard to the interior fringes in the shadow of a narrow body or in the case of a narrow aperture. The limits of the integral vary all the while, and it is therefore impossible to give general results applicable to every case, so that one is obliged to determine the *maxima* and *minima* for each particular case, using

the table, which gives the numerical values of $\int dv \cos qv^2$ and $\int dv \sin qv^2$. I propose to give the results of all the computations of this kind which, up to the present, I have made for the purpose of testing the theory. They are very long, and I have not been able to finish as many as I had desired, but this lack in quantity is, perhaps, compensated by the variety of the cases which I have studied, for in trying the theory on the observations, I have, by preference, selected cases in which the disposition of the fringes is somewhat unusual.

72. And, first, I propose to consider the case of a narrow aperture which presents at once the case of exterior and interior fringes.

Let C be a luminous point, AG a narrow aperture whose edges A and G are straight and parallel; let BD be the central projection of this aperture upon the plane in which the fringes are observed, and P be any point in this plane at which the intensity is to be determined. For this purpose we must integrate

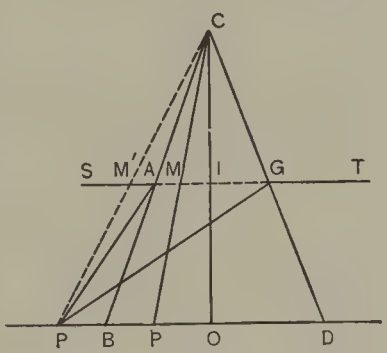


Fig. 20

$$\int dz \cos\left(2q\frac{z^2(a+b)}{ab\lambda}\right)$$

and

$$\int dz \sin\left(2q\frac{z^2(a+b)}{ab\lambda}\right)$$

between the limits A and G, afterwards taking the sum of the squares of these integrals. This will give us the intensity of light at the point P, but we must not forget that the origin from which z is measured lies upon the direct line CP, and that therefore the two limits A and G correspond to $z=\mathrm{MG}$ and $z=-\mathrm{AM}$. The next step is to compute the corresponding values of v from the formula

$$v=z\sqrt{\frac{2(a+b)}{ab\lambda}},$$

or

$$v=x\sqrt{\frac{2a}{(a+b)b\lambda}},$$

where x is the distance from the point P to the edge of the geometrical shadow. From the table of integrals,

$$\int dv \cos qv^2 \text{ and } \int dv \sin qv^2,$$

we then find the values most nearly corresponding to those of v.

Let us call t the difference between the value for which the integral is desired and the number i [*for which it is computed*] in the table. The proper integral can then be found by means of the formulæ of approximation.

$$\int_o^{i+t} dv \cos qv^2 = \int_o^i dv \cos qv^2 + \frac{1}{2iq}(\sin qi\,(i+2t) - \sin qi^2)$$

$$\int_o^{i+t} dv \sin qv^2 = \int_o^i dv \sin qv^2 + \frac{1}{2iq}(-\cos qi\,(i+2t) + \cos qi^2).$$

Having made this computation for the two values of v which represent the edges A and G of the aperture, if the point is inside we add these integrals; if, however, it is on the outside, we subtract them; and, lastly, take the sum of the squares of the two numbers thus found. In like manner, one finds the intensity of the light for any other point whose position is given, and in comparing these various results the positions of *maxima* and *minima* may be found.

[*Half a page concerning the method of interpolation is here omitted.*]

73. In order to apply this method of computation to the observations, I first determined the tabulary value of c, that is to say, the size of the aperture, by means of the formula

$$v = c\sqrt{\frac{2(a+b)}{ab\lambda}},$$

so that I thus obtained the tabular interval between the limits. By a few easy trials I find between what numbers of the table the *maxima* and *minima* lie; afterwards I determine their position more accurately by the process which I have just described. Having thus obtained the values of v corresponding to *maxima* or *minima*, I subtract them from the half of the tabulary value of c, in order to refer them to the middle of the aperture. And, last of all, the formula

$$x = v\sqrt{\frac{(a+b)b\lambda}{2a}}$$

gives me the distance of these same *maxima* or *minima* from the middle of the projection of aperture, which is the point of reference used in my observations.

COMPARISON OF THEORY AND EXPERIMENT

REGARDING THE POSITIONS OF MAXIMA AND MINIMA IN THE FRINGES PRODUCED BY A NARROW APERTURE

Number of bright or dark bands counted from middle	Approximate value of v counted from edge of aperture	Corresponding intensity	Value of v corresponding to *maxima* or *minima*	Distance of *maxima* or *minima* from projection of centre of aperture		Difference
				Computed	Observed	
FIRST OBSERVATION						
	m.	m.	mm.			
$a=2.010$; $b=0.617$; $c=0.50$; tabulary value of $c=1.288$						
				mm.	mm.	mm.
1. Minimum	+0.812	0.03495	+0.913	0.79	0.77	+0.02
	+0.912	0.01645				
	+1.012	0.03406				
2. Minimum	+2.412	0.00238	+2.463	1.58	1.58	0.00
	+2.512	0.00235				
	+2.612	0.00541				
THIRD OBSERVATION						
	m.	m.	mm.			
$a=2.010$; $b=0.401$; $c=1.00$; tabulary value of $c=3.062$						
				mm.	mm.	mm.
1. Minimum	−1.262	2.2575	−1.181	0.14	0.16	−0.02
	−1.162	2.2153				
	−1.100	2.2577				
2. Minimum	−0.300	0.7135	−0.215	0.51	0.48	+0.03
	−0.262	0.6925				
	−0.162	0.6950				
3. Minimum	+0.400	0.1501	+0.431	0.77	0.76	+0.01
	+0.438	0.1477				
	+0.500	0.1604				
4. Minimum	+0.938	0.0799	+1.084	1.02	1.01	+ .01
	+1.038	0.0417				
	+1.138	0.0432				
5. Minimum	+1.800	0.0170	+1.736	1.28	1.28	0
	+1.738	0.0128				
	+1.700	0.0141				
FIFTH OBSERVATION						
	m.	m.	mm.			
$a=2.010$; $b=0.492$; $c=1.50$; tabulary value of $c=4.224$						
				mm.	mm.	mm.
1. Maximum	−1.300	2.7239	−1.168	0.42	0.43	−0.01
	−1.200	3.0466				
	−1.100	2.9780				

[*The second, fourth, and sixth observations are omitted.*]

Evidently theory and observations agree in general quite well, although in the second and fourth observations the disagreement is quite marked and rather more than one would expect from the size of the fringes; for in the second observation the individual measures differ at most by 0.04 mm., and the fourth observation, which I have already described, agrees perfectly, as has been seen, with another experiment in which the same fringes appear. This disagreement, therefore, can only be explained by assuming that the theory is wrong or that constant errors have entered the observations through optical illusion.

74. Our theory rests upon a hypothesis which is at once so simple and so inherently probable, and which besides has been so strikingly verified by many varied experiments, that one can scarcely doubt the truth of the fundamental principle. It is quite possible that this anomaly is only apparent, and that the eye does not correctly estimate the position of the *minima* in question. We must remember that they are not very sharp, and that they are always bounded on each side by two bright bands of very different intensities. Now, in order to determine the position of the *minimum*, my eye must include a part of each of these two bands, so that that part of the dark band on the side next the brightest appears to me darker still on account of its environment; thus attracting, as it were, the apparent *minimum* to its side, and, indeed, all the discrepancies lie in this direction. That the eye includes a sufficiently large portion of the fringes for correctly estimating the position of *maxima* and *minima* is evident from the fact that in repeating the fourth observation, using a diaphragm of small aperture in the focus of the micrometer eye-piece, nothing was left but a band which was uniformly dark and in which the minimum was no longer distinguishable. If I have succeeded in getting the correct positions of the minima in the exterior fringes even in regions of poor definition, it is owing to the fact that the bright bands between which these are included differ very slightly in intensity; and if, in the case of the narrow aperture and cylindrical lens, experiment and theory happen to agree in spite of great differences of intensity between two adjoining bright bands, this is because the dark band, especially in the first and second orders, is almost perfectly black. In general, whenever the *maximum* or *minimum* is very sharp, I find ex-

periment and calculation in thorough agreement. In the fifth observation, for instance, I measure the distance from the centre to the *maximum* of the first order because this bright band is very well defined, and I am therefore able to determine its most brilliant point with great precision. The difference between the computed and observed values is indeed only 0.01 mm.

75. But our theory does more than merely give us the positions of *maxima* and *minima*, for it enables us to predict the general appearance of the phenomena, so that, without experimental determination, we can foretell the variations of intensity in the light; thus, for instance, in the fifth observation the part of the shadow corresponding to the middle of the aperture was filled by a large dark band of a tint that was practically uniform up to 0.26 mm. on each side of the centre, after which the intensity of the light increased rapidly so as to form the bright band of the first order which I have just mentioned. Now in computing the intensity of the light within these limits, we find that in fact its intensity varies scarcely at all, but that in passing from these limits to the bright band it increases very rapidly. In the following table are given the results of computation for different points of the dark band and the two bright bands which include it. The position of each point is denoted by the corresponding value of v, measured always from one of the edges of the aperture.

	Number of observation	Value of v	Intensity
	1	1.100	2.9780
	2	1.200	3.0466
	3	1.300	2.7239
	4	1.400	2.2843
Limit of the colored region as determined by observation.	5	1.524	1.9671
	6	1.824	1.9100
	7	2.112	1.9802
The distribution of intensities on the other side of the centre is the same.			

If the distances of these various points from a common origin be plotted as abscissas and the corresponding intensities as ordinates, we shall obtain the curve MCM′, which gives us in fact a picture of the phenomenon just as one finds it in the

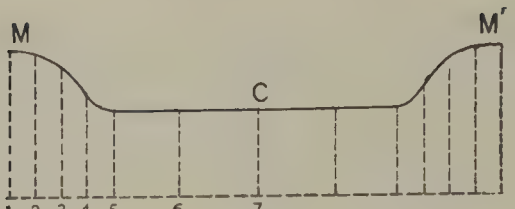


Fig. 21

experiment. I should like to have made similar drawings for all the other observations in order to facilitate the comparison of theory with experiment, but the length of the computation and the time at my disposal did not permit.

[*Five pages, in which the case of a narrow opaque obstacle is discussed, are here omitted.*]

79. I have now applied the principle of Huygens to the three general classes of phenomena in which diffraction occurs, namely, first, to the fringes produced by a screen whose edges are straight and infinitely long, and which is so large that the light passes practically only one edge of the screen; secondly, to the fringes produced by a system of two similar screens brought very near together; thirdly, to those fringes which accompany the shadow of a very narrow screen.*

Comparing observations with the predictions of the theory, I have shown that it suffices to explain the most diverse phenomena, and that the general expression for the intensity of light derived from it gives us a faithful picture of the phenomena, even when they are most bizarre and apparently irregular.

Besides the three general classes, one might devise a large number of others by combining these among themselves. The theory would doubtless apply here with the same success and the same ease. The computation would be more tedious in proportion as the variety of limits assigned to the integrals became greater and greater; the experiments would also demand more complicated apparatus.

80. In the first section of this memoir I have described a phenomenon which results from a combination of two of the principal cases of diffraction, namely, the fringes produced by

* I do not here include those fringes which are produced by the biprism, or two mirrors slightly inclined to each other, for, properly speaking, these are not diffraction effects, since they are not produced by rays which are diffracted or inflected, but by two pencils which are regularly reflected or refracted.

light in passing through two apertures, each very narrow and each near to the other. Having prepared a sheet of copper in the form drawn in Fig. 15, I noted that when the large fringes produced by each of the slits CEC′E′ and DFD′F′, expanding as I moved away from the screen, had filled the shadow of CDFE so that it contained only the bright band of the first order, the interference bands resulting from the two pencils of light became much sharper and brighter than the interior fringes of the part ABCD. The lower part, CEDF, which was at first brighter than the other, became darker the farther I went away from the screen, but its fringes continued to show colors which in white light were purer and bands which in homogenous light were sharper. With the simple apparatus which I employed one could not obtain exact measures, and I have not therefore carried out the computations for this experiment; accordingly I limit myself to the explanation of these phenomena by means of some general considerations.

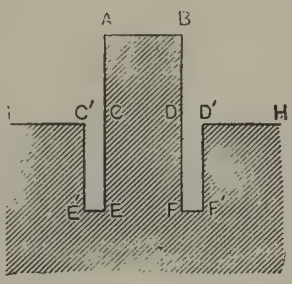


Fig. 15

Let L be the luminous point, and IK the horizontal projection of the part AEBF of the screen represented in Fig 15. P is any point in the interior of the shadow lying upon the straight line LO. From the point L as centre, and with a radius equal to LI, describe the arc IMM′, representing the incident wave. Now with a point P as centre, and with a radius equal to IP, describe the arc I*mm*′. The various distances between these two arcs give us the differences of path traversed by the secondary waves meeting at P. We shall first consider the upper part of the screen—that is, the case where the wave IMM′ is not intercepted on the other side of the point I. Let us now imagine this wave divided into a large number of small arcs, IM, MM′, etc., in such a manner that the straight lines drawn to P from any two consecutive points of division differ by half a wave-length; and, for sake of simplicity, let us suppose that the

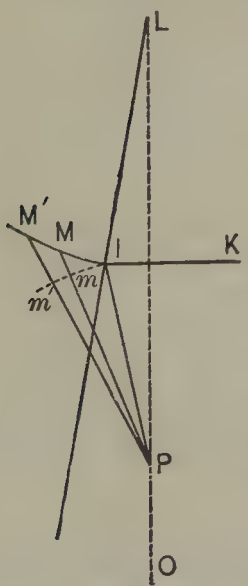


Fig. 22

point P lies well within the edge of the shadow, or, what is the same thing, let us imagine the ray IP sufficiently inclined to the incident ray to make these arcs practically equal. Then each of these arcs, excepting the one at the end IM, will lie between two others, which will combine to annul its effect at the point P. In the case of the arc IM, which lies at the extreme edge of the wave, we have, however, an exception; for this arc loses only one-half its intensity by interference with the vibrations of the neighboring arc, MM′. If, therefore, we intercept this arc [MM′] and all the rest of the incident wave, the light which is received by the point P will actually be increased;* this is precisely the effect which, at a certain distance, is produced by the part of the screen G′C′E″ (Fig. 15). But in proportion as the point P (Fig. 22) recedes from the opaque screen, the arc I*mm′* approaches the wave IMM′; and in the case where the luminous point L is at an infinite distance, these two approach indefinitely near to each other. The divisions M, M′, etc., being determined by the separation of these two arcs, keep spreading apart from the point I in proportion as the arcs approach each other. It follows, therefore, that the part MI of the incident wave will grow larger and larger, and the rays from this part passing the edge C (Fig. 15) retain at least half their intensity in the region behind the upper part of the screen. But in the lower part of the screen the aperture *CEC′E′* does not increase in size, so that if the luminous point is far enough away the effective arc IM (Fig. 22) will finally become so large compared with this aperture that [*most of the rays from* MI *are intercepted by* GC′E′, *and hence*] the point will receive less light in the lower part of the shadow than in the upper.

Let us now pass to the consideration of fringes produced by the meeting of rays coming from both edges of the screen, AEBF (Fig. 15). Behind the upper part, ABCD, the inflected light diminishes rapidly in intensity as one recedes from the edge of the geometrical shadow, and therefore all the fringes except those which are very near the middle are produced by two rays of very unequal intensities; consequently the dark

* The light at P would be increased still more if the screen were perforated in such a way as to permit all the arcs of even order to pass through and at the same time intercept all the arcs of odd order.

bands are not very sharp when one uses homogeneous light, and the colors are mixed with gray when one uses white light. Behind the lower part, CEDF, the two pencils of light coming from the slits CEC′E′ and DFD′F′ have a practically uniform intensity throughout a considerable portion of the bright band from each of these apertures; and if these apertures are so narrow compared with the distance between them that the region of uniform intensity in the inflected light includes all the fringes produced by the two pencils, then in those points where the vibrations are in complete discordance the light-waves will almost completely destroy one another; accordingly the dark bands will be very much sharper than in the upper part of the shadow when homogeneous light is employed, and the colors will be very much purer when white light is used. When one looks at these points close up to the screen before the larger fringes which are produced by each slit have spread out into the shadow AEBF, the phenomenon becomes very complicated and changes rapidly with the distance of the magnifying-glass, especially when the distance between the two slits is not very great when compared with their size. It would be interesting to determine by computation the positions of the *maxima* and *minima* of the bright and dark bands, and to compare these results with those of observation. I have no doubt that the theory would thus acquire fresh confirmation.

81. Hitherto we have considered all waves as coming from a single centre, but, in actual experiment, luminous points are always made up of a very large number of centres of vibration, and it is to each one of these by itself that the preceding discussion applies. So long as these are not very widely separated from each other, the fringes which they produce practically coincide, but the dark bands from one overlap the bright bands of the other in proportion as we increase the dimensions of the luminous point, until finally they completely annul each other. In the case of the exterior fringes this effect is more and more appreciable as one gets farther and farther away from the screen, because it increases directly as the distance, while the size of the bright and dark bands increases less rapidly. And this is why a luminous source sufficiently small to produce fringes which, in the near neighborhood of an opaque body, are very sharp will, at a considerable distance from this body, give only ill-defined fringes.

82. It is not necessary that the interposed body should be opaque in order to produce the phenomena of diffraction at its edges; all that is required is that a part of the wave should be retarded with respect to its neighboring parts, but this is exactly what a transparent body does when its refractive index differs appreciably from that of the medium surrounding it; it thus gives rise to fringes which border both the inside and the outside of their shadow. They are exactly like the exterior fringes of opaque bodies when the difference of path between the rays which have traversed the transparent screen and the outside rays contains a considerable number of wave-lengths, because their mutual influence [*interference*] is no longer appreciable and we have simply the addition of two uniform illuminations. But this is not the case when the transparent screen is very even or when its refractive index differs very slightly from that of the surrounding medium, for now the fringes are altered in a very marked way by the mutual influence of those rays which traverse the transparent plate and those which pass its edge. It is from similar reasons that the striæ in layers of mica resulting from slight differences of thicknesses give rise, in white light, to colored fringes in the very peculiar manner described by M. Arago.

83. As to fringes of the kind which we have called interior, they are not to be obtained with a narrow transparent body, because the direct light which traverses it is so much brighter than the inflected rays as to mask the effects of interference; and, besides, the bright and dark bands which this transparent body tends to produce, when considered as a narrow aperture, do not coincide with those which it tends to produce when considered as a small obstacle.

84. The phenomena of diffraction, once explained for the case of homogeneous light, are easily predicted for the case of white light. These fringes come from the superposition of all the bright and dark bands of the various sizes produced by the different kinds of waves which go to make up white light, so that when we have once computed the intensity of each of the principal kinds of rays at the point under consideration, using the proper wave-length, according to the theory which I have just explained, we can find the resultant tint by substituting these values in Newton's empirical formula for determining the result obtained by mixing any set of colored rays.

85. Polished surfaces illuminated by a point-source present a set of diffraction phenomena exactly like those which we observe in direct light. The field of light reflected by a mirror is bordered with fringes similar to those which surround the shadows of bodies. If the surfaces be very narrow or so blackened that only a single bright line remains, or indeed if one inclines the mirror in such a way as to diminish greatly the size of the field [*foot-note here omitted*], the phenomenon of a pencil of light dilated by passing through a very narrow aperture will be reproduced. If a mirror be blackened throughout its entire extent, with the exception of two bright lines, it gives rise to a set of fringes identical with those produced by two parallel slits in an opaque screen. If, instead of blackening the large part of the reflecting surface, one, on the contrary, merely traces a single fine black line, it will produce fringes similar to those observed in the shadow of a narrow screen. In short, the phenomena are absolutely the same as if the mirror were transparent and the rays came from the image of the luminous point. The explanation is very simple; for we know that the image (which lies upon the perpendicular drawn from the luminous point to the mirror, and which is situated at a distance from the surface of the mirror equal to the distance of the luminous point from the mirror), has this remarkable property, namely, that its distance from any point on the surface of the mirror is equal to the distance of the same point from the luminous centre. When, therefore, we consider the rays as originating in the image of a luminous point, we do not alter the difference of path traversed by the elementary waves which produce the fringes, and consequently there is no change in the size, or in the relative intensities, of the bright and dark bands.

I may here remark that the position [*phase*] of the resultant of the secondary waves at any point, depending as it does merely upon differences of path, ought, in the case of reflection, to be the same as if the rays were emitted by the image just mentioned. Consequently, in the case of a polished surface of large area, all the partial resultants will be situated at the same distance from this point, thus making it the centre of the reflected wave.

86. It is by means of these secondary waves that Huygens has explained in such a simple manner the laws of reflection

and refraction, showing that they are phenomena of the same kind as the propagation of light in a homogeneous medium; but his explanation leaves much to be desired. He has not proved that there will be only one system of waves resulting from this multitude of systems of secondary waves, for he has not used the principle of interference. He assumes that the light is appreciable only in those points where the secondary waves coincide [*in phase*] exactly; while the complete absence of any luminous disturbance can occur only when the secondary disturbances are in [*direct*] opposition. It was this, doubtless, that led him to think that light was not inflected to any appreciable extent into shadows, and which prevented him from discovering the phenomena of diffraction, the laws of which his theory could have given him without recourse to experiment.

This theory, when combined with the principle of interference, gives us not only the path of the ray in the particular case where reflection occurs at a polished surface of indefinite extent, but also in those cases where the surface is very narrow or even discontinuous; it shows us how diminution in size of the surface produces the dilation of the reflected ray, and how a system of very narrow mirrors placed side by side and very close together can produce colored images, owing to the mutual influence of pencils of light thus dilated. This is the phenomenon of ruled surfaces. With the same ease it explains the images and colored rings produced by a thin fabric or even an irregular combination of very fine threads or small particles, provided they are almost equal in size, when placed between the eye of the observer and the luminous point.

I think it hardly necessary to emphasize these phenomena, since they are merely combinations of those described above, and since I have attempted to give, for all of them, a general theory.

ON THE ACTION OF RAYS OF POLARIZED LIGHT UPON EACH OTHER*

BY

ARAGO AND FRESNEL

1. Before describing the experiments which form the subject of this memoir it will perhaps be well to recall the exquisite results obtained by Dr. Thomas Young, who, with rare sagacity and characteristic skill, has already studied the effects which rays of light exert upon each other.

First. Two rays of homogeneous light coming *from the same source* and reaching a certain point in space by paths which are *different and slightly unequal* in length, either strengthen one another or annul one another, and produce upon the receiving screen a bright or a dark point according as the difference of path has one value or another.

Second. Two rays always intensify each other at any point for which their paths are equal; if their intensities are added for another point where the difference of path is equal to a quantity d, their intensities will be added also for all differences of path included in the series $2d$, $3d$, etc. The intermediate values $0+\frac{1}{2}d$, $d+\frac{1}{2}d$, $2d+\frac{1}{2}d$, etc., represent the points in which the rays annul each other.

Third. The quantity d does not have the same value for all homogeneous rays. In air its value for the extreme red rays of the spectrum is $\frac{66}{100000}$ mm., while for violet rays it is only $\frac{42}{100000}$ mm. For other colors the corresponding values are intermediate between these which we have just given.

The periodicity of color which is seen in Newton's rings, in

* [*Annales de Chimie et de Physique*, t. x., p. 288 (1819).]

halos, etc., seems to depend upon the influence exerted upon one another by rays whose paths at first diverge, and later are so inclined as to again meet; but in order to bring these various phenomena into harmony with the laws just stated, we are forced to *admit* that difference of path alone is not sufficient to determine the mutual action of two rays at their point of meeting except when they are both travelling in the same medium; and it must be recognized also that differences of refractive index, or thickness in the transparent bodies traversed by the respective rays, produce the same effect as difference of path. In this journal, vol. i., p. 199, there is described a *direct* experiment due to M. Arago, which shows the same thing, and proves also that a transparent body *diminishes* the speed of light traversing it in the ratio of the sine of the angle of incidence to the sine of the angle of refraction; so that in all the *phenomena of interference* * two different media produce similar effects when their thicknesses are in inverse ratio to their refractive indices. These considerations at once suggest a new method for measuring slight differences of refrangibility.

2. While we were trying to determine what accuracy was attainable by this method, one of us (M. Arago) thought that it would be interesting to find out whether the actions which ordinary rays exert one upon another were in any way modified when two previously polarized pencils of light were made to interfere. We know that if a narrow body be illuminated by light coming from a point-source, its shadow is bordered on the outside by a series of fringes produced by the interference of the direct light with the rays inflected near the opaque body. It is known also that a part of this same light, passing into the geometrical shadow from the two opposite sides of the body, there gives rise to fringes of the same kind.

Now the fact was easily recognized that these two systems of fringes are absolutely the same, whether the incident light has received no modification whatever, or whether it has been polarized previous to incidence. Rays which are *polarized, in one plane*, therefore mutually affect one another in the same manner as rays of ordinary light.

* This is the name which Mr. Young has given to the phenomena produced by the meeting of two or more rays of light.

3. It was still to be determined whether two rays *originally polarized* at right angles would not produce phenomena of the same kind when they met inside the geometrical shadow of an opaque body. For this purpose we placed in front of the point-source* sometimes a rhomb of calc-spar, sometimes an achromatic prism of rock crystal, and thus obtained two luminous points. In each case we had a divergent pencil, and these two pencils were polarized at right angles. Behind the two radiant points and midway of the space between them was placed a cylinder of metal. In this manner a part of the polarized light from the first pencil reached the interior of the shadow *via* the right-hand side of the cylinder; while a part of the light from the second pencil, polarized in a plane at right angles to the first, entered the shadow from the left-hand side of the cylinder. Some of these rays met along the line joining the centre of the cylinder and the middle point of the straight line drawn from one luminous point to the other. Here these rays had traversed equal paths, and one might expect them to produce fringes. On the contrary, not the slightest trace of fringes could be seen, even with a magnifying-glass. In fact, the rays here cross without either affecting the other. The only fringes which make their appearance in this experiment arise from the interference of rays which come from only one of the radiant points and enter the shadow from each side of the cylinder. Those which we tried to produce by the interference of rays polarized at right angles to each other would have occupied a position intermediate between those just mentioned.

Since the images which we employed were not very widely separated, the thicknesses of crystal traversed by the ordinary and the extraordinary rays must have been very nearly equal. Nevertheless, similar experiments have already shown us, only too frequently, how sensitive the phenomena of interference are to the slightest difference of speed in the rays, to the length of path, and to the refractive index of the medium. No argument was needed, therefore, to convince us of the necessity of repeating these experiments under conditions which would eliminate these various sources of inaccuracy. This has been attempted by each of us.

* For all the experiments described in this paper our source of light was the focus of a small magnifying-glass.

4. M. Fresnel at once devised two distinctly different methods. The principle of interference shows us that pencils of light from two luminous points, originally from a single point-source, produce bright and dark bands at points of intersection even though no opaque body be interposed. (See *Annales de Chimie et de Physique*, t. i., p. 332.)

To solve the problem it is then only necessary to determine whether, when *two* images are produced by placing a rhomb of calc-spar in front of a luminous point, they will behave in this same way; but since, from the theory of double refraction, we know that the extraordinary ray traverses carbonate of lime more rapidly than the ordinary ray, it becomes necessary to compensate this extra speed before the two rays are allowed to intersect. In order to accomplish this a method was employed which has been described by M. Arago in this journal, vol i., p. 199. M. Fresnel placed in the path of the extraordinary pencil *alone* a plate of glass whose thickness had been determined by computation in such a way that, under perpendicular incidence, this pencil lost nearly all the ground which, in the crystal, it had gained over the ordinary ray. By slightly inclining the plate the compensation could be made exact. In spite of these precautions, the two rays, polarized at right angles, gave not the slightest trace of interference bands.

In another experiment, M. Fresnel compensated for the difference of speed in the two rays by allowing them each to fall upon a small unsilvered mirror whose thickness had been so *computed* that the extraordinary ray, when reflected at the second face, lost by twice traversing the glass more than it had gained in traversing the crystal; a gradual inclination of the plate brought about complete compensation.

Under no angle of incidence, however, would the ordinary rays, reflected at the first surface, interfere with the rays reflected from the second surface to produce bands.

5. In order to avoid the theoretical consideration introduced into the preceding experiment, and to maintain the original intensity of the light, M. Fresnel adopted the following method: A rhombohedron of calc-spar was sawed through the middle, and the two parts were placed one in front of the other with their principal sections at right angles. In this position, the ordinary ray from the first crystal was refracted as an extraordinary ray in the second; while, conversely, the extraor-

dinary ray in the first crystal suffered ordinary refraction in the second. On viewing a luminous point through this combination, one sees only a double image. Each pencil has experienced in succession the two kinds of refraction. The sum of the paths of each pencil through the two crystals ought, therefore, to be equal, since by hypothesis the crystals have the same thickness; so that everything is compensated, both as regards speed and length of path. Nevertheless, two systems of rays polarized at right angles never gave rise to any interference fringes. Lest the two parts of the rhombohedron did not have quite the same thickness, we took pains in each test to vary slightly and slowly the angle of incidence at the face of the second crystal.

6. The method devised by M. Arago for solving this same problem was independent of double refraction. It has been known for a long time that if one cuts two very narrow slits close together in a thin screen and illuminates them by a single luminous point, there will be produced behind the screen a series of bright bands resulting from the meeting of the rays passing through the right-hand slit with those passing through the left. In order to polarize at right angles the rays passing through these two apertures, M. Arago at first thought of using a thin piece of agate, sawed through the middle and placed one piece in front of each slit, in such a way that the edges formerly meeting along the line of the cut are now at right angles to each other. This arrangement ought certainly to produce the effect expected. But not having at hand a suitable piece of agate, M. Arago proposed to supply its place by two piles of plates, of proper thickness, built up from sheets of mica.

For this purpose we selected fifteen plates as clear as possible and superposed them. This pile was next cut in two by use of a sharp tool. So that now we had two piles of plates of *almost exactly* the same thickness, at least in those parts bordering on the line of bisection; and this would be true even if the component plates had been perceptibly wedge-shaped. The light transmitted by these plates was almost completely polarized when the angle of incidence was about thirty degrees. And it was exactly at this angle that the plates were inclined when they were placed in front of the slits in the copper screen.

When the two planes of incidence were parallel, *i. e.*, when the plates were inclined in the same direction, up and down, for instance, one could very distinctly see the interference bands produced by the two polarized pencils. In fact, they behave exactly as two rays of ordinary light. But if one of the piles be rotated about the incident ray until the two planes of incidence are at right angles to each other, the first pile, say, remaining inclined up and down while the second is inclined from right to left, then the two emergent pencils will be polarized at right angles to each other and will not, on meeting, produce any interference bands.

The pains we took to make these two piles of equal thickness would indicate that we also took care in placing them before the slits to have the light traverse those parts which were originally contiguous. But all difficulties of this kind are really solved by the fact that the two rays when polarized in the same plane interfere like ordinary light. Moreover, we could not produce interference by slowly and gradually changing the inclination of one of the plates so long as the planes of incidence were at right angles.

7. The same day that we tried the combination of these two piles we also tried an experiment suggested by M. Fresnel an experiment which, it must be confessed, is less direct than the preceding, but which is also more easily performed and which demonstrates equally well the impossibility of producing fringes by bringing together rays polarized at right angles to each other.

In front of a sheet of copper in which are cut two slits we placed, for instance, a thin plate of selenite. Since this is a doubly refracting crystal, there will be two pencils of light polarized at right angles passing through each slit. Now if rays polarized in one plane can affect rays polarized in a plane at right angles, we should expect with this arrangement to see three distinct systems of fringes. The *ordinary* rays from the right-hand slit would combine with the *ordinary* rays from the left-hand slit to form a first system symmetrical with respect to the line bisecting the space between the two slits. The bands formed by the two *extraordinary* pencils would fall in the same position as the preceding, increasing their intensity, but remaining indistinguishable from them. As to those which would result from the action of the *ordinary* rays from the

right upon the *extraordinary* from the left, and conversely, it is clear that they would form a system to the right and to the left of the central band. The distance of either of these systems from the centre would increase with the thickness of the plate, for, as we have seen, difference of speed is quite as effective as difference of path in shifting the position of fringes. Now, since the fringes in the centre are the only ones visible, even though the plate of selenite be so thin as not to shift the other two systems very much, we must conclude that rays of light polarized at right angles do not affect one another.

8. In order to verify this conclusion, suppose that we cut the selenite plate in two, and that we place one half in front of the first slit and the other half in front of the other slit; and instead of placing their axes parallel as in the case of a single plate, let us put them at right angles to each other. In this way the *ordinary* ray coming through the right-hand slit will be polarized in the same plane as the *extraordinary* ray from the left-hand slit, and *vice versa.* These rays will then form fringes; but their speeds in the crystal will not be equal, and they will not lie symmetrically about the middle of the space between the two slits. Central fringes will be produced only by ordinary or extraordinary rays from the one slit meeting rays from the other slit which are polarized in the same plane. But when the two parts of crystal are arranged as we have here supposed them, those rays which are polarized at right angles to each other ought not to affect one another. One would, therefore, see simply the first two systems of fringes separated by an interval of white or of some uniform shade.

[*An unimportant foot-note is here omitted.*]

If, without changing the experiment in any other respect, we simply set the two plates of selenite so that their axes make an angle of 45° instead of 90°, we should at once see three systems of fringes; for now, since their planes of polarization are no longer at right angles, each pencil from the right will interfere with the two pencils from the left, and *vice versa.* It should be observed also that the middle system is the most intense, resulting, as it does, from the exact superposition of interference bands of polarized pencils of the same kind.

9. Let us return to the combination of the two piles and imagine that the planes of incidence are mutually perpendicular, so that the two pencils are polarized at right angles to each

other. Between the copper screen and the eye place a doubly refracting crystal in such a way that its principal section makes an angle of 45° with the planes of incidence. In accordance with the well-known laws of double refraction, the rays which are transmitted by the piles will afterwards, in passing through the crystal, each be divided into two others. These two will be of equal intensity, will be polarized in planes which are mutually perpendicular, and one of these planes will coincide with the principal section of the crystal. One might therefore expect to see, in this experiment, one series of fringes due to the meeting of the ordinary pencil from the right with the ordinary pencil from the left, and a second series similar to the preceding, but arising from the interference of the two extraordinary pencils. Such, however, is not the case; for these four pencils meet and produce only a uniform illumination, showing not the slightest interference.*

This experiment shows that two rays originally polarized at right angles to each other may subsequently be brought into the same plane of polarization without again acquiring the power of interference.

10. In order to produce interference between two rays polarized at right angles and afterwards reduced to the same plane it is necessary that they should originally have been polarized in one and the same plane. This is shown by the following experiment, which was devised by M. Fresnel.

A plate of selenite, backed with a sheet of copper in which two apertures have been made, is illuminated by a pencil of polarized light coming from a point-source and striking the selenite plate at perpendicular incidence. The axis of the plate makes an angle of 45° with the original plane of polarization. As in all similar experiments the shadow of the copper screen is observed with a magnifying-glass; but in this case

* If the plate interposed between the copper screen and the eye were so thin as to only slightly separate the two images, one might explain the absence of interference as follows: *viz.*, suppose the two systems of bands are superposed in such a fashion that the bright bands of one system coincide with the dark bands of the other system, and *vice versa*. But the insufficiency of this explanation is shown by placing a rhombohedron of Iceland spar between the eye and the preceding crystal. In certain positions this Iceland spar separates the two systems of bands, because they are polarized at right angles. But even under these circumstances one sees no trace of bands.

a rhombohedron of Iceland spar, in which [*the separation of images due to*] double refraction is *perceptible*, is placed in front of the focus.

The principal section of the Iceland spar makes an angle of 45° with that of the plate of selenite. Accordingly we find in each image three systems of fringes, one falling exactly in the middle of the shadow, the others being situated on the right and left respectively.

Let us now consider one of these two images, say the ordinary, and see what gives rise to these three systems of bands.

The pencils which pass through the two slits are polarized in the same plane, but on emergence from the plate of selenite they are divided into two pencils polarized at right angles. Since double refraction in this plate is inappreciable, the ordinary and extraordinary pencils each follow practically the same route, though with different speeds.

Each of these double pencils, say the one from the right-hand slit, will be divided, in passing through the Iceland spar, into four pencils, two ordinary and two extraordinary; but, as a matter of fact, one will see only two, since components in the same plane will coincide. It is also evident, from the well-known laws of double refraction and from the relative positions of the selenite and the Iceland spar, that at emergence from this latter crystal the ordinary pencil will be composed partly of the ray which was ordinary in the selenite and partly of the ray which was extraordinary; while the other two components of these same rays go to form the extraordinary image which we are not now considering. The pencil which emerges from the left-hand slit behaves in the same way. We see, in fact, that the ordinary pencil coming either from the right or left hand slit will, after traversing the two crystals in this new instrument, be composed partly of light which has followed the ordinary path in each crystal and partly of light which started out as an extraordinary ray.

Rays coming from the two slits and following the ordinary path through each of the two crystals will have traversed routes of the same length and with the same speed. On meeting, they ought, therefore, to give rise to central bands. The same is true of rays which have pursued the extraordinary path both in the selenite and in the Iceland spar. The bands

in the middle of the shadow result, therefore, from the superposition of these two different systems.

Now as to that portion of light from the right-hand slit which has traversed the selenite as an extraordinary ray, for instance, but passed the Iceland spar as an ordinary ray, it is evident that it will have traversed a path which in length is equal to that of the left-hand pencil which made the whole trip as an ordinary ray. But since in the selenite the speeds are different, those points where they meet to form fringes will not lie symmetrically between the two slits, but will be shifted to the right, *i. e.*, to the side opposite the ray which for a while travelled as an extraordinary ray, but now travels more slowly. Finally, as a last combination, we have interference between that component of the right-hand pencil which traversed both crystals as an ordinary ray and that component of the left-hand pencil which in the selenite was an extraordinary ray and in the Iceland spar an ordinary ray. This interference gives rise to a system of bands situated on the left of the centre.

We have now explained the paths of the rays which meet to form the three systems of fringes in the experiment under discussion. And it may be remarked that the right and left systems were produced by the interference of rays which were previously polarized at right angles in the selenite and afterwards reduced to the same plane in the Iceland spar. Two rays polarized at right angles and later reduced to the same plane of polarization can, then, meet and produce interference bands; but for this purpose it is an essential condition that the rays should ORIGINALLY *have been polarized in the same plane.*

So far we have not considered the interaction of the two pencils which suffered extraordinary refraction in the Iceland spar. These pencils also furnish three systems of bands, but they are separated from the others. If we allow all the conditions of the experiment to remain the same, except that we substitute for the Iceland spar a plate of selenite or quartz which does not give two distinct images, the six systems, instead of being reduced to three by superposition, will result in one central system. This remarkable fact shows, *first*, that the fringes resulting from the interference of the ordinary rays are complementary to those produced by the interference of the extraordinary rays; and, *secondly*, that these two sys-

tems are so located that a bright band in the one system corresponds to a dark band in the other system. Were these two conditions not satisfied, one would not find uniform and continuous illumination on each side of the central fringes. We meet here, then, the same difference of half a wave-length that is found in the phenomena of colored rings.

From the experiments just described we may, therefore, infer the following facts :

(1.) Two rays of light *polarized at right angles* do not produce any effect upon each other under the same circumstances in which two rays of ordinary light produce destructive interference.

(2.) Rays of light polarized in the same plane interfere like rays of ordinary light ; so that in these two kinds of light the phenomena of interference are absolutely identical.

(3.) Two rays which were *originally polarized at right angles* may be brought to the same plane of polarization *without thereby acquiring the ability to interfere.*

(4.) Two rays of light polarized at right angles and afterwards brought into the same plane of polarization interfere like ordinary light provided they were originally polarized in the same plane.

(5.) In the phenomena of interference produced by rays which have experienced double refraction the position of the interference bands is determined not only by difference of path and difference of speed, but in some cases, as above indicated, it is necessary to take into account also a difference of one-half a wave-length.

All these laws are, as we have seen, based directly upon experimental evidence. In starting from the phenomena of crystalline plates, they can be derived more simply ; but then we have to assume that the colors of the plates when illuminated by polarized light are produced by the interference of several systems of waves. The evidence which we have just presented has the advantage of establishing the same laws quite independently of hypothesis.

Biographical Sketch

Augustin Jean Fresnel was born in Normandy in 1788, and died near Paris in 1827.

As a child he was quite the reverse of precocious; but at the age of sixteen he was ready to enter the *École Polytechnique* at Paris, where he received sound mathematical training and attracted to himself the attention of Legendre. His education was completed at the *École des Ponts et Chausées* where he received an engineer's training. Several years were next spent in professional work in various parts of France.

In 1816, through the influence of Arago, he received an appointment in Paris, where he remained during the rest of his life. When we recall that his first studies in optics date from 1814, his accomplishments during the eleven years of his Paris residence must ever fill us with wonder. New ideas were not only rapidly acquired, but were also rapidly perfected. They were at once submitted to the test of experiment and as quickly received elegant mathematical description.

The wave-theory of light had lacked neither merit nor able support; Grimaldi, Hooke, Huygens, and Young had been its advocates; but it was only in the hands of Fresnel that the problem and its solution received such clear and simple statement as to command acceptance. The work of Fresnel lies exclusively in the domain of optics, each of his investigations falling into one of two distinct groups, *viz.*, the kinematics of light and the dynamics of light.

His earlier papers deal with questions of diffraction, interference, and polarization, in which the chief factors of the discussion are displacements, velocities, and squares of velocities —the quantities of kinematics.

His later papers, however, refer more to the medium through which luminous energy is transferred; they deal with the forces of elasticity here brought into play, and seek to determine the speed of light as a function of the mechanical properties of the matter through which the light travels; they deal, in short, with the dynamics of light.

But the particular achievements with which the name of Fresnel must always be associated are

(1.) The introduction of the idea of transverse vibrations.

(2.) The combination of the principle of Huygens with that of interference.

An excellent and appreciative sketch of Fresnel will be found in Arago's *Notices Biographiques*, vol. i. It is here that he paraphrases Newton's remark concerning Cotes by saying "*que nous savons quelque chose quoique Fresnel ait peu vécu.*"

Between the years 1866 and 1870 the French government published the works of Fresnel in three worthy quarto volumes, ably edited by Senarmont, Verdet, and the author's brother, Léonor Fresnel.

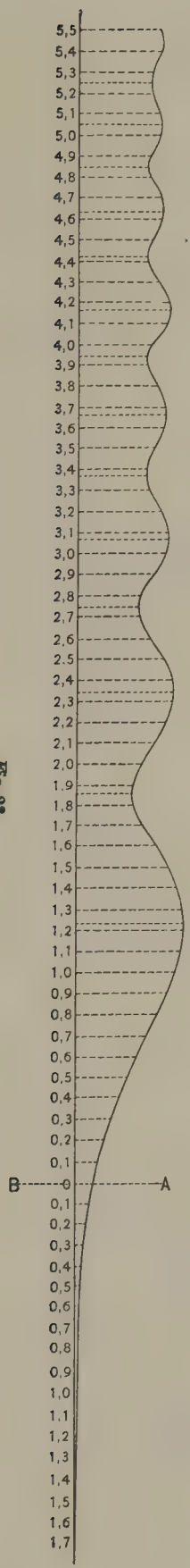

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Fig. 23

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THE END

PRISMATIC AND DIFFRACTION SPECTRA

Translated and Edited by J.S. Ames

HARPER'S SCIENTIFIC MEMOIRS

EDITED BY

J. S. AMES, PH.D.

PROFESSOR OF PHYSICS IN JOHNS HOPKINS UNIVERSITY

II.

PRISMATIC AND DIFFRACTION SPECTRA

PRISMATIC AND DIFFRACTION SPECTRA

MEMOIRS BY JOSEPH VON FRAUNHOFER

TRANSLATED AND EDITED

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NEW YORK AND LONDON
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PREFACE

THE spectrum of the sun was first observed, in 1666, by Newton, who allowed light coming from a small round opening in a shutter to pass through a glass prism. This spectrum was most impure ; and a pure spectrum was not obtained until, in 1802, Wollaston repeated Newton's experiment, replacing the round opening by a slit parallel to the edge of the prism. He observed several dark lines crossing the spectrum, which limited, as he thought, the different spectral colors. His description of this discovery is given at the end of this volume.

Working independently of Wollaston, Fraunhofer, in 1814, rediscovered the lines in the solar spectrum, which now bear his name. He at first used a slit and prism ; but, later, he discovered that the same phenomena could be obtained by means of gratings made up of wires or ruled on glass. The papers of Fraunhofer in which he describes these results are printed in full in this volume. The great merit of Fraunhofer's work is the systematic, logical method by which he proceeds from investigation to investigation.

The most important contributions of Fraunhofer to the science of Spectrum Analysis are :

1. The application of the objective prism to the study of spectra of the sun, the stars, flames, etc.
2. The discovery of the principle of plane gratings.
3. The discussion of the effect upon spectra of
   a. Periodic errors in the ruling of gratings.
   b. The shape of the groove.
   c. The relative widths of opaque and open spaces.
4. The first measurements of the wave-lengths of various solar lines.

5. The discovery of the agreement in wave-length of lines in the spectra of flames and of certain stars with those in the solar spectrum.

All modern work in spectroscopy is based upon that of Fraunhofer, and a brief bibliography of the most important contributions is appended to this volume.

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[*Introduction is omitted.*]

It would be most advantageous if one could measure for every kind of glass the dispersion for every color ; but the different colors in the spectrum have no definite limits, and so this cannot be determined immediately from the color-image. The uncertainty is so great that the experiments are useless. It could be done more exactly if colored pieces of glass or colored liquids could be found which would transmit light of only one color—*e. g.*, one which transmitted only blue, another only red, etc.; but I have not been so fortunate as to discover any such substances. With all I tried, the white light which was transmitted was broken up into all colors, the only difference being that the particular color which the glass or the liquid had was the strongest in the spectrum. Colored flames, also, which are obtained by the combustion of alcohol, sulphur, etc., do not give in their spectra homogeneous light which corresponds to their color ; yet with these, as well as with oil and tallow light, and in general with light of all flames, I found in the spectra a bright, sharply defined streak in the region between the red and the green, which is in exactly the same position in all the spectra, and which will be most useful in what follows. This bright band appears to be formed by rays which are not dispersed further by the prism, and are therefore homogeneous. There is a similar streak in the green, which is, however, not so well defined and is much more feeble, so that in some cases it is recognized with difficulty ; on this account it cannot be of much service.

[*The experiments on the dispersion of various prisms are omitted.*]

In order to determine more accurately the refraction of the different colored beams, partly also to see if the action of the refracting medium was the same on sunlight as on artificial light, I devised an apparatus which could be used with sunlight,

just as the one before described was with lamplight. This was soon seen to be superfluous.

In the window-shutter of a darkened room I made a narrow opening—about 15 seconds broad and 36 minutes high—and through this I allowed sunlight to fall on a prism of flint-glass which stood upon the theodolite described before. [*The dimensions are not given.*] The theodolite was 24 feet from the window, and the angle of the prism was about 60°. The prism was so placed in front of the objective of the theodolite-telescope that the angle of incidence of the light was equal to the angle at which the beam emerged. I wished to see if in the color-image from sunlight there was a bright band similar to that observed in the color-image of lamplight. But instead of this I saw with the telescope an almost countless number of strong and weak vertical lines, which are, however, darker than the rest of the color-image; some appeared to be almost perfectly black. If the prism was turned so as to increase the angle of incidence, these lines vanished; they disappear also if the angle of incidence is made smaller. For increased angle of incidence, however, these lines become visible again if the telescope is made shorter; while, for a smaller angle of incidence, the eye-piece must be drawn out considerably in order to make the lines reappear. If the eye-piece was so placed that the lines in the red portion of the color-image could be plainly seen, then, in order to see the lines in the violet portion, it must be pushed in slightly. If the opening through which the light entered was made broader, the fine lines ceased to be clearly seen, and vanished entirely if the opening was made 40 seconds wide. If the opening was made 1 minute wide, even the broad lines could not be seen plainly. The distances apart of the lines, and all their relations to each other, remained unchanged, both when the width of the opening in the window-shutter was altered and when the distance of the theodolite from the opening was changed. The prism could be of any kind of refractive material, and its angle might be large or small; yet the lines remained always visible, and only in proportion to the size of the color-image did they become stronger or weaker, and therefore were observed more easily or with more difficulty.

The relations of these lines and streaks among themselves appeared to be the same with every refracting substance; so that, for instance, one particular band is found in every case

only in the blue; another is found only in the red; and one can, therefore, at once recognize which line he is observing. These lines can be recognized also in the spectra formed by both the ordinary and the extraordinary rays of Iceland spar. The strongest lines do not in any way mark the limits of the various colors; there is almost always the same color on both sides of a line, and the passage from one color into another cannot be noted.

With reference to these lines the color-image is as shown in Fig. 5. It is, however, impossible to show on this scale all the lines and their intensities. (The red end of the color-image is in the neighborhood of A; the violet end is near I.) It is, however, impossible to set a definite limit at either end, although it is easier at the red than at the violet. Direct sunlight, or sunlight reflected by a mirror, seems to have its limits, on the one hand, somewhere between G and H; on the other, at B; yet with sunlight of great intensity the color-image becomes half again as long. In order, however, to see this great spreading-out of the spectrum, the light from the space between C and G must be prevented from entering the eye, because the impression which the light from the extremities of the color-image makes upon the eye is very weak, and is destroyed by the rest of the light. At A there is easily recognized a sharply defined line; yet this is not the limit of the red color, for it proceeds much beyond. At *a* there are heaped together many lines which form a band; B is sharply defined and is of noticeable thickness. In the space between B and C there can be counted 9 very fine, sharply defined lines. The line C is of considerable strength, and, like B, is very black. In the space between C and D there can be counted 30 very fine lines; but these (with two exceptions), like those between B and C, can be plainly seen only with strong magnification or with prisms which have great dispersion; they are, moreover, very sharply defined. D consists of two strong lines which are separated by a bright line. Between D and E there can be counted some 84 lines of varying intensities. E itself consists of several lines, of which the one in the middle is somewhat stronger than the rest. Between E and *b* are about 24 lines. At *b* there are 3 very strong lines, two of which are separated by only a narrow bright line; they are among the strongest lines in the spectrum. In the space between *b* and F there can be counted about 52 lines; F is

fairly strong. Between F and G there are about 185 lines of different strengths. At G there are massed together many lines, among which several are distinguished by their intensity. In the space between G and H there are about 190 lines, whose intensities differ greatly. The two bands at H are most remarkable; they are almost exactly equal, and each consists of many lines; in the middle of each there is a strong line which is very black. From H to I the lines are equally numerous. In the space between B and H there can be counted, therefore, about 574 lines, of which only the strong ones appear on the drawing. The distances apart of the strongest lines were measured by the theodolite and transferred according to scale directly to the drawing; the weak lines, however, were drawn in, without exact measurement, simply as they were seen in the color-image.

I have convinced myself by many experiments and by varying the methods that these lines and bands are due to the nature of sunlight, and do not arise from diffraction, illusion, etc. If light from a lamp is allowed to pass through the same narrow opening in the window-shutter, none of these lines are observed, only the bright line R [*referred to before*], which, however, comes exactly in the same place as the line D (Fig. 5), so that the indices of refraction of the rays D and R are the same. The reason why the lines fade away, or even entirely vanish, when the opening at the window is made too wide is not difficult to give. The stronger lines have a width of from five to ten seconds; so, if the opening of the window is not so narrow that the light which passes through can be regarded as belonging to one ray, or if the angular width of the opening is much more than that of the line, the image of one and the same line is repeated several times side by side, and consequently becomes indistinct, or vanishes entirely if the opening is made too wide. The reason why the lines and bands are not seen when the prism is turned, unless the telescope is made shorter or longer, will be seen from the following considerations:

It is only when rays fall upon a prism in such a way that the angle of incidence equals that of emergence that they proceed, so far as divergence is concerned, just as they fall upon the prism; if the angle of incidence is greater, the rays, after refraction through the prism, diverge from a more remote point; if the angle is smaller, the rays diverge from a nearer point.

The reason is that the rays which pass through near the edge of the prism have a shorter path through the prism to traverse than those which pass through farther from the edge. This does not change, it is true, the angle of the refracted rays ; but the sides of the triangle for the emerging rays become greater in the one case, less in the other. This difference should vanish when the rays fall upon the prism parallel to each other, which is in accord with experiment. Since the violet rays through the objective of the theodolite telescope have a shorter focal length than the red rays, it is evident why the eye-piece must be displaced in order to see plainly the lines in the different colors.

Since the lines and bands in the color-image have only a very small width, it is evident that the apparatus must be most perfect in order to avoid all aberrations which could make the lines indistinct or entirely scatter them. The faces of the prism must therefore be perfectly plane. The glass to be used in such prisms should be entirely free from waves and streaks ; with English flint-glass, which is never free from streaks, only the stronger lines are seen. Common glass and English crown-glass contain many streaks, even if they are not visible to the naked eye. If one does not possess a prism of perfect flint-glass, it is better to choose a fluid which has a great dispersive power—*e. g.*, aniseed oil—in order to see the lines ; yet in this case the prismatic vessel must have faces which are perfectly plane parallel. With all prisms the faces should make an angle of 90°, or nearly so, with the base ; this must be placed horizontal, in front of the telescope, if the axis of the latter is horizontal. The narrow opening through which the light enters must be exactly vertical, etc. It is easy to understand why the lines become indistinct if one or the other of these precautions is neglected.

Since the lines and bands are seen in the color-image of every refracting medium of uniform density, I have used them in order to measure the refraction of a medium for each colored ray ; and this could be done with great accuracy, because the majority of the lines are sharply defined. Since with refracting media which have feeble dispersion or with prisms of small angles it is only with difficulty that the fine lines can be recognized even with strong magnification, I selected in these experiments, for all refracting media, the stronger lines, *viz.*,

B, C, D, E, F, G, and H. The lines at *b* I did not take, because it is too near F; and I tried to come more in the middle between D and F. Since the eye-piece must be displaced in order to see distinctly the lines in the different colors, no large arcs such as BH could be measured, *only the small ones* such as BC, CD, etc.

[*Ten pages, containing descriptions of experiments on different kinds of glass, are omitted.*]

On seeing the number of lines and bands in the spectrum of sunlight, one can perhaps with difficulty avoid the conjecture that diffraction at the narrow opening in the window-shutter has something to do with these lines, although the experiments which have been described do not in the least indicate this, but, on the contrary, refute it. Partly in order to be perfectly sure on this point, partly also in order to make further observations, I changed the experiment in the following manner:

If sunlight coming from a small *round* opening, 15 seconds in diameter, in the window-shutter is allowed to fall upon a prism placed in front of the theodolite telescope, it is clear that the color-image which is seen through the telescope can have only an inappreciable width, and therefore will form only a line; in a colored line, however, no fine cross-lines can be seen. In order to see the many lines in this color-image, all that is necessary is to make the spectrum broader without altering its length in the least. I accomplished this by placing against the objective a piece of glass which was plane on one side and curved on the other, so as to be a portion of a cylinder of large diameter. The axis of the cylinder was placed parallel to the base of the prism; consequently, the length of the spectrum could not be changed, and only its breadth was increased. With this arrangement I recognized again in the spectrum all the lines exactly as they are seen when the light comes through a long, narrow opening.

I applied this form of apparatus at night-time to observe Venus directly, *without making the light pass through a small opening;* and I discovered in the spectrum of this light the same lines as those which appear in sunlight. Since, however, the light from Venus is feeble in comparison with sunlight reflected from a mirror, the intensity of the violet and the extreme red rays are very weak; and on this account even the stronger lines in both these colors are recognized only with

difficulty, but in the other colors they are very easily distinguished. I have seen the lines D, E, *b*, F perfectly defined (Fig. 5), and have even recognized that *b* consists of two lines, one weak and one strong; but the fact that the stronger one itself consists of two I could not verify owing to lack of light. For the same reason the other finer lines could not be distinguished satisfactorily. I have convinced myself by an approximate measurement of the arcs DE and EF that the light from Venus is in this respect of the same nature as sunlight.

With this same apparatus I made observations also on the light of some fixed stars of the first magnitude. Since, however, the light of these stars is much weaker than that of Venus, it is natural that the brightness of the spectrum should be much less. In spite of this I have seen with certainty in the spectrum of Sirius three broad bands which appear to have no connection with those of sunlight; one of these bands is in the green, two are in the blue. In the spectra of other fixed stars of the first magnitude one can recognize bands; yet these stars, with respect to these bands, seem to differ among themselves. Since the objective of the telescope has an aperture of only 13 lines*, it is clear that these observations can be repeated with much greater accuracy. I intend to repeat them with suitable alterations, and with a larger objective, in order to induce, perhaps, some skilled investigator to continue the experiments. Such a continuation is all the more to be desired, because the experiments would serve at the same time for the accurate comparison of the refraction of the light of the fixed stars with that of sunlight.

The light of electricity is, with respect to these lines and bands, markedly different from sunlight, and also from the light of flames. Several lines are found in the spectrum of this light; some are very bright, and among these one in the green is almost dazzlingly bright in comparison with the rest of the spectrum. There is another line in the orange which is not quite so bright; it appears to have the same color as the bright line in the spectrum of lamplight. If, however, the angle of refraction is measured, it is found that its light is refracted much more—about as much as the yellow rays of lamplight. Near the red end of the spectrum there is a line

[* 1 *centimetre equals* 4.43296 *lines.*]

which is not very bright; its light is refracted, so far as I could be sure, exactly the same as that of the bright line of lamplight. In the rest of the spectrum four other lines can be recognized easily.*

If lamplight is allowed to pass through a very narrow opening of 15 to 30 seconds' width and then to fall upon a strongly dispersive prism placed in front of a telescope, it is seen that the reddish-yellow bright line of this spectrum consists of two very fine bright lines which in intensity and distance apart are like the two dark lines D (Fig. 5). But, regardless of the width of the slit, if the point of the flame and the lower blue end are covered, and so only the brightest portion of the flame exposed, the reddish-yellow lines of the spectrum appear less bright, and therefore are recognized with more difficulty. These lines, consequently, seem to be formed mainly by the light from the ends of the flame, especially by that from the lower one.

In the spectrum of the light caused by the combustion of hydrogen and alcohol, the reddish-green line is very bright in comparison with the other portions. When sulphur is burned it is seen with great difficulty.

I intend to repeat these experiments which have reference to the perfecting of achromatic telescopes, using a new instrument by means of which I hope to obtain at least twice the accuracy. I shall be able also to make new experiments with this instrument for which my present one is not adapted, and which will be of interest perhaps for practical optics.

In all my experiments I could, owing to lack of time, pay attention to only those matters which appeared to have a bearing upon practical optics. I could either not touch other questions, or at most not follow them very far. Since the path thus traced in optical experiments seems to promise to lead to interesting results, it is greatly to be desired that skilled investigators should devote attention to it.

* In order to use the electric light I placed two conductors one-half an inch apart, one being joined to an electrical machine, the other to the earth, and connected them with a fine glass fibre. The light then appeared to pass continuously over the fibre, and the latter formed a fine luminous line.

New Modification of Light by the Mutual Influence and the Diffraction of the Rays, and the Laws of this Modification.

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Schumacher's *Astronomische Abhandlungen*, II., 46–112, 1823.

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New Modification of Light by the Mutual Influence and the Diffraction of the Rays, and the Laws of this Modification.

All experiments in which the eye of the investigator is provided with good optical instruments are distinguished, as is well known, by a high degree of precision; and some of the most important discoveries could not have been made without these instruments. Up to the present time, in experiments on diffraction there has been no instrument, except a magnifying-glass, which could be used with profit; and this may perhaps be one of the reasons why in this field of physical optics we are so backward, and why we know so little of the laws of this modification of light. Since at small angles of inclination refraction and reflection of light are altered by diffraction, and since in many other cases diffraction plays an important part, which may often be unnoticed, it is most to be desired that these laws should be exactly known; and this is specially so because a knowledge of them makes the nature of light itself better known at the same time.

If sunlight is admitted into a darkened room through a small opening and falls upon a dark screen some distance away, which has a narrow aperture, and if the light which passes through this slit is allowed to fall upon a white surface or a piece of ground-glass placed a short distance behind the screen, one sees, as is well known, that the illuminated portion of the white surface is larger than the narrow slit in the screen, and that it has colored edges—in short, that the light through the slit is inflected or diffracted. The narrower the openings, so much the greater is the inflection. The shadow of every body which is placed in a beam of sunlight entering a darkened room through a small opening is bounded by fringes of color which are, moreover, for any given distance of the surface on which the shadow is received, of the same size for bodies of all

kinds of matter. The shadow of a narrow object, such as a hair, has, in addition to the outer fringes, others within the shadow, which change with the thickness of the hair, but in other respects are similar to the outer ones.* Since the colored fringes are very small, and since most of the light is lost through absorption at the surface on which the shadow is cast, no great accuracy could be expected with the methods which have been used up to this time to observe diffraction phenomena; and this is all the more true because by these methods it is impossible to measure the angles of inflection of the light which alone can make us acquainted with the laws of diffraction. Up to the present, these angles from which the path of the diffracted light can be learned have been calculated from the dimensions of the colored bands and their distance from the diffracting body; but assumptions have been made which, as we shall see, do not agree with the truth, and which, therefore, give false results.

The number of different optical phenomena has become in our time so great that caution must be taken so as to avoid being deceived, and also to refer the phenomena always to the simple laws. This is more necessary in the case of diffraction, as we shall see, than in all the other phenomena. I shall, therefore, report the experiments which I have made for the determination of the laws of diffraction of light in an order which is different from that in which I actually performed them, by which procedure many experiments become superfluous and a better understanding will be reached.

Diffraction of Light through a Single Opening.

In order to receive in the eye all the light diffracted through a narrow opening, and to see the phenomena strongly magnified; still more, in order to directly measure the inflection of the light, I placed in front of the objective of a theodolite-telescope a screen in which there was a narrow vertical opening which could be made wider or narrower by means of a screw. By means of a heliostat I threw sunlight into a darkened room

* All that is known on the subject of diffraction may be found in Biot's "Traité de Physique, Exp. et Math.," T. 4, p. 743; and in the *Göttinger Commentaren*, Vol. IV., p. 49.

through a narrow slit so that it fell upon this screen, through whose opening the light was therefore diffracted. I could then observe through the telescope the phenomena produced by the diffraction, magnified, and yet seen with sufficient brightness; and at the same time I could measure the angles of inflection of the light by means of the theodolite.

The colors which are produced by the diffraction of light through a single opening are arranged in an order similar to that of the colors of Newton's rings, which are produced by the contact of two slightly convex pieces of glass; with this difference, that with the latter a black spot is seen in the centre, while it is not with the former. Fig. III., Table I., will help the description. If the telescope of the theodolite is so adjusted that on removing the screen which has the diffraction-slit the slit at the heliostat is focused on the micrometer cross-hairs, and if then the screen—whose slit must be very narrow—is placed in front of the objective, there will be seen in the centre of the field a white band $L^{I}L^{I}$; and the cross-hairs will be in the middle of this band at K. This band becomes yellow near each side, and finally red. In the space $L^{I}L^{II}$ there is a vivid color-spectrum, which is indigo near L^{I}, then blue, green, yellow, and near L^{II} red. The color-spectrum in the space $L^{II}L^{III}$ is much less intense than that in $L^{I}L^{II}$; the arrangement of its colors is as follows: Near L^{II} blue, then green, yellow, and near L^{III} red. The spectrum in the space $L^{III}L^{IV}$ is still weaker than the last; near L^{III} it is green; near L^{IV}, red. There then follow a great number of spectra which grow continually weaker until they can be no longer distinguished, and then can be seen only a horizontal strip of light which is, however, stretched out through a great distance. The spectra just described are exactly the same on the two sides of K—*i. e.*, they are symmetrical. The transitions from one color into another are not sharply defined, but imperceptible, and the same thing is true of the spectra.

The instrument with which I observed and measured the angles is essentially a 12-inch repeating theodolite whose verniers read to 4″. In the middle of the circle and above it there is a plane horizontal disk 6 inches in diameter which turns on its axis, and whose centre lies exactly on the axis of the theodolite. It has its own divisions, which read to 10″. At the middle of this disk is placed the screen through which the

light is diffracted, and which therefore stands in the axis of the theodolite, so that the corrections to the angles as measured, which otherwise would have to be made, owing to the distance of the diffracting body from the axis, are not necessary. The divisions marked on the disk serve to measure, if it is necessary, the angle of incidence of the light, etc. Beyond the disk, at a distance of $3\frac{1}{3}$ inches* from the centre, the telescope begins. Its objective has an aperture of 20 lines and a focal length of 16.9 inches, and it is clamped to the alidade of the 12-inch circle and suitably counterbalanced. The axis of the telescope is parallel to the plane of the circle and accurately horizontal. I used a magnifying power of 30 and 50. The whole instrument is isolated from the floor. In the prolongation of the optical axis, $463\frac{1}{2}$ inches from the middle of the theodolite, is placed the heliostat, whose hour motion can be regulated by a screw and a rod which is attached to it and which reaches to the theodolite, so that one can at will increase or decrease the intensity of the sunlight. The opening at the heliostat is vertical, 2 inches long, and can be made wider or narrower. I ordinarily used it with a width of only 0.01 to 0.02 inch.

I measured the width of the opening in the screen by means of an achromatic microscope which I devised for this purpose, because this quantity must be known most exactly. At the foot of this microscope there is a sliding-piece which can be moved in one direction by a fine screw having nearly 88 turns to a Paris inch ; the screen is so fastened to this sliding-piece that its opening, which is to be measured, comes vertically above the screw. There are cross-hairs in the eye-piece of the microscope, which can be seen at the same time as the object viewed. By means of the screw which moves the sliding piece, first one edge and then the other of the object is made to coincide with one edge of the cross-hair, and the position of the screw is read each time. The difference in the readings is the diameter of the object, in terms of the pitch of the screw, independently of the construction of the optical parts of the microscope, the magnification, etc. Since the head of the screw is divided by a vernier into 1000 parts, it is possible to measure the diameter of a sharply defined object to at least 0.00002 of an inch ; in many cases to 0.00001. I generally used an objec-

* [1 *Paris inch* = 2.70700 *cm.* = 1.06577 *inches.*]

tive with which the microscope magnifies the diameter of the object 110 times.

Since it is impossible to find a fixed point of reference in the color-spectrum arising from diffraction through a single narrow opening, I took, in order to measure the angles of deflection, the transition from one spectrum into another—that is, L^{I}, L^{II}, L^{III}, etc., or the red end of each spectrum. I measured the distances $L^{I}L^{I}$, $L^{II}L^{II}$, etc., repeating at least three times. The halves of these are, therefore, the deflections from the centre—that is, KL^{I}, KL^{II}, etc. I shall designate by L^{I}, L^{II}, etc., the angles of these deflections. All spectra which are due to diffraction through a single opening I shall call spectra of the *first class*, merely to distinguish them from other kinds which will be discussed later. The following table contains the angles of inflection of light through openings of different widths. I designate this width by γ; it is expressed always in fractions of a Paris inch. I call the arithmetical mean of L^{I}, $\frac{L^{II}}{2}$, $\frac{L^{III}}{3}$, etc., L.

No.	γ	L^{I}	L^{II}	L^{III}	L^{IV}	L	L_{γ}
							0.0000
1	0.11545	37″.58	1′ 15″.5	1′ 53″	....	37″.66	210
2	0.06098	1′ 11″.6	2′ 22″.7	3′ 31″.7	4′ 44″.7	1′ 11″.17	210
3	0.03690	1′ 57″.1	3′ 53″.3	5′ 48″.3	....	1′ 56″.6	209
4	0.02346	3′ 4″	6′ 7″.7	9′ 16″.3	....	3′ 4″.43	210
5	0.01237	5′ 48″.5	11′ 38″	17′ 26″.5	23′ 14″.7	5′ 48″.7	209
6	0.01210	6′	12′ 1″	18′ 14″	24′ 9″	6′ 1″.84	212
7	0.01020	6′ 56″	13′ 56″	20′ 54″	....	6′ 57″.3	206
8	0.00671	11′ 6″	22′ 12″.7	33′ 14″	44′ 35″	11′ 6″.4	217
9	0.00642	11′ 11″	22′ 18″	33′ 43″	44′ 58″	11′ 12″.2	209
10	0.00337	21′ 3″	42′ 16″	1° 4′	....	21′ 10″.3	207
11	0.00308	23′ 31″	47′ 6″	1° 10′ 43″	....	23′ 32″.7	211
12	0.00218	33′ 30″	1° 7′ 40″	.....	....	33′ 40″	213
13	0.00215	35′ 24″.7	1° 10′ 16″	.....	....	35′ 17″	220
14	0.00114	1° 4′ 53″	.....	.....	....	1° 4′ 53″	215

The angles recorded in this table are all given exactly as I observed them, without any corrections, and it is not difficult, therefore, to determine the limits of accuracy. Since the points of transition from one spectrum into another are not sharply defined, recourse must be had, within certain limits, to estimation. In long spectra—*i.e.*, when the opening in the screen is narrow—these limiting positions lie farther apart, and the angles will not agree among themselves as well as in the case of a large opening in the screen and small spectra. The

relative accuracy is, however, nearly the same. Within the limits of error, the following conclusions may be drawn from the table:

With single openings of different widths the angles of inflection of the light are inversely proportional to the widths of the opening.

In light diffracted through a narrow opening the distances from the centre to the red rays of the various spectra on both sides form an arithmetical series in which the common difference is equal to the first term.

It will appear from later experiments that this law holds for the other colored rays, and that it is true for the spectra which are far from the axis as well.

In the general case, for any width of opening, measured in fractions of a Paris inch and designated by γ, where L^{I}, L^{II}, L^{III}, etc., are the arcs for radius 1, we have:

$$L^{I} = \frac{0.0000211}{\gamma},$$

$$L^{II} = 2 . \frac{0.0000211}{\gamma},$$

$$L^{III} = 3 . \frac{0.0000211}{\gamma}, \text{ etc.}$$

In order to see whether the spectra which arise from diffraction consist of homogeneous light, I fastened in front of the eye-piece of the theodolite-telescope [*i.e., between the eye-piece and the eye of the observer*] a small flint-glass prism of about 20°, in such a manner that the axis of the prism was horizontal and the edge pointed down. If, with an eye-piece like this, one views a homogeneous spectrum such as is obtained when a good prism is placed in front of the objective, the cross-hairs of the eye-piece will be seen in every color. If, however, there is no homogeneous light in the field of view, the horizontal thread will disappear. The explanation is not difficult. If the spectra produced by diffraction through a single opening are brought into the field, no trace of the horizontal thread is seen in the first and second spectra; in the third it is just perceptible; in the fourth it is more definite, although still indistinct. This indistinctness becomes

gradually less in the succeeding spectra, so that far from the middle of the field the horizontal thread can be seen fairly defined. Accordingly, the spectra near the axis consist of non-homogeneous light; those farther away become more and more homogeneous.

The lower horizontal red end of the first spectrum appears blue when viewed through the prism; the upper blue end of this spectrum, however, appears red, which also proves that this spectrum consists of non-homogeneous light; for in a spectrum formed by a prism no blue light can be produced out of red, nor red from blue. Since the light is refracted through the eye-piece prism, and since rays are refracted differently—*e.g.*, the blue more than the red—it, therefore, happens that, when there is in the field of view a homogeneous [*i.e.*, *pure*] spectrum, which without the prism would be horizontal, the thread of the cross-hair which would have been horizontal is no longer so, but is lower at the end where it points towards the more refrangible rays, and higher at the other; it, therefore, has an inclined position which may be easily noticed. Since in the spectra formed by diffraction those which are far from the axis overlap, and since a portion of each spectrum encroaches upon the one which precedes and the one which follows, as is apparent from the second of the above laws, the inclined position of the cross-hair serves to convince one of their existence and furnishes a method of counting them. For if there are several overlapping spectra in the field of view, there will be seen as many oblique cross-hairs as there are spectra. I shall return to this matter later.

[*The case is then discussed when the two halves of the screen forming the slit are no longer in the same plane, but one is displaced normally with reference to the other.*]

In all the above experiments the light at the heliostat fell upon a narrow vertical opening, so that only one beam of light entered, or, in other words, so that the light apparently came from a luminous line. The reason for this is easily understood; in every other case each beam would form its own spectrum, and there would be as many of these overlapping as there were beams. If, for instance, the light came from a luminous surface whose angular width was greater than that of the spectra, it would be impossible to distinguish any spectra through a narrow opening in the screen; because the rays

coming from the right-hand side of the luminous surface would produce red in the same place where the rays from the left side would produce blue, etc.; and the light would be all confused, and consequently white again. Since, however, light is diffracted by every narrow opening, one might imagine that the light falling upon the slit in the screen had already been modified by diffraction at the slit near the heliostat. Although this doubt disappears, either by a consideration of the diameter of the sun or on certain other grounds, nevertheless I have investigated the subject experimentally. All that is necessary is that the light should come as if from a luminous line. I prepared, therefore, a lens, 2 inches long, $\frac{1}{3}$ inch wide, which was plane upon one side, and upon the other was a portion of a cylinder of 0.66 inch diameter. [*Parallel*] light falling upon this lens proceeds after refraction as if it came from a line at a distance of 0.62 inch from the lens. The opening at the heliostat was made $\frac{1}{4}$ inch wide, and the cylindrical lens was placed in front of it. If the path of the light through the lens is traced, it is seen that no light passing around the edge of the slit at the heliostat can fall upon the screen which is in front of the theodolite-telescope, and consequently no diffracted light can reach it. Using this cylindrical lens, both the spectra due to diffraction through a small opening and their dimensions are exactly the same as they were when the narrow slit was used at the heliostat.

By means of a screen which contains a long, narrow opening, light is diffracted in one direction only; by my screen this direction was horizontal, because the opening was vertical. A screen containing an opening which is not narrow, but is, for instance, as high as it is broad, will produce diffraction in a vertical direction also. It is easily understood that in this case the incident light must not come from a luminous line, because the diffraction in a vertical direction would not be observed, for the same reasons as I have given above. The light must come through an opening at the heliostat, which is as broad as it is high. I, therefore, allowed the light, as a rule, to pass through a round opening 0.04 to 0.08 inch in diameter. If a round opening is used, and if the screen with a long vertical opening is placed in front of the theodolite-telescope, the spectra, as can be easily predicted, have a very small height, but in a horizontal direction they are exactly as they were when the light

passed through a long vertical opening at the heliostat. There is seen, therefore, only a horizontal line, in which the colors alternate as described above, and which is narrower in proportion as the round opening at the heliostat is made smaller. This opening should not be made too small, however, because if the light is to be diffracted in other directions also, there would not be sufficient brightness.

If the light passes through a round opening at the heliostat, and if there is placed in front of the theodolite-telescope a screen containing a rectangular opening, which has perfectly straight sides and sharp corners, and which is, *e.g.*, as high as it is wide, a colored cross will be seen in the telescope, in which the colors alternate vertically as well as horizontally in the same way as they did in light diffracted by a long, narrow opening. In the corners of this colored cross there are seen still feebler color-spectra, a, b, c, d, Table II., Fig 2. [*This figure is omitted.*] The explanation of these spectra which are seen only in the corners will be apparent from experiments which will be discussed further on. The dimensions of the colors which form the cross are the same as when a long, narrow opening of the same width is used: *viz.*, $L^{I} = \frac{0.0000211}{\gamma}$, $L^{II} = 2.\ \frac{0.0000211}{\gamma}$, etc., in both a vertical and a horizontal direction. If the rectangular opening in the screen is not as broad as it is high, the vertical spectra of the cross have a different width from the horizontal ones; and the weak subsidiary spectra in the corners are altered accordingly. By means of a screen, then, the width of whose rectangular opening is less than its height, there is produced a colored cross whose vertical arm is composed of spectra smaller than the horizontal ones, in the inverse ratio of the height to the width.

[*Diffraction through a circular opening is then discussed.*]

Mutual Action of a Great Number of Diffracted Beams.

In order to make a great number of beams, diffracted exactly alike, fall upon the whole surface of the theodolite-objective, I stretched upon a frame a great many wires of the same thickness, parallel to each other and at the same distances apart; the light was then diffracted through the inter-

vening spaces. In order to be sure that the wires were exactly parallel and at equal distances from each other, I cut on two opposite ends of a rectangular frame a fine screw-thread, which had 169 turns, nearly, to a Paris inch; I stretched the wires in the spirals of this screw, and I could then be sure that they were exactly parallel and at equal distances apart.

Through a vertical opening at the heliostat, 2 inches high and 0.01 inch wide, I directed an intense beam of sunlight upon the objective of the theodolite-telescope, and placed the grating at the middle of the theodolite-disk. The grating consisted of about 260 parallel wires, which were 0.002021 inch thick and whose edges were 0.003862 inch apart. I took care to see that no light except that coming through the grating fell upon the objective. Since the small openings between the threads diffract the light, all the light which fell upon the objective was diffracted to the same extent. I was most surprised to observe that the phenomena seen through the telescope when the grating was used were entirely different from those observed by diffraction through a single opening. The opening at the heliostat is seen exactly as it would be through the telescope if the grating were away; and at some distance from this image, on both sides, are seen a great number of colored spectra, which are exactly like those seen through a good prism; they become longer the farther they are from the axis, but they decrease in intensity. Fig. 1, Table I., represents some of these spectra. In A is seen the opening at the heliostat, colorless and sharply defined, just as it would appear if the grating were removed. On both sides of A the phenomena are perfectly symmetrical. If the apparatus is well made, there is no light in the space AH^{I}. The first spectrum is in the space $H^{I}C^{I}$; H^{I} is the violet end, C^{I} the red. There is no light in the space $C^{I}H^{II}$. The second spectrum is in the space $H^{II}C^{II}$; it is twice as long as the first, and the order of the colors is the same; it is somewhat less intense than the first. In the space between C^{II} and F^{IV} comes the third spectrum; a portion of its violet rays, however, overlaps the red of the second, and the red of the third overlaps the blue of the fourth. The intensity of the third spectrum is again less than that of the second. Between F^{IV} and D^{IV} is the fourth spectrum, whose blue end overlaps the third, and whose red end overlaps the fifth spectrum. Other spectra follow, which become gradually weaker, and of which

13 on each side of A can be counted without difficulty if the apparatus is perfected. One can convince himself easily of the existence of a still greater number; and the only reason why they cannot be counted is because they become longer and longer, and consequently overlap more and more.

If the eye-piece of the telescope is so focused that, if the grating were away, the opening at the heliostat would appear perfectly sharp, there are seen in the spectra produced by the grating the same lines and streaks which I discovered in the spectrum of sunlight produced by a good prism.* This is of special interest, because it makes it possible, as we shall see, to determine with a great degree of accuracy the laws of this modification of light produced by the mutual action of a great number of diffracted beams. In the drawing I have represented in each spectrum only the strongest of these lines, those with which we shall have to do hereafter; there is seen, however, especially in the longer spectra, a great number of lines, exactly as with a prism. The relative intensity of the lines and their arrangement are the same as with a prism. It is only with reference to the relative spaces which in any one spectrum are occupied by different colors that there is a striking difference between the spectra produced by the grating and the prism. On this account, and because with some kinds of gratings the spectra are very small, one must be well acquainted with the lines formed by a prism in order to know at once, in spectra of every size, with which bands or lines he has to do. This is all the more necessary because in the grating-spectra which are far from the axis the lines are superimposed.

I shall call these spectra which are produced by a grating of parallel threads "*perfect spectra of the second class*," so as to distinguish them from others which arise from the mutual action of a smaller number of beams, and in which the bands and lines are not seen. These spectra have different properties, and I shall call them "*imperfect spectra of the second class*."

In order to vary the conditions of the phenomena as much as possible, I made gratings of wires of different thicknesses and

* I have described them in a memoir printed in the *Denkschriften der k. b. Akademie der Wissenschaften* for the year 1814–15, which bears the title, "Determination of the refractive and dispersive power of different kinds of glass with reference to the perfecting of achromatic telescopes." [*This volume, page* 2.]

with different spaces. For this purpose I made a fine screw on which there were nearly 340 turns to the inch. I also drew parallel straight lines at equal intervals on a piece of plane glass covered with gold-foil, through which spectra were obtained exactly as they were through wire gratings.

The size of the spectra of the second class, which are produced by a grating, does not depend upon the width of the spaces nor upon the thickness of the wires, but only upon the sum of the two, or, what is the same thing, upon the distance apart of the centres of the spaces. The less this distance is, the larger are the spectra. The finer, therefore, the screw in whose grooves the thread is stretched, the larger is the spectrum. Consequently, for these spectra of the second class it is entirely immaterial what the thickness of the wire or the width of the openings is. A hair, silver wire, or gold wire can be stretched in the screw-threads; the kind of matter is of no importance. Care must be taken, however, to have the thread of uniform thickness, and particularly to have it stretched straight in order that the openings may be of the same width throughout their entire length. If wire is used, great care must be taken in stretching it over the screws, because it bends so easily. It is difficult to use hairs, because they rarely have a uniform thickness.

If the grooves of the screw upon which the threads are wound are somewhat large—*i.e.*, if the centres of the openings lie far apart—the spectra are small, as noted above; and consequently all come in a small space. If with these coarse grooves the wires are thick, and therefore the width of the openings are correspondingly small, one can see that where the perfect spectra of the second class cease—or, better, become weaker—another kind of spectrum begins. These are much longer; and the lines and bands which occur in the prismatic spectrum are not seen in them. They change only with the width of the grating-openings, and behave like spectra of the first class, which are produced by diffraction through a single narrow opening; and, therefore, I designate them by L^{I}, L^{II}, etc.

We shall see that the spectra of the first class are visible with nearly all perfect gratings regardless of whether the wires are wound on fine or on coarse grooves. It often happens that some of the spectra of the first class coincide with the spec-

tra of the second class, and so modify their intensity. We shall see the connection between these wonderful phenomena from the experiments.

If, when a grating is used, the prism described above in the case of diffraction through a single opening is placed, as there described, in front of the eye-piece, we see that the perfect spectra of the second class consist of homogeneous light, and that beginning with the third, owing to the increase in their length, they overlap at the places of transition from one spectrum into another. On account of the unequal refraction of the different colored beams through the eye-piece prism, the overlapping spectra will be in part separated and appear as is shown in Fig. 4, Table II. [*This figure is omitted.*] We see, for instance, the red end of the third spectrum at C^{III}, and the lines which belong to this color may be definitely recognized; likewise we see the violet end of the third spectrum at H^{III}, and the lines belonging to it. The same thing is true of spectra which lie farther from the axis. Since spectra become longer the farther they are from the axis A, and since for an eye-piece prism of definite angle the height $C^{I}d$ [*the difference in refraction, i.e., the dispersion of C and H by the prism*] is the same for all spectra, the slant of the upper and lower edges of spectra far from the centre must be less steep than that of those near the centre. As will be seen from the observations, glass has an action upon rays of different color different from that of a grating in air. This is the reason why the lower and upper edges of the spectra viewed through the prismatic eye-piece are seen to be non-rectilinear. The horizontal cross-hair of the micrometer is seen well defined in all perfect spectra of the second class, and may be used here also to count the number of the spectra far from the centre, which, on account of their great length and small inclination, are distinguished with difficulty, even with the eye-piece prism.

If the light passes through a cylindrical lens at the heliostat, the phenomena produced by the grating remain the same as when the light passed through a narrow opening.

In the experiments which follow I have used the same system of names for the different lines of the spectra as I did in the prismatic spectrum, viz., C, D, E, F, G, H.* For the first

* [*A summary of the preceding paper is given in the note, which is omitted.*]

spectrum I shall use the symbols C^I, D^I, E^I, etc.; for the second, C^{II}, D^{II}, E^{II}, etc. I shall call the thickness of the wire δ, and the width of an opening γ. These quantities are given in terms of a Paris inch. The arithmetical mean of, *e. g.*, C^I, $C^{II}/_2$, $C^{III}/_3$, etc., I call C; that of D^I, $D^{II}/_2$, $D^{III}/_3$, etc., D, etc. By means of the theodolite I have measured, at least six times, the angular distance of the two symmetrical spectra for each color, or, more often, that of the lines named above. Since the spectrum lines are sharply defined, great accuracy can be obtained with perfect gratings. I give all the angles, exactly as I measured them, without any correction. The grating was always placed at the middle of the horizontal disk of the theodolite. All angles, *e. g.*, C^I, D^I, E^I, etc., are the simple distances from the axis A. In the products $(\gamma + \delta)C$, etc., I have used the sine of the angles. However, with such small angles as these it is entirely immaterial which is used, the sine or the arc.

GRATING No. 1.

γ =0.000628	δ =0.001324
B^I = 44′ 45″	E^{III}=1° 42′ 42″.7
C^I = 42′ 42″.3	E^{IV}=2° 16′ 59″.7
C^{II} =1° 25′ 25″	F^I = 31′ 32″.6
D^I = 38′ 19″.3	F^{II} =1° 3′ 10″
D^{II} =1° 16′ 38″	F^{III}=1° 34′ 44″
D^{III}=1° 55′	G^I = 27′ 57″.3
D^{IV}=2° 33′ 14″.7	G^{II} = 55′ 51″.7
E^I = 34′ 12″.6	H^I = 25′ 42″.3
E^{II} =1° 8′ 28″.3	H^{II} = 51′ 31″.7
B = 44′ 45″	F = 31′ 34″.1
C = 42′ 42″.4	G = 27′ 56″.5
D = 38′ 19″.2	H = 25′ 44″
E = 34′ 14″	

$(\gamma+\delta)$ B=0.00002541	$(\gamma+\delta)$ F=0.00001792
$(\gamma+\delta)$ C=0.00002425	$(\gamma+\delta)$ G=0.00001587
$(\gamma+\delta)$ D=0.00002176	$(\gamma+\delta)$ H=0.00001461
$(\gamma+\delta)$ E=0.00001944	

[*In the following tables only the results are given.*]

GRATING No. 2.

γ =0.001112	δ =0.001817
$(\gamma+\delta)$ B=0.00002541	$(\gamma+\delta)$ F=0.00001795
$(\gamma+\delta)$ C=0.00002426	$(\gamma+\delta)$ G=0.00001587
$(\gamma+\delta)$ D=0.00002177	$(\gamma+\delta)$ H=0.00001464
$(\gamma+\delta)$ E=0.00001946	

GRATING No. 3.

$\gamma = 0.000972$	$\delta = 0.001964$
$(\gamma+\delta)$ C $= 0.00002425$	$(\gamma+\delta)$ F $= 0.00001788$
$(\gamma+\delta)$ D $= 0.00002176$	$(\gamma+\delta)$ G $= 0.00001580$
$(\gamma+\delta)$ E $= 0.00001942$	$(\gamma+\delta)$ H $= 0.00001450$

GRATING No. 4.

$\gamma = 0.000549$	$\delta = 0.003359$
$(\gamma+\delta)$ B $= 0.00002542$	$(\gamma+\delta)$ F $= 0.00001794$
$(\gamma+\delta)$ C $= 0.00002426$	$(\gamma+\delta)$ G $= 0.00001586$
$(\gamma+\delta)$ D $= 0.00002178$	$(\gamma+\delta)$ H $= 0.00001457$
$(\gamma+\delta)$ E $= 0.00001947$	

GRATING No. 5.

$\gamma = 0.003862$	$\delta = 0.002021$
$(\gamma+\delta)$ C $= 0.00002423$	$(\gamma+\delta)$ F $= 0.00001786$
$(\gamma+\delta)$ D $= 0.00002174$	$(\gamma+\delta)$ G $= 0.00001578$
$(\gamma+\delta)$ E $= 0.00001938$	$(\gamma+\delta)$ H $= 0.00001420$

GRATING No. 6.

$\gamma = 0.001036$	$\delta = 0.006759$
$(\gamma=\delta)$ C $= 0.00002422$	$(\gamma+\delta)$ F $= 0.00001785$
$(\gamma+\delta)$ D $= 0.00002175$	$(\gamma+\delta)$ G $= 0.00001591$
$(\gamma+\delta)$ E $= 0.00001942$	$(\gamma+\delta)$ H $= 0.00001451$

GRATING No. 7.

$\gamma = 0.00567$	$\delta = 0.00610$
$(\gamma+\delta)$ D $= 0.00002174$	$(\gamma+\delta)$ F $= 0.00001784$
$(\gamma+\delta)$ E $= 0.00001940$	

GRATING No. 8.

$\gamma = 0.014256$	$\delta = 0.003299$
$(\gamma+\delta)$ D $= 0.00002174$	

GRATING No. 9.

$\gamma = 0.013470$	$\delta = 0.006999$
$(\gamma+\delta)$ D $= 0.00002173$	

GRATING No. 10.

$\gamma = 0.002878$	$\delta = 0.022486$
$(\gamma+\delta)$ D $= 0.00002173$	

A very minute change in the distance apart of the wires or in the openings produces a proportionally great change in the spectra; therefore a small inequality in the distance apart of the centres of the threads will cause a marked indistinctness

in the lines of the spectra. However great may be the accuracy of construction of the gratings, it has its limits; and this is the reason why even with fine gratings certain lines in some spectra cannot be seen with such clearness as will permit their position to be accurately observed. This was the case in grating No. 1 with line B^{II} and the lines in the fifth and following spectra; in No. 2, with the line C^{I} and some others; in No. 3, with B^{I}, B^{II}, etc. The lines B and H are the hardest to see in every spectrum and with every grating; because they lie nearly at the ends of the spectrum, and the intensity of their light is small in comparison with that of the other lines of the spectrum.

The grating with which the greatest number of spectra could be measured with accuracy was No. 4. For some spectra I have used the eye-piece prism in order to determine the position of such lines as were covered by other spectra; these are C^{III}, C^{IV}, G^{III}, H^{III}, which are not visible except by means of the prism. On account of this mutual overlapping only those lines can be seen in the spectra far away from the axis which are contained in the most intense portion; this is in the neighborhood of the line E. All the spectra observed with grating No. 4 have peculiar properties; namely, the spectra near E^{VI} and E^{VII} become weaker, E^{VIII} cannot be seen, but those that follow are again visible; yet in each of these last a different color seems to prevail. If we calculate the location of L^{I} for a single opening, making $\gamma = 0.000549$ — *i. e.*, the distance apart of the wires of the grating—we find that this falls almost where E^{VIII} should be. We will see later that this is also the explanation of the phenomenon described.

With grating No. 5 the fourth spectrum is almost three times as bright as the third. The explanation of this is to be sought in the fact that with this grating L^{I} falls in the third spectrum.

With grating No. 6 E^{VII} and E^{VIII} cannot be seen. At each of the E's that follow a different color prevails, *viz.*, by E^{IX}, blue; E^{X}, bright blue; E^{XI}, green; E^{XII}, yellow; and E^{XIII}, orange. For this grating L^{I} falls in the space where E^{VII} and E^{VIII} ought to be. The colors which are prominent in E^{IX}, etc., are, with reference to their places, those which would be seen in the second spectrum of the first class if the opening were 0.006759, which is the width of the openings of grating No. 6.

With grating No. 7 the third spectrum is only half as bright as the second.

Since with grating No. 8 the first spectrum occupies a space of about two minutes, the lines cannot be seen even if the spectrum is magnified fifty times. In the third and fourth spectra D was visible, but the other lines were not seen sharply enough to allow of measurement. The fifth spectrum is almost invisible; the sixth is weak, the seventh is much less bright than the sixth. With this grating one can distinguish most plainly the spectra of the first class. In order to calculate their positions and compare them with observations, the width of the single opening must be taken as δ instead of γ with this grating, in which the thickness of the wires is less than the width of the openings; and the same is true in all other such cases. The explanation will appear from special experiments which I have made on the question.

With grating No. 9, also, the lines could not be seen in the first spectrum. The third is almost invisible—there is scarcely a faint trace of its existence; similarly with the sixth and ninth spectra. In the spaces where these spectra should fall, L^{I}, L^{II}, etc., come; but in order to calculate their positions, δ must be taken in place of γ for the width of the opening.

With grating No. 10 the lines could first be clearly seen in the fourth spectrum. The eighth spectrum is less bright than the tenth, and the ninth seems to be absent; similarly the eighteenth is apparently invisible. For this grating, also, L^{I} and L^{II} fall in the spaces where the spectra are wanting.

From the close agreement of the values of $(\gamma+\delta)$ D, etc., with different gratings, one can judge of the degree of precision, which is not unimportant. If any one has doubt as to the possibility of such accuracy with reference to the quantities γ and δ, he need only realize that the method is to measure, say, 100 turns of the screw on which the threads are wound, by means of the microscope described above, and to divide the measured distance by the number of grooves taken; by which process $\gamma+\delta$ is often measured accurately to the sixth decimal place.

The following laws are deduced at once from the experiments with various gratings:

With two different gratings constructed of wires of uniform thickness and having a constant width of opening, the size of the

spectra which arise owing to the mutual action of a great number of beams diffracted through the narrow openings, and their distances from the axis, vary inversely as the distance between the centres of two openings, or, what is the same thing, as $\gamma + \delta$.

With perfect spectra of the second class the distances [from the axis] *of rays of the same color in different spectra form an arithmetical series whose common difference is equal to the first term.*

With a grating the thickness of whose wires and the width of whose openings are expressed in fractions of a Paris inch,

$$B=\frac{0.00002541}{\gamma+\delta}, \quad C=\frac{0.00002425}{\gamma+\delta}, \quad D=\frac{0.00002175}{\gamma+\delta},$$

$$E=\frac{0.00001943}{\gamma+\delta}, \quad F=\frac{0.00001789}{\gamma+\delta}, \quad G=\frac{0.00001585}{\gamma+\delta},$$

$$H=\frac{0.00001451}{\gamma+\delta}.$$

The relative spaces occupied by the different colors in any one spectrum from a grating are remarkable. For instance, the space CD is to the space GH nearly as 2 1; while in a spectrum produced by a flint-glass prism of only 27° the ratio of these spaces is almost 1 : 2, and with a prism of water nearly as 2 : 3.

I have already called attention to the fact that in order to see the lines in the spectra of the second class, the eye-piece of the telescope must be accurately focused in such a way that, if the grating were away, the vertical opening at the heliostat would be clearly seen. A small displacement of the eye-piece makes the lines indistinct or invisible. The rays, therefore, after they are modified by the grating, diverge from a point which is at the same distance from the grating as is the opening at the heliostat.

If a grating is placed at a considerable distance in front of the objective in such a manner that the rays falling upon the telescope from the heliostat must pass through the grating, the lines of the spectra appear, with the eye-piece in the position described above, just as they would if the grating were close to the objective ; the distances of the colored beams from the axis

are, however, found to be smaller. The reason for this may be seen at once. [*Explanation omitted.*]

If light is allowed to pass through two identical gratings at the objective—*i.e.*, if two identical gratings, one behind the other, are placed in front of the telescope—the dimensions of the spectra are the same as they would be with one of the gratings. If two different gratings are used, the distances of the spectra from the axis are what they would be if the finer of the two gratings was placed by itself in front of the objective.

Mutual Action of Two, Three, Etc., Diffracted Beams.

If by means of two screens whose edges are straight and vertical the exposed surface of a grating is reduced to the interval of one opening, and if, when placed in front of the telescope, sunlight is allowed through this opening only, the spectra are the same, of course, as those obtained by means of a single narrow opening of the same width. The spectra are, therefore, of the first class, as represented by Fig. III., Table I. If the two screens before the grating are moved back so that light passes through two openings, and consequently two diffracted beams fall upon the objective, there is seen through the telescope a new sort of spectrum in the space which $L^{I}L^{I}$ occupied before, as is shown at M^{I}, M^{II}, etc., in Fig. II. I shall call these spectra "*imperfect spectra of the second class.*" With reference to the colors and their order of occurrence, this space $M^{I}M^{I}$ is similar to $L^{I}L^{I}$ in spectra of the first class; the space $M^{I}M^{II}$ is like $L^{I}L^{II}$, etc.; they are visible in that space only in which, with a single opening, $L^{I}L^{I}$ comes. Beyond this space the spectra are exactly like those given by a single narrow opening. Accordingly, if two diffracted beams fall upon the objective, one sees the imperfect spectra of the second class and the spectra of the first class at the same time. We shall see below what relation there is between the dimensions of these imperfect spectra of the second class and the width of the openings, etc. I shall designate the red ends of these spectra by M^{I}, M^{II}, M^{III}, etc.

If the two screens in front of the grating are so placed as to allow light to pass through three openings, the space $M^{I}M^{I}$, Fig. II., is divided into new spectra almost in the same manner

as the space $L^{I}L^{I}$ was before. These new spectra are similar to the others in the order of their colors, and are present only in the space $M^{I}M^{I}$. I shall call these new spectra "*spectra of the third class*," and shall designate their red ends by N^{I}, N^{II}, N^{III}, etc. Beyond the space $M^{I}M^{I}$ the spectra of the second class appear almost exactly as they did when there were two diffracted beams. However, with certain gratings their distances from the axis were changed slightly. The spectra of the first class appear also as they did when there were two diffracted beams. There are seen, then, in this case three different kinds of spectra, *viz.*, third class, imperfect second class, and first class. The experiments described below will show what relation exists between the dimensions of the spectra of the third class and the width of the openings.

With four diffracted beams there are seen spectra of the first class, the imperfect second class, and the third class; but the last are noticeably smaller than they were when there were three diffracted beams. The spectra of the second class have changed only slightly.

With five diffracted beams the spectra of the third class are smaller than with four, while those of the second class have changed only a little. With six diffracted beams the spectra of the third class are smaller than with five; with seven beams, smaller than with six, etc., until they finally become so small that they can be distinguished no longer, and there is seen only a bright, colorless line which has the same appearance exactly as the opening at the heliostat would have if the grating were away. The spectra of the second class have in the mean time, owing to the increase in the number of diffracted beams, gradually changed both in their constitution and in their distances from the axis, and have approached more nearly in every respect the perfect spectra of the second class. The lines of the spectra become visible, however, and the light becomes homogeneous, only when a great number of equally diffracted beams at equal distances apart mutually influence each other.

In the following experiments, N^{I}, N^{II}, N^{III}, etc., are, as I said above, the red ends of each spectrum of the third class; just as M^{I}, M^{II}, etc., are the red ends of each imperfect spectrum of the second class. This method of writing symbols has also been used in the spectra of the first class. What I said there also

in regard to the accuracy of the angles holds true here. The angles M^{I}, M^{II}, etc., are always the distances from the axis. [*Experiments omitted.*] By careful consideration of the degree of accuracy of the above experiments, it follows, within the limits of error, that:

Using one and the same grating, but a different number of openings, the distances of the spectra of the third class from the axis and their dimensions vary inversely as the number of diffracted beams, i.e., as the number of grating openings, beginning with three.

With different gratings and with the same number of openings, the distances of the spectra of the third class from the axis and their dimensions vary inversely as $\gamma+\delta$.

For spectra of the third class, the distances from the axis form an arithmetical series whose common difference equals the first term.

Further:

FOR THREE BEAMS.

$$N^{I}=\frac{0.0000208}{3\,(\gamma+\delta)},\quad N^{II}=2\,\frac{0.0000208}{3\,(\gamma+\delta)}.$$

FOR FOUR BEAMS.

$$N^{I}=\frac{0.0000208}{4\,(\gamma+\delta)},\quad N^{II}=2\,\frac{0.0000208}{4\,(\gamma+\delta)},\quad N^{III}=3\,\frac{0.0000208}{4\,(\gamma+\delta)}.$$

FOR FIVE BEAMS.

$$N^{I}=\frac{0.0000208}{5\,(\gamma+\delta)},\quad N^{II}=2\,\frac{0.0000208}{5\,(\gamma+\delta)},\quad N^{III}=3\,\frac{0.0000208}{5\,(\gamma+\delta)}.$$

The space KM^{I}, Fig. II., Table I., which is seen when there is mutual action of two diffracted beams, contains the spectra of the third class if there are three beams, as I have stated above; therefore, with three, four, etc., beams, M^{I} can no longer be seen; and in these cases M^{II} is the red end of the first spectrum of the second class, a fact which must not be overlooked when comparing perfect and imperfect spectra. Since the spectra of the third class which are contained in the space KM^{I} become smaller as the number of beams which mutually influence each other increases, and since the spectra be-

come ultimately so small that they cannot be distinguished, thus producing in K only a bright, colorless line, the space between K and M^{I} must be without light; because, with an increase in the number of beams, the spectra of the second class do not change their position and magnitude, comparatively.

With grating No. 9, M^{IV} is invisible because it falls nearly at L^{I}. Since in this grating γ is greater than δ, the latter must be used instead of the former in calculating L^{I}. Why this is so will appear at once. In order to produce spectra of the first class, one must have two edges lying close together, by which the light is diffracted. It is not necessary that these two edges should face each other; they can also be turned reversed, provided only that they lie near enough, as is the case with a narrow metal strip, a thread, or a wire. In these cases, however, the spectra of the first class cannot be seen plainly, because they occur at the same places where non-diffracted white light comes. By means of a telescope, however, one can easily convince himself of their existence. For this purpose I stretched a wire 0.02287 inch thick across the middle of an opening in a screen, which was $\frac{1}{4}$ inch wide, and placed it in front of a telescope, with the thread vertical. The spectra of the first class which must arise, owing to the width of the opening in the screen—$\gamma = \frac{1}{4}$ inch—would be too small to notice, and the opening at the heliostat must appear through the telescope almost as if the screen were away; if, however, the thread stretched across the opening produces spectra, these must be seen on either side of the opening at the heliostat; this is actually the case. Owing to the width of the opening at the heliostat, it is seen so brightly that the eye can scarcely bear it; and on each side are seen spectra of the first class. On account of the intensity of light in the middle, L^{I} could not be measured; but the following two could be. I found $L^{II} = 6' 16''$ and $L^{III} = 9' 30''$. If we assume the above thickness of the wire as the value of γ, in order to calculate L^{II} and L^{III}, angles are obtained which agree as closely with those observed as could be expected under the circumstances.* In order, therefore, to produce spectra of the first class, it is not necessary that the edges which diffract the light should face each other; they

* These experiments deserve to be continued, because, in some cases, there are discrepancies which exceed the limits of probable error.

can be reversed, as is the case with a wire. With grating No. 9 the edges of the wires are nearer than those of the intervening spaces; therefore, the former produce the spectra of the first class.

The imperfect spectra of the second class are often modified by the position of spectra of the first class; and these modifications depend, besides, upon the increase or decrease in the number of the beams which mutually interfere. The exact law of these small changes cannot be deduced from the experiments so far made, and other experiments are therefore necessary. This much, however, can be seen from the above, that with different gratings the distances of the imperfect spectra of the second class, from the axis, and their dimensions, are nearly inversely as $\gamma+\delta$; further, that with two beams M^{I} is considerably smaller than the differences $M^{II}-M^{I}$, $M^{III}-M^{II}$, etc.; and therefore, with reference to the succession of the distances of the colored beams from the axis, these spectra are markedly different both from those which arise from a round opening and from those which come from a long, narrow opening. With gratings where δ is greater than γ, it is not difficult to deduce a law for the imperfect spectra of the second class—for instance, with gratings No. 6 and No. 10.

The angles L^{I}, L^{II}, etc., cannot be determined with great accuracy when there are two, three, etc., diffracted beams; therefore no certain conclusion can be drawn from the variations in these angles with three, four, etc., diffracted beams in the case of grating No. 10. One reason lies in the fact that it is almost impossible to make three or four intervening openings exactly equal, although the centres of the wires may be at equal distances from each other. We know from the experiments with one diffracted beam how great the variation in the distances of the spectra is, if, with an opening already small, the width changes even slightly; therefore with three diffracted beams the distances of the spectra of the first class from the axis may easily be found to be different from what they are with two beams, etc. This inequality [*of the spaces*] has much less influence upon spectra of the second and third class.

Mutual Action of Beams Diffracted in Water and other Refracting Media.

[*Experiments omitted.*]

The conclusion is: *In different refracting media, with identical gratings, the sines of the angles of the diffracted beams vary inversely as the indices of refraction.*

Mutual Action of Beams Diffracted by Reflection.

[*Experiments omitted.*]

Mutual Action of Beams Diffracted through Round and Quadrangular Openings.

[*Experiments omitted.*]

When seen under a microscope the barbs of most birds' feathers contain regularly arranged fine openings. If one looks through the barbs at a brilliantly illuminated point which is not too near, he can see, even with the naked eye, colored spectra, which have a definite position. If such a feather is placed in front of a telescope and light coming from a round opening near a heliostat is allowed to pass through it, spectra of the first and second class are seen. The spectra which were seen with the naked eye were those of the first class, which are very large but of feeble intensity; and therefore, when seen through the telescope, which magnifies greatly, they may be easily overlooked, if attention is not paid to their distance from the axis.

With some gratings made of parallel wires, one sees through the telescope, beyond the space which is occupied by the length of the spectra—that is, in the dark portion of the field—what are apparently the wires of the grating itself, which is, however, impossible, if one considers the path of the light; one might perhaps believe that this light is due to interval reflections at the surfaces of the objective. But this is not the case; for the eye-piece can be pushed in or drawn out an inch, and the lines still remain visible. These lines have also a definite color; namely, one is always a reddish yellow, the second is blueish green, the third is again reddish yellow, etc. I shall return to this subject at another opportunity.

The phenomenon which is seen through a telescope when light is allowed through an opening of the shape of an equilateral triangle placed in front of the objective is also interesting.

It may be surprising that so many phenomena could up to this time have escaped the notice of investigators; and that, for instance, the simple law, that in diffraction through a single opening the inflection of the rays varies inversely as the width of the opening, was not only not found, but that results were obtained differing widely from this. The explanation lies in the mode of observation. One would fall into similar errors if, for instance, he tried to determine the path of colored light through glass lenses by intercepting and measuring this refracted light at different distances. This mode of observation is to blame for the fact that the phenomena of the mutual influence of beams of light have escaped the attention of investigators,* phenomena which must be known before the laws of diffraction can be learned. For if light coming through a grating is intercepted by a white surface or a piece of ground-glass, one cannot see, even on a small scale, what is observed through a telescope when used with the grating; and in general nothing is seen.

It is remarkable that the observed laws for the mutual action and diffraction of beams can be deduced from the principles of wave-motion; that, from a knowledge merely of the angle of diffraction of the light and of the distances at which the beams interfere, the magnitude of a vibration of light, for every color, can be expressed by means of an extremely simple equation, and that these determinations, under the most varying conditions, agree to a high degree of accuracy; further, that these same principles furnish an explanation of the cause of the origin of the lines and bands which are seen in a prismatic spectrum, etc. On another occasion I shall make known the theory of interference and diffraction of beams of light.

The phenomena of interference and diffraction of light are,

* T. Young had already observed that the colored fringes which are seen in the interior of the shadow of a hair vanish if one edge is covered; so that the beams of light going by both edges must combine to produce the interior color-bands.

as we see from the laws deduced, infinite in number; and all that was previously known amounts to only a few special cases. The theory will also make us acquainted with these phenomena which can be subjected to no new investigation following the path which I have taken.*

I cannot repeat too often that all apparatus to be used in these experiments must be perfect to a high degree; and one can judge of the degree of this perfection from the ratio of the dimensions of the grating to the dimensions of the spectra, etc. An inequality or imperfection, apparently unimportant, can produce a great indistinctness or even the entire vanishing of the phenomena; therefore, one must consider well everything that can be of injurious influence. More than with all other optical phenomena, one must be on one's guard in these experiments against optical illusions.

It will be reward enough for me if, by the publication of the present experiments, I have directed the attention of investigators to this subject, which still promises much for physical optics and appears to open a new field.

* Among these are the color-fringes which are seen in the shadow of a single edge of a body; also the phenomena which Herr Hofrath Meyer observed long ago and described in the *Göttinger Commentaren*, Vol. IV., p. 49.

Short Account of the Results of New Experiments on the Laws of Light, and their Theory.

Gilbert's *Annalen der Physik*, Band 74, p. 337–378. *Edinburgh Journal of Science*, VII., VIII., 1827, 1828.

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Short Account of the Results of New Experiments on the Laws of Light, and their Theory.

Read in the Mathematical-Physical Section of the Academy, June 14, 1823.

I published a year ago, in a memoir which is printed in the eighth volume of the *Denkschriften* of the Royal Bavarian Academy of Sciences, an account of some new experiments concerning *diffraction* of light and those phenomena which occur owing to the mutual action of diffracted beams of light; and the *laws* which can be deduced from these experiments were also developed. Guided by these deductions, I have continued these experiments since then; and what follows is an account of those of my results which are suited for communication in a brief description. I must, however, assume as known the larger portion of what the above-mentioned memoir contains on this subject.

[*Résumé of previous papers is omitted.*]

In this memoir I shall designate by B, C, . . . H *colored rays* of different kinds: B is a red ray which lies near the end of the spectrum; C is deeper in the red; D is orange-colored; E, green; F, blue; G, indigo; H, violet. In every spectrum of sunlight which consists of *perfectly homogeneous* light there are found in the places named fixed lines or streaks which are distinguished from the other countless lines of the spectrum either by their intensity or by their position.*

The angles from the axis through which the rays B, C, etc., are deflected by the grating I designate by B^{I}, C^{I}, etc., in the *first* spectrum, which is nearest the axis; by B^{II}, C^{II}, etc., in the *second;* by B^{III}, C^{III}, etc., in the *third* from the axis, etc. From the experiments which are described in detail in my

* [*A long note giving the conditions for securing a pure prismatic spectrum is omitted.*]

second memoir, it follows that, with all gratings, if γ denotes the width of a single grating space, and δ the width of a single wire, expressed in fractions of a Paris inch, the arcs of these angles are as follows, the radius being taken equal to 1:

$$B^{I}=\frac{0.00002541}{\gamma+\delta}, \quad C^{I}=\frac{0.00002425}{\gamma+\delta}, \quad D^{I}=\frac{0.00002175}{\gamma+\delta},$$

$$E^{I}=\frac{0.00001943'}{\gamma+\delta}, \quad F^{I}=\frac{0.00001789}{\gamma+\delta}, \quad G^{I}=\frac{0.00001585}{\gamma+\delta},$$

$$H^{I}=\frac{0.00001451}{\gamma+\delta}.$$

Further, that

$$B^{II}=2\,B^{I},\ C^{II}=2\,C^{I},\ \text{etc.}$$
$$B^{III}=3\,B^{I},\ C^{III}=3\,C^{I},\ \text{etc.}$$
$$\text{Etc.}$$

The numerator in these general expressions is a number which is absolutely constant for each definite colored ray, but different for different rays, however varied the cases are, and which refers, as is easily seen, to a definite absolute scale; in this case to the Paris inch. If for each colored ray this number is designated by ω, and if the angle of deflection of one and the same ray in the first spectrum is designated by ϑ^{I}, in the second by ϑ^{II}, in the third by ϑ^{III}, etc., then,

$$\vartheta^{I}=\frac{\omega}{\gamma+\delta}; \quad \vartheta^{II}=\frac{2\,\omega}{\gamma+\delta}, \text{ etc.}$$

And if, further, ν denotes the number which gives the order of the spectrum (for the axis $\nu=0$; for the first spectrum $\nu=1$; for the second, $\nu=2$, etc.; and it can never be a fraction); and if, for the sake of brevity, we call the sum of the width of one grating opening and of one line of the grating, or $\gamma+\delta$, equal to ε, we have as a general formula

$$\text{(I.)} \qquad \vartheta^{(\nu)}=\frac{\nu\omega}{\varepsilon}.$$

According to the results of the previous experiments (and as is shown by the general formula (I.) which is derived from them), the angles of deflection of the same colored beams in

the series of spectra formed by the grating are in the ratio of the numbers 0, 1, 2, 3, etc. The experiments from which these results are deduced gave, however, such small angles that for them the sine, tangent, and arc do not sensibly differ. With my finest grating, where $\varepsilon = 0.001952$ inch, D^1 was equal to 38′ 19″.3. If the angles were larger, that is, if the gratings were many times finer, one might think it probable, from certain considerations, that the arcs themselves do not have the relations given above, but that some trigonometrical function of them does. In accordance with the theory which will be discussed later, the *sines* of the angles should have the given relation if the light falls normally upon the grating. Partly in order to verify this directly by experiment, partly because the laws of this modification of light can be deduced with more accuracy from larger spectra, and conclusions as to the theory of these phenomena can be drawn with more certainty, it was greatly to be desired, it seemed to me, that gratings should be made much finer than those which I used in my earlier experiments. The distances apart of the lines or wires of such fine gratings must, however, be exactly equal to such a high degree that the fixed lines of the spectra can be seen ; otherwise the distances of the colors from the axis cannot be measured.

It will not be considered possible by any one who is acquainted with the difficulties of the work to prepare a screw much finer than that which I made for my earlier experiments. With a special apparatus I succeeded in scratching parallel lines upon a piece of plane glass, covered with gold-foil, which were at such distances from each other that ε was equal to 0.00114 inch. If one tries to scratch lines closer together than this, no gold is left on the glass, and there are, consequently, no apertures. The spectra produced by such a grating, where $\varepsilon = 0.00114$, are much larger than those obtained before, and the fixed lines were plainly seen in them ; but the results obtained were still far from being able to decide the question at issue.

It is entirely immaterial in a grating which is to be used for these experiments whether the threads out of which it consists are opaque, translucent, or transparent. A grating of glass fibres, for instance, produces these phenomena as well as one made out of metal wires. I covered, therefore, one side of a piece of plane glass with a layer of grease so thin that it

could scarcely be recognized by the eye alone; in this grease I scratched parallel lines which were only half as far apart as the finest of those ruled on the gold-foil. With this grating I obtained spectra in which the fixed lines could be plainly seen, and which were therefore suited for measurements of the distances from the axis. In no layer of grease or varnish is it possible to rule finer parallel lines than these, which shall be at equal intervals.

It was only by means of the diamond that I succeeded in producing finer gratings. A machine specially made for the purpose enabled me, by using a diamond-point, to rule lines in the surface of a piece of plane glass which were almost perfectly parallel. If one is so fortunate as to find a good diamond-point, lines can be ruled with this machine which are so close together that they cannot be distinguished by the strongest compound microscope. It is, however, not enough to be able to rule in a given space a great number of lines which shall still be separate from each other; these lines must be ruled so accurately at equal distances apart that the variation shall not amount at the most to the one-hundredth of this distance. By means of my machine I have obtained a grating in which $\epsilon=0.0001223$ inch, and whose lines are so evenly spaced that the fixed lines of the first and second spectra obtained with it can be plainly seen.*

* By means of my machine one can rule parallel lines which are separated by spaces as wide as themselves, and which are so close together that 3200 of them lie in a Paris inch; but I have not yet succeeded in making their distances from each other so exactly equal that in this distance, which is 0.00003125 inch, errors of one part in a hundred—*i.e.*, of 0.00000031—are not present; and perhaps this would not be possible for the hand of man, whatever machine is used. Since no use can be made of 100 or 200 parallel lines, and since with such fine gratings there must be several thousand lines in order to produce intense perfect spectra, it is evident that it was largely a question of fortune, with $\epsilon=0.0001223$, to find a diamond-point which could rule several thousand such fine lines without changing. So far, I have succeeded in making only one grating as fine as this. If the diamond-point changes during the ruling, the previous work is useless. Without one's intending it, the point often makes stronger or weaker lines. Even with the most powerful microscope one cannot be sure whether the point is suited to rule proper lines. A diamond which appears less pointed than another often rules the finer lines; and therefore a suitable point can be found by experiment only. What renders the matter more difficult is that a slight change in the inclination or in the position of the diamond,

By means of this grating spectra are obtained which are as long as those obtained from large prisms; and even in the first spectrum the D line (in the orange) can be seen practically double, so that the space between can be measured. And since with this grating D^{I}, for instance, equals 10° 14′, the law of modification of the light produced by it can be deduced with great accuracy.

When the light fell normally upon the grating in which $\varepsilon = 0.0001223$, I obtained

$C^{I} = 11° 25' 20''$	$F^{I} = 8° 26' 6''$
$C^{II} = 23° 19' 42''$	$F^{II} = 17° 3' 34''$
$D^{I} = 10° 14' 31''$	$G^{I} = 7° 27' 19''$
$D^{II} = 20° 49' 44''$	$G^{II} = 15° 3' 9''$
$E^{I} = 9° 9'$	$H^{I} = 6° 52' 36''$
$E^{II} = 18° 32' 34''$	

The angles are so large that the arcs, sines, and tangents are sensibly different. Since the instrument with which the angles are measured gives, without repeating the observation, readings accurate to 4″, one can easily decide how trustworthy the readings are.

The third spectrum, the fourth, and the following were seen satisfactorily in these experiments with this grating; but the fixed lines in the various colors could not be recognized with sufficient precision to have their distances from the axis measured as accurately as in the first and second.*

with reference to the plane of the glass often seriously influences the strength of the lines. Since every line must be drawn separately and with the greatest care, one can easily understand how much time and patience is required in order to rule a few thousand lines with sufficient accuracy.

* However great the accuracy of the ruling with reference to the distances apart of the lines of the grating, it naturally has its limits, and perfect accuracy cannot be expected; a statement which is true of everything made by the hand of man. The amount of the displacement or blurring of the fixed lines of the different spectra which would be produced by a small inequality in the intervals of the parallel lines of a grating, can be deduced from the equation (I.) $\vartheta^{(\nu)} = \frac{\nu\omega}{\varepsilon}$. For it follows from it that $d\vartheta^{(\nu)} = -d\varepsilon\frac{\nu\omega}{\varepsilon^2}$; consequently an equality which amounts to only one-hundredth of the very small distance ε (*i.e.*, $d\varepsilon = 0.00000122$ inches) will pro-

The measurements given for the first and second spectras are sufficient, however, to allow one to decide upon the law of this modification.

Since the distance between the parallel lines, ruled on the *glass grating*, as I shall call it, must be known most accurately, and since these lines are seen with great difficulty even with the strongest microscope, and in any case cannot be counted, I try to rule the first and last line of the grating somewhat stronger than the others, and I measure their distance apart by means of a microscopic apparatus prepared for the purpose. The ruling-machine counts the lines as they are ruled ; and so I know how many there are in the ruled space ; *e.g.*, in the grating just discussed there are 3601 lines. Therefore, from a knowledge of the distance of the first line from the last, the distance between the centres of two—that is, ε—can be determined to a high degree of accuracy. The ratio, too, of the thickness of a scratch to the width of the opening between two lines can be very closely determined. If the lines were so wide that one touched the other, and consequently there was no aperture between, no light could be regularly reflected from the ruled surface ; it would be scattered as from a ground surface. If the apertures were as wide as the scratches, the ruled surface could reflect only half as much light as an unruled surface of the same size. The ratio, therefore, of the light regularly reflected from the ruled glass surface to that which is reflected by an unruled glass surface of the same size is the same as that of the width of the aperture between two scratches to the width of the lines [*i.e.*, *to* ε].* It need scarcely be noted that in the experiments made in this

duce a displacement of the fixed line D, in the first spectrum, of 6′ 5″; in the second, of 12′ 10″; in the third, of 18′ 15″, etc. This is the reason why the fixed lines may be well defined in the first and second spectra, and yet not be so in the third, fourth, etc. Whoever has to do with small magnitudes, not merely in numbers, but in experiments, will know how to estimate this accuracy, and will have an idea as to the difficulty with which it is obtained.

* Even if the amount of the reflected light could be accurately measured, this conclusion must be regarded as only approximately true for reasons to be discussed later. With gratings for which ε is still smaller than this it would be less true. In the experiments described in this paper the determination of this ratio is of no importance whatever ; the magnitude of ε is alone of interest.

way care must be taken to have no light reflected from the second surface of the glass.

Using another grating, where ε was equal to 0.0005919 inch, I obtained, with light at normal incidence,

$C^{I} = 2° 20' 57''$	$E^{V} = 9° 28' 3''$
$D^{I} = 2° 6' 30''$	$F^{I} = 1° 44' 19''$
$D^{II} = 4° 13' 7''$	$F^{II} = 3° 28' 45''$
$D^{III} = 6° 20' 7''$	$F^{III} = 5° 13' 23''$
$D^{IV} = 8° 27' 43''$	$F^{IV} = 6° 58' 18''$
$D^{V} = 10° 35' 53''$	$G^{I} = 1° 32' 22''$
$E^{I} = 1° 53' 7''$	$G^{II} = 3° 4' 57''$
$E^{II} = 3° 46' 17''$	$G^{III} = 4° 37' 30''$
$E^{III} = 5° 39' 50''$	$H^{I} = 1° 25'$
$E^{IV} = 7° 33' 41''$	$H^{II} = 2° 50' 11''$

All the observations, with both glass gratings, can be expressed very approximately by the equation

$$\text{(II.)} \quad \sin \vartheta(\gamma) = \frac{\nu\omega}{\varepsilon}.$$

If the light is incident *normally*, therefore, the *sines* of the distances of a colored ray from the axis in the different consecutive spectra are in the ratio of the numbers 0, 1, 2, 3, 4, etc.*

The glass grating, $\varepsilon = 0.0005919$, has the remarkable property that its spectra on one side the axis are more than twice as intense as those on the other. The scratches of this grating are visible under a microscope, but it is impossible to distinguish

* If it were not the sines, but the angles, of deflection of a colored ray in the different spectra which had the given ratio, then with the finer grating, where $D^{I} = 10° 14' 31''$, D^{II} should equal $20° 29' 2''$; according to experiment, however, $D^{II} = 20° 49' 44''$—that is, is 20 minutes greater. Consequently, the sines of the angles have the given relation. Calculation leads to a difference in the seconds, which, although small, is larger than can be ascribed to an error of observation. Whether this difference is to be ascribed to a slight imperfection in the grating, or whether it lies in the nature of the phenomena, can be decided only by a series of experiments with different fine gratings. I give in the table the angles exactly as they were observed, not allowing myself any correction whatever. I determined these angles very often, and always by repeating six times; yet I obtained angles for the bright colors which agree nearly always to within one second; notwithstanding which, small constant errors could cause the differences in the seconds which are mentioned above.

in them any special form. I sought the explanation in the idea that, on ruling the lines, the diamond-point had such a position with reference to the plane glass that one edge of each scratch was sharp, the other not so well defined; and I think that this hypothesis is confirmed by the following experiment: On a piece of plane glass, over which there was a thin layer of grease, I ruled parallel lines in such a way that with every line the grease on one side was sharply defined, on the other less so; and I obtained phenomena, in fact, through this grating which were like those seen through the glass grating mentioned.

If the incident light does not fall perpendicularly upon the grating, but is in the plane perpendicular to the lines of the grating, and *inclined* to the normal to the grating, the action is as if for this incident light the distance apart of the parallel lines—that is, ϵ—was smaller than when the incidence was normal, in the ratio of the radius to the cosine of the angle of incidence. The distances of the spectra from the axis (ϑ) must therefore become greater as the angle of incidence increases; because (as equation (II.) shows) the sines of this distance increase in the same ratio as ϵ decreases. If we designate, therefore, by σ the *angle of incidence—i.e.*, the angle between the incident beam and the normal to the grating—we might think we could safely conclude from equation (II.)—*i.e.*, $\sin \vartheta^{(\nu)} = \frac{\nu\omega}{\epsilon}$—that the following is true: $\sin (\vartheta)^{\nu} = \frac{\nu\omega}{\epsilon . \cos \sigma}$. But from the theory of these phenomena, which will be discussed later, it may be predicted that in this case the spectra on the two sides of the axis will be no longer symmetrical; that therefore, *e.g.*, D^1 must be larger on one side of the axis than upon the other. This is confirmed by experiment, as appears below. With gratings in which ϵ is not very small, the difference is not marked, it is true;* it is, however, exceptionally large with the grating in which $\epsilon = 0.0001223$ inch. For instance, with this, for $\sigma = 55°$, D^1 on the one side of the axis equals $15° 6'$, and on the other, $D^1 = 30° 33'$.

* In fact, with coarse gratings, if σ is not very large, one can use the expression $\sin \vartheta^{(\nu)} = \frac{\nu\omega}{\epsilon . \cos \sigma}$ with sufficient accuracy, as I did in my memoir *A New Modification*, etc. We shall, however, find below an exact expression which is equally simple.

If in the theory of spectra of the first class the non-symmetrical ones, which consist of non-homogeneous light, and which contain no fixed lines,* are of importance, the *non-symmetrical spectra of the second class* are of immensely greater theoretical interest, because they show the fixed lines; and therefore the laws of this modification of light can be exactly deduced, and the theory of these phenomena can be most closely verified.† I give here the results of numerous experiments on the colored rays D and F. In order to indicate on which side of the axis ϑ lies—the angle measuring the distance of the colored ray from the axis—I mark with $-$I, $-$II, . . ., those spectra which lie upon that side of the axis on which the incident light is inclined towards the face of the grating; and with $+$I, $+$II, . . ., the spectra on the opposite side, where the inclined incident beam makes obtuse angles with the grating, so that D^{-I}, D^{+I} or D^{-II}, D^{+II}, . . ., mean the opposite positions of D in the first or second spectrum, etc. By σ is to be understood the angle of incidence in the sense described above—*i.e.*, the angle included between the incident ray and the normal to the grating.

EXPERIMENTS WITH THE GLASS GRATING $\varepsilon=0.0001223$.

$\sigma=55°$

$D^{-I}=15°\ 6'\ 36''$	$F^{-I}=12°\ 44'\ 40''$
$D^{+I}=30°\ 33'\ 10''$	$F^{+I}=19°\ 58'\ 54''$
$D^{-II}=27°\ 23'\ 18''$	$F^{-II}=23°\ 16'\ 50''$

$\sigma=50°$

$D^{-I}=13°\ 58'\ 12''$	$F^{-I}=11°\ 43'\ 53''$
$D^{+I}=20°\ 42'\ 51''$	$F^{+I}=15°\ 53'\ 10''$
$D^{-II}=25°\ 46'\ 20''$	$F^{-II}=21°\ 47'\ 36''$

$\sigma=45°$

$D^{-I}=13°\ 2'\ 37''$	$F^{-I}=10°\ 54'\ 55''$
$D^{+I}=17°\ 14'\ 14''$	$F^{+I}=13°\ 37'\ 38''$
$D^{-II}=24°\ 25'\ 30''$	$F^{-II}=20°\ 33'\ 39''$

* *New Modification of Light*, etc.

† A simple hypothesis can be easily devised to satisfy all simple laws. For instance, the laws of refraction and reflection can be deduced from a great number of hypotheses. It is, therefore, good fortune if a new law is found which is apparently very complicated, because it has at least the advantage of limiting more closely the field of the hypotheses concerning light.

$\sigma = 40°$

$D^{-I} = 12° 17' 34''$	$F^{-I} = 10° 15' 29''$
$D^{+I} = 15° 8' 52''$	$F^{+I} = 12° 8' 12''$
$D^{-II} = 23° 18' 54''$	$F^{-II} = 19° 32' 57''$

$\sigma = 30°$

$D^{-I} = 11° 12' 22''$	$F^{-I} = 9° 18' 36''$
$D^{+I} = 12° 40' 30''$	$F^{+I} = 10° 17' 34''$
$D^{-II} = 21° 42' 5''$	$F^{-II} = 18° 4' 35''$
$D^{+II} = 28° 50' 5''$	$F^{+II} = 22° 30' 10''$

$\sigma = 20°$

$D^{-I} = 10° 33' 2''$	$F^{-I} = 8° 44' 10''$
$D^{+I} = 11° 19' 23''$	$F^{+I} = 9° 15' 22''$
$D^{-II} = 20° 46' 54''$	$F^{-II} = 17° 12' 45''$
$D^{+II} = 24° 14' 30''$	$F^{+II} = 19° 27'$

In these experiments, when measuring the first angle, the instrument could not be used in such a way as to allow repetition. These first angles are, therefore, in the seconds, somewhat less exact than when the light has normal incidence. Since, however, the differences between the two sides of the axis amount to several degrees, a few seconds do not affect the matter.

In all the experiments of this kind D^{-I} and D^{+I} are different, as is seen; similarly with F^{-I} and F^{+I}; and the same was true of the other colored rays, for which I have not given the observations. If the light is not incident normally, therefore, the spectra produced by the grating are no longer symmetrical on the two sides of the axis; and the difference in their positions is so marked when the angle of incidence is great that it could be detected even if it were only one-hundredth part as much. It would be, however, extremely difficult for the most clear-sighted physicist to deduce a *law* for these phenomena directly from the experimental results.

Let τ denote the angle made with the plane of the grating by a colored beam after diffraction (with normally incident light, therefore the complement of ϑ), and y a straight line drawn perpendicular to the plane of the grating from the micrometer-threads of the observing-telescope.* All of my observations can

* If a is the distance from the grating to the micrometer-thread (and therefore to the place where the image of the phenomenon is formed), $y = a \sin \tau$. [*a is measured along the beam of light.*]

then be expressed, with the accuracy obtainable in experiments of this kind, by the following formula:

$$\text{(III.)}\ \operatorname{tang} \tau(\pm\nu) = \frac{\sqrt{[\varepsilon^2 - (\varepsilon.\sin\sigma \pm \nu\omega)^2].[4y^2 + \varepsilon^2 - (\varepsilon.\sin\sigma \pm \nu\omega)^2]}}{2y\,(\varepsilon.\sin\sigma \pm \nu\omega)}$$

I have deduced this equation, without any approximation, from the principles of *Interference* which were proposed in 1802 by Dr. Thomas Young, and afterwards fully justified by the painstaking labors of Arago and Fresnel. In this formula ω denotes the *length of a light-wave.* Although this quantity is extremely small, we can deduce it with a high degree of accuracy from the experiments which are described in my memoir, *New Modification of Light*, etc.; and the results of which for the different colored rays are given in general formulæ on page 30. From the experiments with glass gratings we learn this quantity so exactly that, for the bright colors, hardly one-thousandth portion of ω can be uncertain. From the experiments with the finer gratings we obtain, by means of the angles for the first spectrum with normal incidence of the light, if (Cω) denotes the length of a light-wave for the ray C, (Dω) for the ray D, etc.,

$$\left.\begin{array}{l} (\mathrm{C}\omega) = 0.00002422 \\ (\mathrm{D}\omega) = 0.00002175 \\ (\mathrm{E}\omega) = 0.00001945 \\ (\mathrm{F}\omega) = 0.00001794 \\ (\mathrm{G}\omega) = 0.00001587 \\ (\mathrm{H}\omega) = 0.00001464 \end{array}\right\} \text{in fractions of a Paris inch.*}$$

[*Reduced to centimetres, this gives for* D *the wave-length* 0.00005888 *cm.* 1 *Paris inch*=2.70700 *cm.*]

* The fixed line B, near the red end, could not be seen well enough, owing to the great width of the image, to allow its position to be determined accurately. I shall try to make a greater number of fine gratings, in order to determine more accurately, where it is possible, the value of ω for the different colored rays; yet this value is now known sufficiently closely to decide whether the theory is confirmed by the experiments. If the values of (Dω), (Eω), etc., are deduced from the experiments with the coarse glass gratings, they are found to be somewhat greater in the eighth decimal place than those given by the finer gratings; this small difference simply points to a less exact determination of ε, which is more accurately known

It will strike one from equation (III.) that the value of τ depends somewhat upon y—*i.e.*, upon the distance at which the image of this phenomenon is formed; that therefore, according to the principles of interference, this light thus modified does not proceed exactly in a straight line, but in a curved one. The equation of this curved line, developed without any approximation, is

$$(\text{IV.}) \quad x^2[4\varepsilon^2-4(\varepsilon.\sin\sigma\pm\nu\omega)^2]=4y^2(\varepsilon.\sin\sigma\pm\nu\omega)^2 + [\varepsilon^2-(\varepsilon.\sin\sigma\pm\nu\omega)^2](\varepsilon.\sin\sigma\pm\nu\omega).*$$

Since these phenomena can be observed only with a telescope, which must have great power if the distances are to be measured with some accuracy, and therefore cannot be very short, y is always large in comparison with ω and ε. With my instrument, the distance of the micrometer-threads from the grating is 21.43 inches. It would be only when y is not very large in comparison with ω that the value of τ could vary with that of y; but if in equation (III.) y is put one-half as great as it actually was in my experiments, and then twice or three times as great, the same value of τ is always obtained.† As the equation shows, the value of $\varepsilon^2-(\varepsilon.\sin\sigma\pm\nu\omega)^2$ vanishes, then, in com-

with the finer gratings. I do not need to recall the fact that the accuracy of the determination of the value of ε, however great it is by the help of the means which I use, has its limits. I hope to increase it still more in the future.

* Equations (III.) and (IV.) are developed for the case when the incident rays may be regarded as parallel. If the distance of the source of light were not great in comparison with ε, instead of $\varepsilon.\sin\sigma$ in these equations one would have to write $\frac{\varepsilon.\sin(\sigma+\frac{1}{2}\beta)}{\cos\frac{1}{2}\beta}$, where β is defined by the equation $\sin\beta=\frac{\varepsilon.\cos\sigma}{a}$. In this last equation a denotes the distance of the source of light from the grating.

† I measured these angles also with a telescope of 4-inch focal length; but, as was to be expected from the equation, no other changes were found than those which must be ascribed to the diminished power; they were sometimes positive, sometimes negative. If a curvature of the modified light were noticeable, it might be in the case when the distance of the source of light is not so great; but in that case the experiments are subject to many difficulties, and demand a different apparatus, which must be perfect to a high degree.

parison with $4y^2$; and so it can be neglected. We therefore have, approximately,

$$\text{tang } \tau^{(\pm\nu)} = \frac{\sqrt{[\varepsilon^2 - (\varepsilon . \sin\sigma \pm \nu\omega)^2]}}{\varepsilon . \sin\sigma \pm \nu\omega},$$

or (V.) $$\cos \tau^{(\pm\nu)} = \frac{\varepsilon . \sin\sigma \pm \nu\omega}{\varepsilon}.$$

This equation represents the experiments on page 49, with unsymmetrical spectra, as closely as does equation (III.). In both equations the sign + gives the position of the colored rays on one side of the axis in the different spectra; the − sign, those on the opposite side. In making comparisons, it must not be forgotten that in the experiments the angular distances from the axis are measured, but that τ expresses the angles measured from the plane of the grating. It is scarcely necessary to remark that in the special cases when, for instance, the above equation is used for the ray C, instead of ω, (Cω) is to be introduced; for the ray D, (Dω), etc. In these cases I designate for the ray C the corresponding τ by (Cτ); that for D, by (Dτ), etc. Equation (V.) becomes, accordingly, in these cases,

$$\cos (C\tau)^{(\pm\nu)} = \frac{\varepsilon . \sin\sigma \pm \nu(C\omega)}{\varepsilon};$$

$$\cos (D\tau)^{(\pm\nu)} = \frac{\varepsilon . \sin\sigma \pm \nu(D\omega)}{\varepsilon}, \text{ etc.}$$

If the incidence of the rays upon the grating is normal, $\sin\sigma = 0$, and equation (V.) becomes

$$\cos \tau^{(\pm\nu)} = \frac{\pm\nu\omega}{\varepsilon}.$$

Since in this case $\cos\tau = \sin\vartheta$, this is the equation (II.) deduced directly from the experiments on page 47; therefore those experiments also confirm the theory.

In any *other medium* than air I denote the index of refraction, *e.g.*, of the ray C by (Cn), of the ray D by (Dn), etc. It follows at once from the experiments on light diffracted in different media* that in the above equation (Cω) is to be replaced by $\frac{(C\omega)}{(Cn)}$; Dω, by $\frac{(D\omega)}{(Dn)}$, etc.

* *New Modification of Light*, etc., p. 36.

If the unruled surface of a glass grating is covered with *black resin varnish,* which has nearly the same refracting power as the glass, then, as is well known, no light will be reflected from this side into the interior of the glass, and only the ruled surface of the glass reflects light. If light from an apparently very narrow opening in the window-shutter is incident upon the grating, and if, in the manner described, the light *reflected from the ruled surface* reaches the objective of the telescope, the phenomena are exactly the same as if the light, at the same inclination, had been transmitted *through* the grating—*viz.*, unsymmetrical spectra of the *second* class are seen. The intensity of these spectra is also great enough to allow the distances of the different fixed lines to be determined with considerable accuracy. I have made a great many experiments on these spectra arising from *reflection* at different angles of incidence, the details of which I pass over. Equation (V.) represents all the results as satisfactorily as was to be expected. By the development, from the principles of interference, of the formula for the phenomena due to reflection, the same expression is obtained as for transmitted light—*viz.*, without approximations, equation (III.). In this way, also, the theory is confirmed by experiment.

It is most remarkable that for a certain angle of incidence a part of one spectrum arising from reflection consists of *completely polarized light.* This angle of incidence is very different for the different spectra, and, still more notably, even for the different colors of one and the same spectrum. With the glass grating $\varepsilon = 0.0001223$, $(E\tau)^{+I}$ is polarized—*i. e.*, the *green* part of the first spectrum, if $\sigma = 49°$; $(E\tau)^{(+II)}$, or the green part of the second spectrum on the same side, if $\sigma = 40°$; finally, $(E\tau)^{(-I)}$, or the green part of the first spectrum on the opposite side of the axis, if $\sigma = 69°$. If $(E\tau)^{(+I)}$ is completely polarized, the other colors of this spectrum are not completely so. With $(E\tau)^{(+II)}$ this is less the case ; and σ can be sensibly changed while this color remains polarized. For no angle of incidence is $(E\tau)^{-I}$ as completely polarized as is $(E\tau)^{(+I)}$. With a grating in which ε is greater than in this one, the angles of incidence must be entirely different if the above-mentioned spectra are to be polarized.*

* It would be premature to wish to deduce the law for this phenomenon from a small number of observations. This can be done with certainty

It is seen from equation (V.) that, if $\varepsilon < (\varepsilon . \sin \sigma \pm \nu\omega)$, $\cos \tau^{(\pm\nu)} > 1$, which is impossible. With normal incidence ε must be greater than $\nu\omega$, if $\tau^{(\nu)}$ is to be visible—*i.e.*, possible. If ε is less than ω, *no colored beam is any longer visible,* however the light is incident; and there remains only the white light in the axis—*viz.*, $\cos \tau^{(o)} = \sin \sigma$. This holds true for both transmitted and reflected beams, because the equations are the same for both. If $\varepsilon = \omega$, $\tau^{(1)}$ would be 90°. If, therefore, a grating were made in which the distance apart of the centres of two of the parallel lines is smaller than ω, no spectrum at all would be seen, only the white beam at the axis.

We cannot believe that the polishing, which is done artificially upon glass, etc., is mathematically perfect. If the polish consists of unevennesses, which, in reference to their distances apart, are smaller than ω, they are of no detriment to either transmitted or reflected light, and no colors of any kind could be produced by them. Further, there would be no means of making these inequalities visible.* If the small inequalities had an action on the light—*e.g.*, by reflection according to the known laws—the rays would be scattered most irregularly, because the radii of curvature of these small unevennesses must be very small, and any regular reflection would be impossible. If a reflecting surface consists of unevennesses whose distances apart are smaller than ω, no spectrum is possible, as has been said; and light can be reflected only along the axis. For this ray $\nu = 0$; in which case equation (V.) gives the ordinary *law of reflection—viz.*, $\cos \tau^{(o)} = \sin \sigma$. This law follows, therefore, from interference, without the need of as-

only from a large number of gratings with which ε is widely different. Since in these experiments it is not necessary for the fixed lines of the spectra to be plainly seen, gratings can be made in which ε is sensibly smaller than in the finest ones which I have used up to this time. It is not improbable that the principles of interference will lead perhaps to a theory of the polarization of light. This is neither the place nor the time (at present) for me to communicate my opinions on this matter. Fortunately, other experiments of another kind are possible, which seem to have bearing upon this question; yet they are of a most delicate nature, as are the greater part of all the experiments on this subject.

* We can conclude from this what it is possible for one to see through a microscope. A microscopic object—*e. g.*, one whose diameter is equal to ω, and which consists of two portions—cannot be recognized as consisting of two parts. This shows us the limits of the power of a microscope.

suming a repelling force perpendicular to the reflecting plane.* The fact that at greater angles of incidence more light is reflected than at smaller ones follows just as simply and is in accord with experiment. It is remarkable that according to the exact formula (III.), at distances from the reflecting surface which are not great in comparison with ω—*i.e.*, at very short distances—the angle of reflection can be markedly different from the angle of incidence. One can easily see by an accurate consideration of this equation that at distances which permit one to make exact observations this difference is too small to be thought of; therefore, the ordinary experiments for the determination of the law of reflection can prove nothing against its deduction from the theory of interference.

It is seen, as a result of all the experiments with the different gratings, that the distances of the spectra from the axis increase as the grating-space—that is, ε—becomes smaller; and that, if the spectra are to be homogeneous, this ε must remain constant throughout the whole grating. If these spaces are unequal, the larger ε's will produce small spectra, the smaller ε's large ones, which mingle with each according to the degree of the irregularity. If the irregularity is very great, not even heterogeneous colors will be seen; and the light must be white in the whole region, as experiment shows is the case. It is, nevertheless, interesting to know what phenomena would arise if the grating-spaces were *unequal in a regular manner*—*i.e.*, if the inequality of the distances, whatever it is, is repeated at regular intervals. To this end I covered several pieces of plane glass with gold-foil, and upon them I ruled parallel lines whose distances apart were unequal, but unequal in various regular ways. I can give here in brief only a few of the results of these experiments, which must, besides, be continued further. The spectra which are seen with a telescope through this kind of a grating consist of homogeneous light; and their fixed lines are recognized most plainly, so that their distances from the axis can be accurately measured. If the grating-spaces of a grating with periodic inequalities are denoted by ε', ε'', etc., and if one of the identical portions, each of which

*[*A long note follows in which Fraunhofer upholds the wave-theory against its rivals. He says it accounts for regular refraction, the colors of thin plates and of mother-of-pearl, reflection from a rough surface, etc.*]

contains unequal ε's, is expressed by $\varepsilon'+\varepsilon''+\varepsilon'''+\ldots+\varepsilon^n$, then the results of the experiments are, that with normal incidence the distances of the different spectra from the axis are expressed by the following equation:

$$\sin \vartheta^{(\nu)}=\frac{\nu\omega}{\varepsilon'+\varepsilon''+\varepsilon'''+\ldots+\varepsilon^n}.$$

In whatever manner the ε's follow each other in the portion which contains the unequal parts and which is represented by the division of the equation, even if some of them are equal, the equation remains always the same, provided only that these portions cannot be themselves subdivided into smaller identical portions in which the ε's succeed each other in the same manner and same sense. This phenomenon, arising from gratings with periodic errors, is remarkable on account of the relative intensities of the different spectra, concerning which, however, no general statement can be made in a few words and without a figure. With some gratings of this kind, several spectra, or parts of them, may be entirely wanting, or may have such a feeble intensity that they can be observed with difficulty, while the following ones again are very intense. This has the great advantage, that in this case the fixed lines of those spectra can be observed which cannot be seen with gratings whose grating-spaces, ε, are equal, because they are covered by the neighboring spectra. For instance, with no grating having equal ε's can C^{XII} and F^{XII} be seen; but with a grating which has periodic errors, such that each portion consists of three ε's differing from each other in the ratio 25 : 33 : 42, C^{XII}, D^{XII}, E^{XII}, F^{XII} may be seen so plainly that their distances from the axis can be safely measured. For with such a grating the tenth and eleventh spectra are almost entirely absent. With this grating I saw even E^{XXIV}, and saw it so plainly that its distance from the axis could be measured. The relation between the intensities of the different spectra depends upon the relation between the ε's which follow each other in any one portion, and this is in many cases very complicated.* Similar gratings with peri-

* One can conclude from the experiments mentioned, as well as from others, that many of these phenomena are so complicated—at least, in all likelihood—that they cannot well be described in a few words; and I am for this reason forced to leave untouched much that might be of interest. The further one goes in these experiments, so much the wider becomes the

odic inequalities are obtained if two different gratings—*i.e.*, two in each of which ε is constant, but larger in one than in the other—are placed with their ruled faces together so that their lines are exactly parallel.

[*A page concerning gratings with concentric circular rulings is here omitted.*]

Addendum Concerning the Spectra of Flames, Moon and Star Light, and the Electric Light.

As is well known, the prismatic *color-spectrum* of the light coming from a *flame* (*lamplight*) does not show the dark fixed lines which are present in the spectrum of sunlight; instead of them there is in the orange a bright line which is prominent above the rest of the spectrum, is double, and is at the same place where in sunlight the double line D is found. The spectrum obtained from the light of a flame which is blown with a *blast-tube* contains several prominent bright lines. Of still greater interest for optical experiments is the fact that, by skilful blowing of the flame, the light of the *front half* of the flame can be dispersed no further by the prism, and, consequently, is simple homogeneous light. This light has, so far as I have investigated it, the same refrangibility as the D ray of sunlight. Simple homogeneous light which proceeds in all directions is, for known reasons, very difficult to produce, and can never be obtained with prisms directly; therefore this flame is of great use in many experiments.

By means of the large electrical machine of the physical cabinet of the Royal Academy, I obtained a spectrum of the *electric light*, in which I recognized a greater number of bright lines than I had seen before with weaker light. I determined the relative positions and intensities of the brightest of these.

The light of the *moon* gave me a spectrum which showed in

field which offers itself for new investigations. It is greatly to be regretted that they can be repeated so seldom by any one, owing to the fact that they demand very large, and, in part, expensive apparatus, and also a great deal of time. The fact that the sky must be most favorable makes one lose more time than would be believed, perhaps; which I feel all the more because the demands of business leave me only a few definite days in the month which are free for these investigations.

the brightest colors the same fixed lines as did sunlight, and in exactly the same places.

To observe the *spectra of the light of the fixed stars,* and at the same time to *determine the refrangibility of this light,* I prepared a short time ago a suitable apparatus specially adapted to this end, the telescope belonging to it having an objective of 4 inches' aperture. With it I have obtained many interesting results, although the experiments are still far from being completed. The flint-glass prism of this instrument has an angle of 37° 40′, and has the same diameter as the objective. The angle which the incident ray makes with the ray emerging from the prism is about 26°, so that, if the refrangibility of the light of one star were to differ only slightly from that of another, the difference could be very easily detected. In order to measure with certainty this difference, in case such is found, I added a second smaller telescope, which is fastened to the larger one, and cuts it at an angle of about 26°—*i.e.*, at the angle which the ray emerging from the prism makes with the incident ray. Two observers are needed : one observes the arrival of the star at the cross-hairs of the smaller telescope, which views the star directly, not through the prism ; the other observes, through the larger telescope, the arrival of a portion of the spectrum of the same star. This larger instrument has for this purpose a micrometer-screw, whose movable edge is so placed by the observer, by means of the screw, that at the instant when the star passes the cross-hair of the smaller telescope, viewed directly without the prism, one of the fixed lines of the spectrum coincides with the edge in the larger telescope. The micrometer is left unchanged, and the instrument is then pointed at another star, concerning which one wishes to know if its light has the same refrangibility. If, at the instant when this star passes the cross-hair of the smaller telescope, the same color of the spectrum or the same fixed line arrives at the edge of the micrometer in the larger instrument, the refrangibility of the two kinds of light are the same. Since two observers are necessary for these experiments, Herr Soldner had the kindness to make them with me. These experiments are, however, to be regarded as only just begun ; and I must make some important modifications in the instrument in order to obtain still greater accuracy and to save time in the observations.

Up to the present we have found no *fixed star* whose light,

so far as its *refrangibility* is concerned, is sensibly different from that of the *planets*. When the fixed lines of the spectra are seen plainly, one can be certain with this instrument to 10 seconds; and when the fixed lines cannot be seen, one can still be certain for the orange light to $\frac{1}{2}$ minute. Since the total refraction through the prism is 26°, a difference amounting to $\frac{1}{9360}$ of the whole refraction could still be noticed with this instrument, a difference which even with the horizontal refraction in the atmosphere did not amount to $\frac{1}{4}$ second. Up to this time, as is well known, some astronomers have doubted whether the refraction tables for different stars should not be somewhat different; therefore this doubt seems to be removed by the experiment noted. The continuation of this investigation will lead us, I hope, to more complete knowledge.

In order to see the *fixed lines* of the different stars (with this large instrument) the air must be most favorable—a condition which happens rarely to a sufficient extent. The spectra of the light from *Mars* and *Venus* contain the same fixed lines as does sunlight, and in exactly the same places, at least so far as the lines D, E, *b*,* and F are concerned, whose relative positions can be exactly determined. In the spectrum of the light from *Sirius* I could not distinguish fixed lines in the orange and yellow; in the green, however, there is seen a very strong streak; and in the blue there are two other unusually strong streaks, which seem to be unlike any of the lines of planetary light. We have determined their positions with the micrometer. *Castor* gives a spectrum which is like that of Sirius; the streak in the green, in spite of the weak light, was intense enough for me to be able to measure it; and I found it in exactly the same place as it was with Sirius. I could also distinguish the streaks in the blue; but the light was too feeble to allow of measurement. In the spectrum of *Pollux* I recognized many fixed lines which resembled those of Venus; but all were weak. I saw the D line quite plainly, in exactly the same position as with planetary light. *Capella* gives a spectrum in which, at the places D and *b*, the same fixed lines are seen as in sunlight. The spectrum of *Betelgeux* [*a* Orionis] contains countless fixed lines which, with a good atmosphere, are sharply

* The line *b* lies in the green, between E and F; it consists of three strong lines, two of which are nearer than the third. See previous papers.

defined ; and, although at first sight it seems to have no resemblance to the spectrum of Venus, yet similar lines are found in the spectrum of this fixed star in exactly the places where with sunlight D and *b* come. Some lines can be distinguished in the spectrum of *Procyon;* but they are seen with difficulty, and so indistinctly that their positions cannot be determined with certainty. I think I saw a line at the position D in the orange.

Munich, *July* 14, 1823.

Joseph Fraunhofer was born in Straubing, near Munich, March 6, 1787, and died in Munich, June 7, 1826. His father was a master glass-workman, and at the age of twelve he was apprenticed to a glass-polisher in Munich. In 1806 he became the optician in an optical-mechanical institute in Benediktbeuern. While there he invented new methods, machines, and measuring instruments to perfect the construction of achromatic lenses. In the course of this work he was led to the discovery of the lines in the solar spectrum which bear his name. With the aid of a prismatic-objective telescope he studied the spectra of the moon, the planets, and some of the stars. These results were presented to the Royal Academy of Sciences of Bavaria in 1814 and 1815, and were published in 1817, immediately after which he was elected a Member of the Academy. He returned to Munich to live in 1819, where he was engaged in the manufacture of optical instruments. In 1821 he published his experiments on diffraction through one or more openings, and on the spectra obtained by gratings. In 1823 this paper was continued by further experiments with glass gratings and by measurements of the wave-length of various solar lines. In 1825 appeared his memoir on halos, mock-suns, and allied phenomena. In the later years of his life he was appointed Royal Professor, and received numerous honors both from Bavaria and from foreign lands. He was buried in a plot of land given by the city of Munich for the purpose. His tomb bears the simple inscription, "*Approximavit sidera.*"

A Method of Examining Refractive and Dispersive Powers by Prismatic Reflection.

By WILLIAM HYDE WOLLASTON, M.D., F.R.S.

Read June 24, 1802. *Philosophical Transactions*, 1802.

[*A method is described which is based upon the principle of total reflection; and the results of experiments upon various solids and liquids are given. Then follows a brief discussion of the number of distinct colours in the solar spectrum. The author's ideas are based upon the following experiment.*]

If a beam of daylight be admitted into a dark room by a crevice $\frac{1}{20}$ of an inch broad, and received by the eye at a distance of 10 or 12 feet, through a prism of flint-glass, *free from veins*, held near the eye, the beam is seen to be separated into the four following colours only—red, yellowish green, blue, and violet, in the proportions represented in Fig. 3. [*Omitted.*]

The line A that bounds the red side of the spectrum is somewhat confused, which seems in part owing to want of power in the eye to converge red light. The line B, between red and green, in a certain position of the prism, is perfectly distinct; so also are D and E, the two limits of violet. But C, the limit of green and blue, is not so clearly marked as the rest, and there are also on each side of this limit other distinct dark lines—*f* and *g*—either of which, in an imperfect experiment, might be mistaken for the boundary of these colours.

The position of the prism in which the colours are most clearly divided is when the incident light makes about equal angles with two of its sides. I then found that the spaces AB, BC, CD, DE, occupied by them, were nearly as the numbers 16, 23, 36, 25.

Since the properties of these colours to each other have been supposed by Dr. Blair to vary according to the medium by

which they are produced, I have compared with this appearance the coloured images caused by prismatic vessels containing substances supposed by him to differ most in this respect, such as strong but colourless nitric acid, rectified oil of turpentine, very pale oil of sassafras, and Canada balsam, also nearly colourless. With each of these I have found the same arrangement of these four colours, and, in similar positions of the prisms, as nearly as I could judge, the same proportions of them.

But when the inclination of any prism is altered so as to increase the dispersion of the colours, the proportions of them to each other are then also changed, so that the spaces AC and CE, instead of being as before, 39 and 61, may be found altered as far as 42 and 58.*

By candlelight a different set of appearances may be distinguished. When a very narrow line of the blue light at the lower part of the flame is examined alone, in the same manner, through a prism, the spectrum, instead of appearing a series of lights of different hues contiguous, may be seen divided into five images at a distance from each other. The first is broad red, terminated by a bright line of yellow; the second and third are both green; the fourth and fifth are blue, the last of which appears to correspond with the division of blue and violet in the solar spectrum or the line D of Fig. 3.

When the object viewed is a blue line of electric light I have found the spectrum to be also separated into several images, but the phenomena are somewhat different from the preceding. It is, however, needless to describe minutely appearances which vary according to the brilliancy of the light, and which I cannot undertake to explain.

WOLLASTON was born in Norfolkshire, August 6, 1766, and died in London, December 22, 1828. He discovered the two elements palladium and rhodium, and invented a process for rendering platinum malleable. He is best known, perhaps, for his invention of the various instruments named after him—the reflecting goniometer, the cryophorus, a microscopic doublet.

* [*A note on the existence of the infra-red and the ultra-violet spectra.*]

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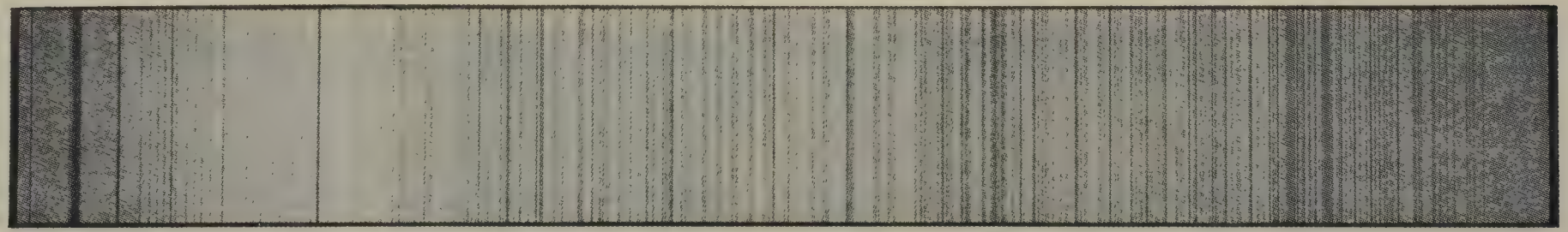

Fig. 5.

THE LAWS OF RADIATION AND ABSORPTION

Translated and Edited by DeWitt Bristol Brace

SCIENTIFIC MEMOIRS

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XV

THE LAWS OF RADIATION AND ABSORPTION

THE LAWS

OF

RADIATION AND ABSORPTION

MEMOIRS BY PRÉVOST, STEWART, KIRCHHOFF,

AND KIRCHHOFF AND BUNSEN.

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Radiation and Absorption.
W. P. 1

PREFACE.

THE attempt of Prévost to explain the experiments of Pictet, of the apparent concentration of *cold* at the focus of a mirror, without attributing the quality of radiation to *cold*, as assumed by Pictet, lead him to the enunciation of the very important principle which he called the *movable equilibrium* of *heat*, now designated as the *theory of exchanges*. Prévost, who was a disciple of le Sage, and who had issued, with many additions, his memoirs, assumed, in addition to a corpuscular fluid caloric, a free corpuscular radiant caloric, the equal interchange of which between neighboring free spaces, constituted heat equilibrium. Any interference with this equilibrium will be reëstablished by the inequalities of the exchanges. On this principle he was able to explain the apparent concentration of *cold* and also to show the inadmissibility of *cold* as an agent susceptible of radiation. He was careful, however, to fortify his principle by showing that the same results would follow on the then distrusted hypothesis of undulatory exchanges, which has been adopted by his successors. Later experimenters, particularly Leslie and De la Provastaye and Desains, confirmed the theory and also showed in many instances quantitative relations between radiation and absorption. But the most important advance was made by Balfour Stewart in establishing, not only a quantitative relation, but also a qualitative or selective one. By the introduction of his ingenious idea of an impervious radiating inclosure he demonstrated the equality between the emissive and the absorptive power of any wave length. We owe to Kirchhoff, however, the first rigorous proof of the celebrated law (usually designated on the Continent as Kirchhoff's law) of the emission and absorption of light and heat, and the application of the same by both Kirchhoff and Bunsen to Spectrum Analysis. The radiation of solids and liquids and gases follows the law exactly when the conditions upon which he founded it are rigorously fulfilled, namely, the complete transformation from one to the other of

radiant energy and their intrinsic heat. We now know that most radiations from gases are not exclusively thermal, but that the substances, cited by Kirchhoff and Bunsen, also give off so called chemical and electrical and fluorescent radiations which Kirchhoff excluded in the proof of his law. In fact none of the gases giving line spectra at temperatures heretofore used do so by simple thermal radiation, but essentially by luminescent actions (chemical, electrical, and photogenic), so that we cannot, in general, apply the law of Kirchhoff of the proportionality between radiation and absorption to either terrestrial or celestial substances. In these cases the principle of resonance usually holds, since in luminescence the radiation of line spectra is accompanied by selective absorption of the same spectral lines, so that the law may be used qualitatively, which is in fact the way Kirchhoff and Bunsen actually attempted to confirm it. The formulation of the complete law for radiations of a *black body* is only given in part by Kirchhoff. The formula of Wien, and more particularly the most recent one of Planck, deduced on theoretical grounds, approximates closely the latest observations on a black body at different temperatures and over different wave lengths.

D. B. Brace.

University of Nebraska.

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MEMOIR ON THE EQUILIBRIUM OF HEAT.

BY

PIERRE PRÉVOST.

Journal de Physique, vol. 38, *pp.* 314–322. *Paris,* 1791.

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MEMOIR ON THE EQUILIBRIUM OF HEAT.

By PIERRE PRÉVOST.

I PROPOSE to analyse and fix with precision the sense of the word *equilibrium* applied to such a fluid as heat. This idea is not exactly defined in the theories which leave questions relative to the nature of this element undecided. If there is any doubt that heat is material, if there is no explanation concerning the contiguity or the noncontiguity of molecules of heat, concerning their mobility or their immobility, the kind of motion, vibratory or translatory, which is attributed to them, it is impossible to arrive at exact and complete ideas of their equilibrium. It results from this that every phenomenon which depends, not upon any equilibrium whatever but upon a specific kind of equilibrium, remains entirely unexplained. And as the imagination determines to some extent, notwithstanding, that which reason wishes to leave undetermined, the true causes are lost sight of, and vain hypotheses are arbitrarily preferred because they are suitable in certain respects and favorable to first appearances.

I will not waste time in discussing the different natures assigned to heat by various physicists. The true constitution of this fluid is connected with the theory of discrete fluids, now known, although it has not been published by its author. For its development and proof I refer to what M. DeLuc has said of it both in his "*Idées sur la météorologie*" and in his *letters* published successively in this Journal: also what I have said of it myself in my essay upon *l'origine des forces magnetiques.* Assuming then the principles of this theory, I shall merely recall them, and use them to establish true ideas on the equilibrium of heat.

I shall afterwards make the application of this theory of the equilibrium of heat to a very remarkable phenomenon which I consider inexplicable without it. This is the phenomenon of the *reflection of cold.* It has been observed by M. Pictet, who

has described it in detail in his *essai sur le feu*. This learned scientist, with whom I have old and valued bonds of friendship, does not at all disapprove of the discussion which I am undertaking although it tends to indicate some inadequacy in the explanation which he, himself, has given to this phenomenon.

It will be seen elsewhere by what I shall say of it, that a complete explanation, such as the theory of M. le Sage furnished, does not enter into the plan which M. Pictet has proposed. I will discuss this phenomenon then, very freely. I shall show that it explains itself without any effort, by the true theory of discrete fluids. I shall also prove that it is not explained at all by the imperfect theories to which physicists commonly limit themselves. I shall close this memoir with two remarks which have some connection with this subject, without being directly related to it.

I.

THEORY OF THE EQUILIBRIUM OF HEAT.

Heat is a discrete fluid. Its elasticity consists in its expansive force. And this is the effect of the movement of its particles. This movement is caused by the impulse of a much more subtile fluid whose effect upon its particles is determined to a certain extent by their form. It is so swift that when heat is freed its translation from one place to another appears instantaneous. It is also sensibly rectilinear, so that perfectly free heat partakes, as far as the movement of its particles is concerned, of all the properties of light, at least so far as our senses can determine in the limited experiments which have thus far been performed.

A discrete fluid whose particles radiate like those of light, may be confined by barriers, but may not be confined by another radiant fluid nor, in consequence, by itself. For it is necessary to conceive of all these fluids as very rare, as having many more void intervals than full ones in the space which they occupy. Light does not stop the passage of light. If this solar emanation is so dense that two luminous currents cannot cross each other without being interrupted, the innumerable crossings and reflections which they experience will destroy entirely its rectilinear direction, and light will lose to our eyes

all its properties which depend upon this direction. What is true of this fluid is true of all radiant fluids. Radiant heat passes through heat, which upon the earth is present in all places, and since it produces no sensible perturbation, it is necessary that these particles should be separated by intervals great relatively to their diameters. It is certain that free radiant heat is a very rare fluid, the particles of which almost never collide with one another and do not disturb sensibly their mutual movements.

In conforming to physical hypotheses, one says ordinarily that heat is coercible by itself : that two contiguous portions of heat have a mutual relation when their temperatures are equal (or as M. Volta has said, when their *tensions* are the same). These expressions are exact, only in so far as they define an appearance. In reality the heat of any portion cannot arrest that of another. These two heats give each other mutually free passage. It would then be wrong for one to conclude from these expressions that two portions of contiguous heat restrain each other mutually, as two bent springs stayed against one another, or as two masses of hair which repel each other by their elasticity.

But in what does the equilibrium of these two portions of contiguous heat consist ? In order to answer this question clearly, I will suppose the two portions to be enclosed in an empty space, terminated on all sides by impenetrable walls. One may represent two cubes applied by one of their faces, forming in consequence a rectangular parallelopiped perfectly hollow, of which the six faces are of the same matter, absolutely solid and without pores. The two portions which I consider are, in this example, the two applied cubes. The heat occupying the interior of this space moves freely there, and assuredly one can see no reason why it should pass with less facility across the boundary of the two portions than across every other section of this space. There are then continual exchanges from one portion to another, and one can affirm (in consideration of the number of particles and their continual motion) that at each observable instant the state and quantity of the heat in each portion are constant. There is then no ceasing of the different particles, which are found at any one

place, but their number and their mean distance in each portion are constant. Concerning their speed, as it is in the same free fluid (consider the constant nature of the cause which produces and renews it continually), it is clear that it does not change : and I shall leave it out of the question, since at the present moment I consider only free radiant heat.

At all times that both portions of the space are found in the circumstances which I have just described, the heat between them is in a state of *equilibrium*. This signifies that the phenomena which manifest their existence remain the same : that if these phenomena change in the same manner and in the same quantity in the two portions, the equilibrium in question will not be disturbed. This would occur if one should remove from the total space, which we are considering, a certain aliquot part of all the heat found there, or indeed if this aliquot part should be added. The identity of the phenomena which implies the equilibrium of heat between these two portions of space is a relative identity, which, as one can see, may exist whatever may be the difference or the absolute inequality.

Let us now suppose that into one of the two portions of space (which I will represent constantly by the two adjoining cubes) one passes suddenly some new heat ; for example, one tenth of all that which is contained in this portion. This heat, instantly placed in motion, spreads immediately throughout all the space where it can penetrate freely. Thus the exchanges between the two portions would be unequal. One would send to the other eleven particles, while the latter would return only ten. This state causes a *rupture* of the equilibrium between the two portions.

By reason of the unequal exchanges one may conceive that the equality would be reëstablished. Thus the rupture of the equilibrium restores very quickly the equilibrium between two portions of free heat.[1]

[1] Suppose that the densities of the heat in our two cubes are as the numbers 1 and 2 (i. e., that one is twice as hot as the other) : suppose further that in one second there passes from the one cube to the other a number of igneous particles which on the whole are as 1 to 10 (so that during this short time there is exchanged one tenth of all the heat).

Absolute equilibrium of free heat is the state of this fluid in a portion of space which receives as much as it allows to escape it.

Relative equilibrium of free heat is the state of this fluid in two portions of space which receive from each other equal quantities of heat, and which are, moreover, in absolute equilibrium, or experience changes precisely equal.

The heat of several portions of space at the same temperature and *near* each other is at once in the two kinds of equilibrium. If one should change the temperature of all the space at the same time, it would destroy the absolute equilibrium, but not the relative equilibrium. Should the temperature of one or of several portions be altered without affecting all, each kind of equilibrium would be destroyed.

If the cause which throws out or which absorbs the heat of any portion is an instantaneous cause, after the action of this cause the relative equilibrium reëstablishes itself incessantly by means of unequal exchanges. And after this reëstablishment the absolute equilibrium remains destroyed, that is to say, the temperature of the place is changed.

If, on the contrary, the cause is permanent, that is to say, if there is opened in any one of the portions of this space a source or a sink which gives out or which absorbs heat incessantly, relative equilibrium tends to establish itself, but does not reëstablish itself entirely during the action of the cause, and absolute equilibrium is constantly destroyed.

II.

APPLICATION OF THE PRECEDING THEORY TO THE PHENOMENON OF REFLECTION OF COLD.

Let us represent two spherical concave mirrors opposite to each other on their axes, and let us suppose placed at their foci

---

After seven seconds the ratio of the densities of the heat in the two cubes will be as 5 to 6. After fourteen seconds, these densities will be as 28 to 29, i. e., very nearly equality : the equilibrium will appear established.

I take this result from a calculation of M. le Sage thirty years since in the case of discrete fluids different from heat.

two bodies precisely equal and similar and of the same substance, which I will call the two *focal bodies*.

To simplify this I will suppose, (1) that all the space where the apparatus is immersed is absolutely cold and receives heat only from part of the two focal bodies, (2) that these are hot and give out radiant heat continuously, (3) that the mirrors reflect the heat but do not absorb it.

With these conditions, it is clear that the heat thrown out by either one of the two focal bodies radiates on all sides. But I shall consider only the part which strikes the mirror of which it is the focus.

This heat is reflected parallel to the axis. Striking the opposite mirror in this direction, it is reflected to the focus of this second mirror and enters as a consequence the body which occupies this focus. Similarly inversely, the heat thrown out by this latter against its mirror enters after two reflections the body which occupies the focus of the first mirror.

Let us suppose, first, the two focal bodies are at the same temperature, or each one sending out in equal intervals of time an equal quantity of radiant heat to its own mirror. The relative equilibrium of heat between the two focal bodies will not be disturbed by this operation; for each of them will receive from the other exactly what it gives up to it. Radiation will exactly compensate absorption.

Now let us change, to a greater or less degree, the temperature of one of the two focal bodies; the exchanges made between them by means of double reflection will cease to be equal: the relative equilibrium will be destroyed. It will tend then to reëstablish itself, and the temperature of these two bodies will approach each other. If additional heat should be thrown upon the first body, for example, a tenth of all that which it has, the second body will make advantageous exchanges with it. For ten particles transmitted by reflection, it will receive eleven by the same means: in this way its heat will be augmented.

If one should withdraw heat from the first body, for example a tenth, the second body will make exchanges at a loss, receiving nine against ten by means of the mirror. It will be cooled.

Such is the result of the theory conforming exactly to that of the ingenious experiments of M. Pictet, in spite of all the

conditions which I have made, since these conditions influence only the quantity of cold or of heat produced by reflection, and not the nature of these actions. It is known that this physicist has observed heat and cold equally reflected in his apparatus, which is such as I have just described. He has not hesitated to explain the reflection of cold just as that of heat in a reciprocal sense: but being limited (conformably to his representation) to explanations drawn at once from experiment, and it not being his purpose, in the important work which he has published, to treat of the constitution of discrete fluids, he has not been able to enter into the details which I have just given. It has resulted from this that the view to which he has come touching the cause of the reflection of cold, founded upon these notions of equilibrium, inapplicable to discrete fluids, is insufficient for the theory, however true as to appearances.

It is certain that when one produces cold at the focus of one of the mirrors, the heat of the thermometer placed at the opposite focus follows the course which M. Pictet traces for it. And this course is what I have just described. But what it is that causes the excess heat of the thermometer to take this course, this physicist has not shown me, because he has not been called upon to consider heat according to its natural constitution. Now if one holds to the ideas of *tension*, of *stress*, in a word, of unvariable equilibrium, he finds that the progress of the phenomenon of heat in the experiment of the reflection of cold remains absolutely inexplicable. I shall now show, (1) that in this hypothesis of unvarying equilibrium no heat ought to pass from the thermometer to its mirror, (2) that if any does pass, this heat should not converge to the focus of the other mirror.

III.

EXCLUSION OF THE INDEPENDENT EXPLANATION OF THIS THEORY.

(I) At the instant when one places a cold body, such as glass, at the focus of one of the mirrors, the heat of all the neighboring bodies passes into it. This cause acts according to the law

of the inverse square of the distance, when we suppose the bodies to be of the same nature, as we do in this instance.

The mirrors employed in the experiment of the reflection of cold were placed ten and one half feet apart. Their curvature was that of a sphere with a radius of nine inches : so that their foci were about four and one half inches from their surface, measured on the axis.

If, then, we consider only the apparatus without taking into account the supports, or the air or the surrounding and neighboring bodies, it is clear that the mirror whose focus is occupied by the glass, being twenty-eight times nearer this cold body than the other mirror, ought to send out to it seven hundred and eighty-four times as much heat in the same time.

Further, the thermometer placed at the focus of this other mirror being nearer the glass than its mirror in the ratio of 26 to 27, ought to set free more heat than a portion of the mirror equal to its bulb in the double inverse ratio (at least for the part of the mirror which lies at the origin of the axis). This ratio is that of 729 to 676, or of about 13 to 12; so that, through the direct influence of the glass, the thermometer loses about a thirteenth more of its heat than if it formed a part of the mirror, at the focus of which it is placed. When the cooling of the first mirror becomes sensibly equal to the second, the thermometer being less distant than the former, is also more affected by the latter in the double inverse ratio of 27 to 28; that is to say, in the ratio of 784 to 729, or of about 14 to 13.

Thus the thermometer is cooled more than its mirror, either directly by the glass, or indirectly by the mirror whose focus this glass occupies. Heat here then is under less *tension* than in the mirror. Consequently it cannot pass from the thermometer to the mirror, nor in consequence, radiate from there to the opposite mirror next the glass. This progress of the phenomena in a system in unvarying equilibrium is contrary to the effect which the cause should occasion. And it is still more inexplicable when we consider the supports of the apparatus and all the surrounding bodies which send heat into the glass, and constantly draw out that of the thermometer, as well as of the opposite mirror: effects independent of reflection and of the particular position of the foci.

(II). To which it is necessary to add, in reconciling the same (which is demonstrated false in the hypothesis I have discussed), that the heat of the thermometer passes in part into its mirror; as there is carried over only what replaces that which escapes, this heat would not be reflected, but absorbed. Now, all the heat which one of the mirrors sends out to the other, aside from that by reflection to the focus, being an irregular radiation, would not converge at the focus of the other mirror. Thus the foci would not be more characteristic than two other points, taken at random between the mirrors, for repeating the experiment of reflection of cold, which is absolutely contrary to the actual observation.

It is apparent, then, that if we refuse to consider heat according to its true constitution as a discrete fluid, whose particles are in motion, and if in consequence we do not arrive at ideas which I have given of the equilibrium of radiant free heat, it is impossible to give any satisfactory explanation (compatible moreover with the principles of sound physics), of this beautiful and remarkable phenomenon of the reflection of cold. The fact is established by an excellent observer, who has very clearly recognized the progress of the phenomena of heat. The discovery of the cause is due to the author of the true theory of discrete fluids.

IV.

SUPPLEMENTARY REMARKS.

(1). Radiant heat is only a part of the heat that escapes from a hot body. Let us suppose that in the preceding experiment the two foci of the mirrors communicate by a metallic bar, terminated at both ends by these foci: if we place at one end of the extremities of this bar an exhaustless source of heat (a red-hot iron, a blast-lamp flame, the focus of a powerful lens): immediately the radiant heat, following the course indicated above, will warm the other extremity of the bar by the double reflection. At the same time the non-radiant heat, creeping gradu-

ally into the contiguous parts of the bar, will slowly heat it and will finally come to the points most distant from the source.

The air being a discrete fluid much more dense[1] than heat, arrests and intercepts the particles of the latter. But being much more rare than the metal, it allows a portion of it to pass, which produces the phenomena of radiant heat. Light, much more rare and subtle than heat, is transmitted in much greater proportion by this same air, the opacity of which is so inconsiderable that it becomes sensible only in very great masses.[2] The transparency or the quantity of fluid transmitted through another fluid, depends upon the rarity and the subtlety of the particles of both fluids. I do not speak here of the affinities and capacities of different bodies for heat. I speak only of the mechanical interception of this fluid by its solid parts. This interception is sufficient only to produce these two kinds of heat or of cold, *radiant* heat, and *nonradiant* heat.

Entangled, further, in the small cavities or in the interstices of solid particles, the heat may or may not recover all the velocity which properly belongs to it, according as these cavities or interstices are or are not sufficiently spacious. When it recovers only a portion of its velocity, it becomes in part insensible or *latent*. When it can recover only a very little of it or none at all, it yields to the affinities of the particles which surround it and *combines* in a thousand ways.

(2). Heat is not the only fluid of its kind. Several discrete fluids are known, radiant and nonradiant.[3] We often have occasion to consider these fluids in the state of equilibrium. The determination of the true sense of this word ought then to be of much importance, independently of the theory of heat.

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[1] The density which I attribute to the air in this instance, consists chiefly in the proximity of its molecules; for a discrete fluid may be composed of very dense particles, but with large spaces between them: so that it could be more permeable than heat, although more dense.

[2] Notice the remarks of M. de Saussure, upon the transparency of the air, in his memoirs upon light. *Académ. de Turin*, 1790.

[3] In the *electrical phenomena*, there are radiations of the corresponding fluid. In *magnetic phenomena* neither of the two magnetic fluids is radiant.

If these remarks and the preceding discussion offer any useful views, if they tend to throw light upon an important class of phenomena, if they suggest any clear ideas upon the method of motion of invisible and subtle fluids which manifest their existence by such diverse appearances : finally, if these conceptions naturally connect themselves with other theories, either already proven, or rendered probable, concerning the various effects of these subtle fluids (such as the phenomena of evaporation, of electricity and of magnetism), is it not the requital of investigating the general theory upon which all these special explanations depend? This theory (I refer to that of M. le Sage of Geneva, upon the nature of discrete fluids) merits the further attention of physicists, since it depends, itself, upon another principle, more general, which has also as a proof of its reliability, the clear and satisfactory explanation of very striking and very general phenomena, absolutely inexplicable without it.

ON RADIANT HEAT.

BY

PIERRE PREVOST.

Geneva, 1809.

(*Selections.*)

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QUESTIONS RELATIVE TO THE NATURE OF CALORIC.

CHAPTER IV. PP. 6—10.

THE word *caloric* (heat) has been originated to explain the cause of heat, with the formally expressed intention of being non-committal as to its nature.

It is desirable to leave it indefinite as to whether heat may be produced by a specific fluid, or, merely, be a movement impressed upon the molecules of a body, without the introduction of any fluid.

Many noted physicists believe there is no specific fluid to which this word *caloric* is applicable. They believe that heat is produced by internal movements of the molecules of a body. More often, however, physicists have recourse to a vibrating ether or to the air, or to some other medium propagating waves to which they attribute the phenomena of heat.

Others believe that *caloric* is a specific fluid, which penetrates the body and produces all the appearances of this kind.

Among the latter, many believe that *caloric* and light are identical. Others are of a contrary opinion.

Some look upon *caloric* as simple ; a smaller number regard it as a compound fluid. Mr. J. A. DeLuc believes that *caloric* is a kind of vapor composed of ponderable matter held in a state of suspension by light. This conception throws some light upon many phenomena and merits serious consideration. Meanwhile, pressed to arrive at the chief object which I have in view, I will refrain from all discussion as to the composition of *caloric.*

I have no desire to repeat here and weigh the general arguments stated on the one side and on the other for sustaining the various views which I have just outlined. I will limit myself to a very few remarks on this subject.

As stated below I propose to consider *caloric* as a specific fluid. I will represent the radiations of this fluid as an emission

and never as an undulation. I believe this conception and this representation to be more conformable than any other to the nature of things and founded upon the soundest principles of general physics. But if those who believe otherwise substitute waves for an emission they may be able, perhaps, to adapt to their opinion the explanations which I give for phenomena of this class. It is no desire of mine that they should attempt it, because I am persuaded that this would be translating a language clear and natural into a language obscure and artificial. But I make this statement to make clear the kind of work which I have undertaken. I do not contest any system, I do not refute any explanation. It is my aim, in limiting myself in my subject, to explain in my own way what seems to me to admit of clear explanation, and to indicate the phenomena of which the explanation remains imperfect. If each one who has an opinion upon the theory of *caloric* will give a concise exposition of his ideas on the subject, and will show how the facts may be coördinated by means of these conceptions: physicists can see at a glance which theory is most satisfactory, or if all should be rejected.

RESUMÉ OF THE PRINCIPLES EXPOUNDED AND OF THE PRINCIPAL CONCLUSIONS WHICH HAVE BEEN DEDUCED FROM THEM.

SECTION IX. pp. 258—261.

HEAT is a discrete fluid; each element of heat follows constantly the same straight line, as long as no obstacle arrests it. Every point of a hot space is constantly traversed throughout by streams of heat.

If we admit this constitution of heat, the following conclusions are inevitable.

The first three require nothing further. The others require the assumption that heat is comparable with light in its movements of reflection and refraction.

1st. conclusion: Free heat is a radiant fluid. Or, as the surface of the body of heat becomes free, each point of the sur-

face of the body is a center to which tend, and from which are carried, in every direction, streams of heat.

2nd. conclusion: The equilibrium of heat between two neighboring free spaces consists in the equality of the exchanges.

3rd. conclusion: When the equilibrium is disturbed, it is reëstablished by unequal exchanges. In a medium of constant temperature, a body that is hotter or colder requires this temperature according to the law that *the periods of time being in arithmetical progression, the differences of temperature are in geometrical progression.*

4th. conclusion: In a space of uniform temperatures, if a reflecting or refracting surface is introduced it has no effect in changing the temperature of any part of this space.

5th. conclusion: In a space of unequal temperature, if there is placed a body which is either hotter or colder and if afterwards a reflecting or refracting surface be introduced, the points, upon which these surfaces direct the rays emanating from this body, will be affected by it, being heated if the body is hotter, or cooled if it is colder.

6th. conclusion: A reflecting body, having been heated or cooled internally, recovers the surrounding temperature more slowly than a nonreflector.

7th. conclusion: A reflecting body, having been heated or cooled internally will have less effect on another body placed at any distance (in heating or cooling it) than a nonreflector would under the same conditions.

All these conclusions have been verified experimentally, except that concerning the refraction of cold. This experiment remains to be made, and I am confident of the result, at least if the refraction of the heat is capable of being observed. This result is indicated in the 4th. and 5th. conclusions, which could in this way be submitted to a new test. It is hardly necessary to indicate in this place the precautions by means of which one would place himself beyond every kind of misobservation.

Biographical Sketch.

Pierre Prévost was born in Geneva, March 3, 1751, and died at the same place on the 9th of April, 1839. He was the son of a clergyman and was educated for a clerical career, but turned his attention to law and later to educational work. He became a professor of philosophy and a member of the Academy of Sciences at Berlin in 1780. Here, through his acquaintance with Lagrange, his attention was directed to science, which later he followed up with his studies on Magnetism and Heat at Geneva where he became professor of physics in 1810. He published much on different subjects, including philology, philosophy, political economy, fine arts, etc. He issued the works of le Sage, supplemented by many additions of his own. His *Du Calorique Rayonnant* appeared at Geneva in 1809 and was an exposition and extension of his theory of exchanges first advanced several years before. The original memoir and later publications appeared in the Journal de Physique and the Phil. Trans. from 1791 to 1802. His remarkable versatility is indicated in the variety of his publications. His most valuable contribution to science is undoubtedly his *Theory of Exchanges* one of the most important principles in the whole range of physical science.

AN ACCOUNT OF SOME EXPERIMENTS ON RADIANT HEAT, INVOLVING AN EXTENSION OF PREVOST'S THEORY OF EXCHANGES.

BY

BALFOUR STEWART.

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1. AN ACCOUNT OF SOME EXPERIMENTS ON RADIANT HEAT, INVOLVING AN EXTENSION OF PREVOST'S THEORY OF EXCHANGES.

By Balfour Stewart, Esq.

Communicated by Professor Forbes.

Read, 15th, March 1858.

Division of Subject.

1. This paper consists of two parts, the first of which is confined to describing the experiments performed; while in the second it is attempted to connect these with certain theoretical views regarding Radiant Heat.

2. The experiments were made with a fourfold object; at least for the sake of clearness, it is well to class them into four distinct groups:—

Group I. Contains those experiments in which the quantities of heat radiated from polished plates of different substances, at a given temperature, are compared with the quantity radiated from a similar surface of lampblack, at the same temperature.

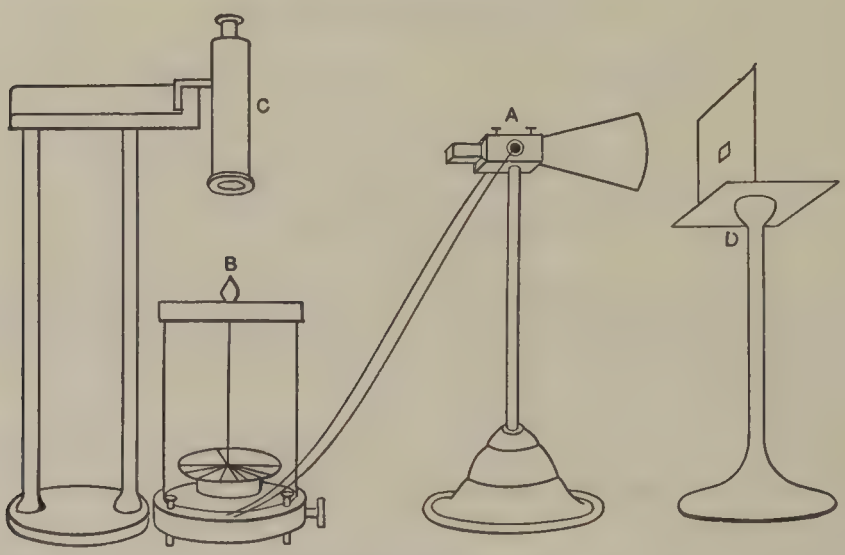
Group II. Those in which the quantities of heat radiated at the same temperature, from polished plates of the same substance, but of different thicknesses, are compared with one another.

Group III. Those in which the radiations, from polished plates of different substances at any temperature, are compared with that from lampblack at the same temperature, with regard to the quality or nature of the heat radiated.

Group IV. Those in which the same comparison is made between the radiations from polished plates of the same substance, but of different thicknesses.

Instruments used, and Method of using them.

3. I am indebted to the kindness of Professor Forbes for the use of a delicate thermo-multiplier, consisting of the sentient pile, and its attached galvanometer and telescope; as well as for much valuable information with regard to the proper method of using the apparatus.



The following arrangement was adopted for the great mass of the experiments:

A. Is the sentient pile, with a polished brass cone attached to it, for collecting the rays of heat.

B. Is the galvanometer, the position of its needle being read to $\frac{1}{10}$th of a degree by the telescope C.

D. Is a screen placed before the mouth of the cone in which there is a small hole or diaphragm .65 inch square. The screen is covered with gilt paper, in order that, should it get slightly heated, it might radiate as little as possible.

The heated body is placed behind the diaphragm, filling up the field of view from the cone; so that every ray reaching the cone from behind the diaphragm comes from the heated body.

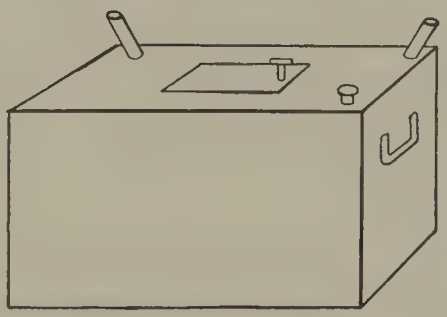
In the following experiments, unless the contrary is mentioned, the distance of the diaphragm from the mouth of the cone is 2 inches.

The dimensions of the cone itself are as follows :

Length of axis, or distance between centre of mouth and pile,.....5 inches.
Diameter of mouth or opening,.....2.6 inches.

The temperature to which the heated body was raised was generally 212°, and the apparatus used for heating it was of the following construction :

It consisted of a tin vessel, having its top, bottom, and sides double (or a box within a box), and furnished on the top with a lid, also double, by means of which the body to be heated was introduced into the interior. Water was poured into the chamber between the outer and inner boxes, and allowed to boil ; and, when the lid was shut, the temperature of the interior was found to rise very nearly to the boiling point ; a thermometer placed in the air of the chamber showing a temperature of 200°, and when lying on the bottom, a temperature of 210°. When an observation was to be made, the hot body was taken out, and that surface which lay on the bottom of the inner chamber placed behind the diaphragm, so as to radiate into the cone. In the following experiments, unless the contrary is mentioned, the body has been heated in this manner.



The first swing of the galvanometer needle was taken as representing the intensity of the heating effect : and Professor Forbes has shown, in a paper read before this Society, 2d May, 1836, that this will hold up to angles of about 20°, which is the maximum deviation used in these experiments.

Observations were always made with as little sunlight as possible ; and under these circumstances, it was ascertained that the stray heat reaching the cone was inappreciable. The needle, it was calculated, reached the limit of its swing about 12

seconds after the heated body had been taken out of the boiling-water apparatus.

Experiments were made to ascertain if the body cooled sensibly during this short period of time, and it was found that its cooling was so trifling as not to interfere in any degree with the results of these observations. In the following experiments, it is therefore assumed that the body remains at its original temperature of 210° while the observation is being made.

Four observations were generally made, and three if they agreed together exceedingly well, but never fewer. Very often the agreement was exact.

First Group of Experiments described.

4. With these remarks, I proceed to describe the experiments belonging to the first group, or those made with the view of comparing the heat radiated from polished plates of different substances with that radiated from a surface of lampblack at the same temperature.

The reason why lampblack was chosen as the standard is obvious; for, it is known from Leslie's observations, that the radiating power of a surface is proportional to its absorbing power. Lampblack, which absorbs all the rays that fall upon it, and therefore possesses the greatest possible absorbing power, will possess also the greatest possible radiating power. The first substance compared with it was glass.

A. *Glass.*—A piece of plate glass, .3 inch thick, having paper coated with lampblack pasted on its surface next the pile, gave a deviation of 18.1. This may be taken as the radiation from lampblack.

Three plates of crown glass, each .05 inch thick, placed one behind the other, gave.....17.7.

A single piece of crown glass of the same thickness, gave.....16.5.

This difference is probably owing to the single plate cooling faster than the three plates. It may be argued that the radiation from the glass is very nearly equal to that from lampblack; and indeed this is already well known.*

* See Leslie's "Inquiry into Nature and Propagation of Heat."

B. *Alum.*—Here the boiling-water apparatus could not be used, since alum becomes calcined at a temperature much below 212° ; but a self-regulating apparatus, invented by the late Mr. Kemp, was employed instead, giving a steady temperature of 98°.

A piece of plate glass .18 inch in thickness, gave....5.0
A piece of alum of the same thickness gave.....5.0

The radiation from the alum may therefore be reckoned equal to that from glass.

C. *Selenite.*—At the temperature of 98°—

A piece of selenite .125 inch in thickness gave....5.1
Under the same circumstances, glass .18 inch thick gave.....5.0

In the boiling-water apparatus,

The same piece of selenite gave.....18.0
While blackened glass gave.....18.5

The radiation from selenite may therefore be reckoned equal to that of alum or glass.

D. *Mica.*—A small box was constructed, having two windows of mica, the thickness of the mica in the one being .0009 inch, and of that in the other.02 inch. This box was filled with mercury (Professor Forbes having suggested the use of that metal, to keep up the temperature, while interfering very little with the radiation). The whole was then set on a glass dish in the boiling-water apparatus.

The radiation from the thin window was 11.2
While that from the thick window was.....12.7

As it would have been manifestly erroneous to compare these with the radiation from the blackened glass lying in contact with the bottom of the apparatus, the thin window was removed, and the blackened paper substituted in place of it.

While the thick mica window gave.....12.7
The blackened paper gave.....13.8

In comparing the radiations from the two windows, they were observed alternately. We see, therefore, that the radiation from mica, especially thin mica, is less than from lamp-

black in the proportion of 11.2 to 13.8, or the heat from thin mica is 80 per cent of that from lampblack.

E. *Rock Salt.*—As in the experiments with rock salt, it was desirable to obtain results of the greatest possible accuracy, the radiation from rock salt was not compared with that from blackened glass; for it was found that glass cooled more rapidly than rock salt.

The following plan was adopted :—

A piece of rock salt .18 inch thick (the temperature as in all the previous examples being about 210°, gave 3.2

A canister with water kept boiling, coated with lampblack.....22.0

In order to estimate how much the rock salt had cooled during the observation, the following experiment was made, without any diaphragm :—

Rock salt .18 inch thick taken to the cone at once, gave.....5.1

After cooling for 15 seconds, it gave.....4.9

It will be seen from this, that were the rock salt, instead of cooling during the 12 seconds necessary for the observation, kept at the temperature of 212°, it would not have given more than 3.3, while the hot-water canister gave 22.0.

5. From these experiments, it appears that glass, alum, and selenite, at low temperatures, have an intensity of radiation very nearly equal to that from lampblack; while mica radiates somewhat less, and rock salt greatly less. This is shown by the following table:

TABLE I.

RADIATING SUBSTANCE.	TEMPERATURE.	
	212°	98°
Lampblack,.....	100	
Glass,.....	98	27
Alum,.....		27
Selenite,.....	98	27
Thick mica,.....	92	
Thin mica,.....	81	
Rock salt.....	15	

Second Group of Experiments described.

6. I now proceed to the second group of experiments, or those designed to compare together the quantities of heat radiated at the same temperature from polished plates of the same substance, but of different thicknesses.

A. *Glass.*—No direct experiment of this kind was made on glass; for although a thick plate gave a somewhat greater radiation than a thin plate, it was imagined that this was due to the unequal cooling of the two plates. Indirectly, however, we may gather that thick glass radiates somewhat more than thin glass, from the following experiment, which belongs more properly to the fourth group:

A plate of crown glass .05 inch thick, being placed before the cone as a screen, and a similar plate .05 inch thick, and 3.75 inches square, being used as the source of heat at a distance of 6 inches, and no diaphragm used, the deviation was 0.95*

But when the source of heat was a similar plate .10 inch thick, the deviation became 1.45

Such a difference cannot be accounted for by the unequal cooling of the plates; and it would seem to indicate that a small quantity of heat from the interior of the thick plate reached the surface; which heat, having already been sifted by its passage through glass, was easily able to pierce the screen.

In another similar experiment,

One piece of crown glass .05 inch thick, gave a deviation of.....1.1

Two plates .05 inch thick, the one behind the other, 1.55

Three such plates,.....1.9

B. and C.—No experiments of this kind were attempted with alum or selenite.

D. *Mica.*—Experiments similar to those already described, only at a distance of 2½ inches from the cone, gave—

---

* Without any screen, it was calculated that the intensity of effect would have been equal to about 150°.

For mica .0009 inch thick (average of two sets of experiments),.....8.2
For mica, .02 inch thick (average of two sets of experiments),.....9.3

The experiments already quoted, which were made at a shorter distance from the pile, gave—

For mica, .0009 inch thick,.....11.2
For mica, .02 inch thick,.....12.7

E. *Rock Salt.*—Three pieces of rock salt were used. Their dimensions were:

	1st Piece.	2nd Piece.	3rd Piece.
Length.....	1.15 inch	2.15 inches	2.5 inches
Breadth.....	1.15 "	1.4 inch	1.4 inch
Thickness...	0.18 "	0.36 "	0.77 "

For these pieces, as well as for the other substances, I am indebted to the kindness of Professor Forbes. When placed behind the diaphragm, the farthest off surface was large enough to fill up the field of view,—that is to say, all rays from the cone striking the nearest surface struck also the surface farthest off ; the distance between the two surfaces being the thickness of the piece.

The following are the means of four sets of experiments :—

Radiation from 1st or thinnest piece.....3.4
" " 2nd or middle piece.....4.3
" " 3rd or thickest piece.....5.3

This proves that more heat is radiated by a thick than by a thin piece of rock salt.

The following experiments were devised by Professor Forbes, to confirm the above results.

(*a.*) The second piece of rock salt was placed obliquely behind the diaphragm, making an angle of 20° with the prolongation of the axis of the cone. A piece of fir wood of the same dimensions was placed in the same way. The two substances being compared in this position, and also in the usual position behind

diaphragm (viz., perpendicular to the direction of the cone's axis), the following was the result :

	Oblique.	Usual position.
Rock salt .36 inch thick,	4.0	4.0
Wood, same size as rock salt,	9.1	14.1

In order that this experiment may be understood, it may be well to mention, that, when the plate was placed obliquely behind the diaphragm it did not quite fill up the field of view. Hence the wood gave out less heat to the cone in this than in its ordinary position.

It appears, therefore, that the radiation from rock salt, in a direction making a small angle with the surface, bears a greater proportion to the corresponding radiation from wood than when both radiations are taken perpendicular to the surface. The reason undoubtedly is, that in the former case the rays come from a greater thickness of the substance, so that their intensity is increased.

β. The middle-sized piece of rock salt was bound tightly to the thickest piece, with a slip of tin foil between, so that the whole might cool as one piece, and thus obviate any objection that might be brought against the results, founded on the unequal cooling of the plates, owing to their thicknesses being different.

The surface of the middle-sized piece facing the pile, gave	6.3
That of the thickest piece, gave	8.1

The plates, therefore, still retain their inequality of radiation; but the amount from each was increased, owing, no doubt, to the reflection and radiation from the tin foil. The radiation from the tin foil may be estimated at 1.0, deducting which, we have 5.3 and 7.1; the increase now being due to reflection from the tin foil.

7. It thus appears, that while the difference between the radiating power of thick and thin glass is so small as not to be capable of being directly observed, there is a perceptible difference between the radiation from thick and thin mica, and a

still more marked difference between the radiation from plates of rock salt of unequal thickness.

But (at least with the thicknesses used) the greatest radiations from mica and rock salt were still below that from lampblack, and the radiation from rock salt greatly so.

The following table exhibits the results of the second group of experiments:

TABLE II.

SUBSTANCE.	RADIATION FROM THICK PLATE.	RADIATION FROM THIN PLATE.
Glass,.....	100	100
Mica,.....	100	89
Rock salt,.....	100	Middle 81 thin 64

Third Group of Experiments described.

8. I now proceed to consider the third group of experiments, or those made with the view of comparing the radiations from various polished surfaces with that from lampblack, as regards the *quality* of the heat; its quality being tested by its capability of transmission through a screen of the same material as the radiating plate.

A. *Glass.*—In an experiment already described, where a plate of crown glass .05 inch thick was used as a screen, and a similar plate of crown glass as a source of heat—

We had.....0.95
A similar plate .1 inch thick as the source of heat, gave.....1.45
Blackened paper attached to a similar surface of plate glass, .3 inch thick, the blackened side being next the pile,.....1.95

Therefore heat from a thin plate of glass is less transmissible through glass than heat from blackened paper.

B. and C.—No experiment of this nature was made with alum or selenite.

D. *Mica.* —The apparatus already described gave—

	Without screen.	With mica screen .0025 inch thick.
For window (the window, it will be borne in mind, is the radiating surface), .0009 inch thick	11.2	2.5
Window .02 inch thick	12.7	3.2
Blackened paper attached to glass lying on the bottom of the boiling-water apparatus, gave	21.0	6.3

We have therefore the proportion of heat passed by mica screen

For heat from thin mica window,	.223
" " " thick " "	.260
" " " blackened paper,	.300

E. *Rock salt.*—The thickest piece of rock salt (thickness .77 inch) being used as a screen, and the diaphragm withdrawn, in order to give greater results; the middle sized piece of rock salt gave—

With screen.	Without screen
6.1	19.6

The same screen stopped 3 rays out of 12 for *ordinary* lampblack heat.

This experiment is sufficient to show that rock salt is much less diathermanous for heat from rock salt than for ordinary heat. The common opinion, that rock salt is equally diathermanous for all descriptions of heat, is therefore untenable.

9. From the third group of experiments it appears, therefore, that heat emitted by glass, mica, or rock salt, is less transmissible through a screen of the same material as the heated plate, than heat from lampblack; this difference being very marked in the case of rock salt.

Fourth Group of Experiments described.

10. I now proceed to the fourth group of experiments, or those made with the view of comparing the radiations of plates of the same substance, but of different thicknesses, with regard to the quality of the heat radiated.

A. *Glass.*—It has been already shown (Art. 8), that heat from crown glass .05 inch thick is less transmissible through glass, than that from crown glass .10 inch thick.

B. and C.—No experiments of the kind were made on alum or selenite.

D. *Mica.*—It has been already shown (Art. 8), that heat from thin mica is less transmissible through a mica screen than heat from thick mica.

E. *Rock salt.*—With a screen of rock salt .18 inch thick, the following result was obtained :

Thickest piece of rock salt, heated to 210°
(thickness .77 inch), gave.....2.5
Middle-sized piece of rock salt, heated to 210°
(thickness .36 inch), gave.....1.7
Thinnest piece of rock salt, heated to 210°
(thickness .18 inch), gave.....1.1
Without any screen, the same pieces gave—
Thickest,.....4.9
Middle-sized,.....4.1
Thinnest,.....3.3
Proportion of heat from thickest piece passed..... .51
Proportion of heat from middle-sized piece passed .41
Proportion of heat from thinnest piece passed..... .33

A similar experiment, with a screen .29 inch thick gave—

	With screen.	Without screen.	Proportion passed.
Thickest piece...	2.6	5.4	.48
Middle-sized, ...	1.8	4.5	.40
Thinnest.....	1.2	3.5	.33

It follows from this, that a screen of rock salt passes heat from thick, more easily than heat from thin rock salt.

11. From this fourth group of experiments, we learn that heat from thick plates of glass, mica, or rock salt, is more easily transmitted by screens of the same nature as the heated plate than heat from thin plates of these materials.

The following table exhibits the results of the third and fourth group of experiments :

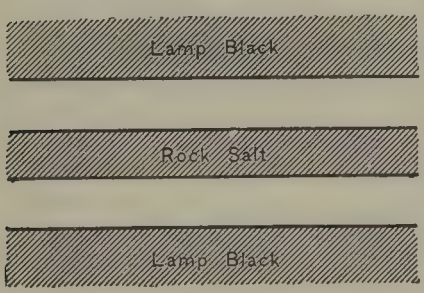
TABLE III.

SOURCE OF HEAT.	No. of Rays out of every 100 that pass through a screen of the same material as the source of Heat in 1st column, the screen being of only one thickness for each material.	No. of Rays of Lampblack Heat out of every 100 that pass through the same screen
Glass (crown $\frac{1}{20}$th inch thick),	0.66	1.33
Glass (crown $\frac{1}{10}$th inch thick),	1.0	
Mica (thickness .0009 inch),	22	30
Mica (thickness .02 inch),	26	
Rock salt (thickness .18 inch),	33	(Art. 12)
Rock salt (thickness .36 inch),	41	82
Rock salt (thickness .77 inch),	50	

Results deducible from the foregoing Experiments.

12. These experiments, as well as others yet to be described, may be explained by Prévost's theory of exchanges, somewhat modified.

In the first place, it would seem to be a consequence of this theory, that radiation must take place from the interior as well as from the surfaces of bodies. For, suppose that we have two indefinitely extended surfaces of lampblack, as in the figure, and between them a plate of rock salt of a certain thickness, also indefinitely extended; and let the whole be kept at the same temperature. Then, since the temperature of the rock salt remains the same, it must radiate as much as it absorbs. But a thicker plate of rock salt, placed under the same circumstances, would absorb more of the heat radiated from the lampblack because each ray would have to pass through a greater depth of the substance of salt; hence a thick plate of rock salt must radiate more than a thin plate. We see likewise, the reason for the small radiative capacity of



rock salt to be its small absorptive capacity. In order to prove this deduction from Prévost's theory experimentally true, the following experiment was devised:

A boiling-water canister, coated with lampblack, was put behind the diaphragm, filling up the field of view, and the three pieces of rock salt heretofore used as sources of heat, were now separately used as screens, being put before the diaphragm, so that the heat from the canister had to pass through their substance before reaching the cone. The following was the result:

	Without any screen.	Screen of Rock salt .18 inch thick.	Screen of Rock salt .36 inch thick.	Screen of Rock salt .77 inch thick.
Radiation from canister,	21.3	17.6	16.8	15.8

The difference between heat absorbed by plate, thickness = .18 inch, and that absorbed by plate, thickness = .36 inch,

is.....1.2
Another similar experiment gives.....0.9 } Mean 1.1

The difference between heat absorbed by plate, thickness = .36 inch and that absorbed by plate, thickness = .77 inch,

is.....1.0
Another similar experiment gives.....1.3 } Mean 1.1

These should nearly correspond to the differences between the radiation from the same place, under their ordinary circumstances of position (if the theory be true which asserts that the absorption of such a plate equals its radiation); accordingly we find that

The difference between heat radiated by plate, thickness.....= .18 inch
And that radiated by plate thickness = .36 inch, } Is 0.9

While the difference between radiation of plate thickness.....= .36 inch
And that of plate thickness.....= .77 inch, } Is 1.0

(Art. 6, mean of four sets of experiments.)

We see, therefore, that there is an agreement between the two sets of differences, as near as can be reasonably expected.

13. If we now suppose a plate of glass, and not a plate of rock salt, placed between surfaces of lampblack, the plate, whether thin or thick, will allow scarcely any heat to pass through it; and, consequently, plates of different thicknesses will all absorb very nearly the same amount, that is, nearly all that enters them. In this case, therefore, the radiation (which is equal to the absorption) will be very slightly increased by an increase of thickness of the plate. Also the amount of heat radiated, being equal to the heat absorbed, will be very nearly as great as that from lampblack.

14. There are, therefore, two peculiarities of the radiation from plates of diathermanous substances, and which are most marked for those substances which are most diathermanous.

1st, That the amount of radiation from such plates is less than that from lampblack.

2d, That the amount of radiation from such plates increases with the thickness of the plate.

The correlation between these different properties of bodies is seen from the following table:

TABLE IV.

Bodies ranked according to their Radiating Capacity (least radiating first).	Bodies ranked according to their Diathermancy (most diathermanous first).	Bodies ranked according to the proportion by which their Radiation is increased by increasing the thickness.
A stratum of heated gas (from Melloni's Experiments),	A stratum of gas.	
Rock salt.	Rock salt.	Rock salt.
Mica.	Mica.	Mica.
Glass.	Glass.	Glass.
Selenite.	Selenite.	....
Alum.	Alum.	....

15. The reason why radiation has hitherto been supposed to be confined to the surface, or to an exceedingly small distance below the surface of a body now becomes obvious. The effect of coating a surface of polished metal with gum, for instance, is

to increase the radiation; but, after a very small thickness of film, an additional coating is powerless to increase the radiation; the reason being, not that radiation is incapable, in all cases, of taking place except at the surface; but because such films being exceedingly impervious to heat of low temperatures, the radiation from them is very little increased by increasing their thickness.

Since, therefore, it appears that radiation takes place from the interior as well as from the surface of bodies, the question arises, are we to suppose each particle of each substance to have, at a given temperature an independent radiation of its own, equal, of course, in all directions? *A priori*, this is the most probable supposition, and it seems likewise to be conformable to experiment.

16. In an experiment already described,

A plate of crown glass .05 inch in thickness being used as a screen, the quantity of heat radiated from crown glass .05 inch thick that passed, was.....0.95

While of that radiated from crown glass, .10 inch thick there passed.....1.45

Another experiment gave—

Quantity of heat from crown glass .05 there passed..1.1

Quantity radiated from two plates of crown glass, each .05 inch thick the one placed loosely behind the other,.....1.55

From this we may infer, that the radiation from two plates of glass placed loosely behind each other is the same as the radiation from a plate of double the thickness, and, consequently, that the radiation from a particle of a substance does not diminish owing to its being placed in the interior.*

17. Let us now refer to the radiation from rock salt:

The radiation from a piece .18 inch thick, was.....3.4

That from a piece .36 inch thick, was.....4.3

That from a piece .77 inch thick, was.....5.3

* The idea of this experiment was derived from a remark of Professor Forbes, who suggested that several plates of rock salt, the one behind the other, might be advantageously substituted for a thick plate of the same material as giving the very same result.

Now if we suppose the radiation of a particle in the interior to be as intense as that of a particle at the surface, why, it may be asked (since rock salt is extremely diathermanous), does not a piece of double thickness give nearly a double radiation and so on, the radiation increasing very nearly as the thickness ?

If we still hold the doctrine of an equal and independent radiation from every particle, we are shut up to the conclusion that rock salt must be comparatively opaque to heat radiated by itself, a result which is abundantly confirmed by experiment.

Thus while the radiation from rock salt .18 inch thick, without any screen, is 3.4, with a screen of rock salt .18 inch thick it becomes 1.1.

If, therefore, we have a piece of rock salt of double the thickness, or .36 inch thick, we should expect that the radiation from it would be $= 3.4 + 1.1 = 4.5$. It is, in fact, 4.3. The difference (0.2) being within the limit of error of observation.

In rock salt, therefore, we may suppose each particle to have an independent radiation of its own, unaffected by its distance from the surface.

18. We see, therefore, that the opacity of rock salt with regard to heat radiated by itself, is a consequent of the admission, that the radiation from rock salt does not increase so rapidly as the thickness increases ; and this again results from the fact, that the absorption of heat by a plate of rock salt does not increase so rapidly as the thickness increases. This, again, is due to the fact, that the first part of the plate of rock salt sifts the heat so that it is more easily transmitted by the second part; and this confirms the result arrived at by Professor Forbes, who, finding that rock salt stopped heat of lower temperature rather more readily than heat of high temperature, concluded that there are a few rays for which rock salt is opaque.*

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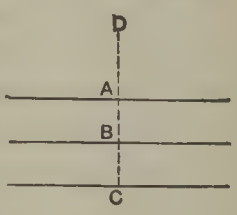
* To take a numerical example, let us suppose the heat from a single plate of rock salt to be = 1, then the heat from a plate four times the thickness, or (which is the same thing) the heat from four single plates, one behind another, should be nearly four times as much or = 4 (if we suppose the heat from each of these four plates to be readily passed by the plates between it and the pile), but the heat from the four-fold plates, instead of being four times as much, is not double of the heat

We conclude, therefore, that every body which sifts heat in its passage through its substance is more opaque with regard to heat radiated by a thin slice of its own substance, than it is with regard to ordinary heat.

19. This conclusion may be also stated thus: We have before proved (Art. 12) that the radiation of a thin slice of any substance equals its absorption; we now add that the heat radiated is the same as that absorbed, with regard to quality as well as quantity.

For this expresses the fact, that substances which sift heat are likewise opaque with respect to heat radiated by themselves. For, since the heat which they absorb is manifestly that kind of heat for which they are opaque, if the description of heat radiated is the same as that absorbed, then they also will be opaque with respect to heat radiated by themselves. Considering, therefore, the heat of any temperature to consist of heterogeneous rays, we may state the law thus: *"The absorption of a plate equals its radiation, and that for every description of heat."*

20. A more rigid demonstration may be given thus: Let AB, and BC be two contiguous, equal, and similar plates in the interior of a substance of indefinite extent, kept at a uniform temperature. The accumulated radiation from the interior impinges on the upper surface of the upper plate; let us take that portion of it which falls on the particle A, in the direction DA. This ray, in passing from A to B, will have been partly absorbed by the substance between A and B, but the radiation of the upper plate being equal to its absorption (since its temperature remains the same), the ray will have been just as much recruited by the united radiation of the



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from the single plate; hence, the heat from any of the interior plates of the compound plate is passed with great loss, by the plates between it and the pile. Now, since the absorption of a plate equals its radiation, the reason why the four-fold plate scarcely radiates twice so much as the single one is, that it scarcely absorbs twice as much; and this again is due to the fact, that the heat after it has passed the first plate of the four-fold plate has become sifted, and passes with little diminution of intensity through the other three plates.

particles between A and B, as it was diminished in intensity by their absorption. It will therefore reach B with the same *intensity* it had at A. But the *quality* of the ray at B will also be the same as its *quality* at A. For, if it were different, then either a greater or less proportion would be absorbed in its passage from B to C, than was absorbed of the equally intense ray at A, in its passage between A and B. The amount of heat absorbed by the particles between B and C would therefore be different from that absorbed by the particles between A and B. But this can not be; for, on the hypothesis of an equal and independent radiation of each particle, the radiation of the particles between B and C is equal to that of the particles between A and B, and their absorption equals their radiation. Hence the radiation impinging on B, in the direction of DB, must be equal in quality as well as in quantity to that impinging upon A; and, consequently, the radiation of the particles between A and B must be equal to their absorption, as regards quality as well as quantity; that is, this equality between the radiation and absorption must hold for every individual description of heat.

21. The following experiment illustrates this law:

The quantity of heat radiated from crown glass screen, .05 inch thick, which passes through a crown glass screen .05 inch thick, ... = 0.95

While that from plate glass .3 inch thick, covered with blackened paper (the blackened paper being next the pile), which passes through the same screen... = 1.95

But if the surface of crown glass .05 inch thick, farthest from the pile, be coated with paper, the polished surface being next the pile, then the amount which passes the screen, ... = 1.85

And if three plates, the one behind the other, of crown glass, each .05 inch thick, be used as the source of heat, the surface farthest from the pile of the farthest off plate only being covered with paper, the amount of radiation which passes the screen, ... = 1.95

Such a plate of glass or series of plates, therefore, by having the farthest off surface coated with paper, gives out heat

similar to that from paper or lampblack; the reason being, that the heat from the paper on the farthest off surface is as much recruited as it is absorbed by its passage through the glass, both as regards quantity and quality; so that the radiation which falls upon the cone is virtually that from paper or lampblack.

22. There is little difficulty in explaining why heat from a thick plate of any substance should pass more readily through a screen of the same substance than that from a thin plate. The reason is, that the heat from the interior of the thick substance, having been sifted in its passage, is, therefore, now more easily able to pass through a screen of the same substance.

23. We see also why, generally speaking, bodies at the same temperature radiate the same quality of heat; let us, for instance, take a tolerably thick plate of glass, and a surface of lampblack, and compare them together. Since the plate of glass absorbs nearly all the rays that fall upon it, it will radiate nearly as much as lampblack; and since the quality of the radiated is the same as the quality of the absorbed heat, its radiated heat will very nearly have the same quality as that which is radiated by lampblack.

The influence of the Reflective and Refractive Powers of Bodies on their Radiation considered.

24. Hitherto in these investigations no account has been taken of reflection at the surfaces of the plates, because—1st, those rays only were considered which passed perpendicularly, or nearly so, through such plates; and, 2d, because the indexes of the refraction for the substances experimented on were not very high.

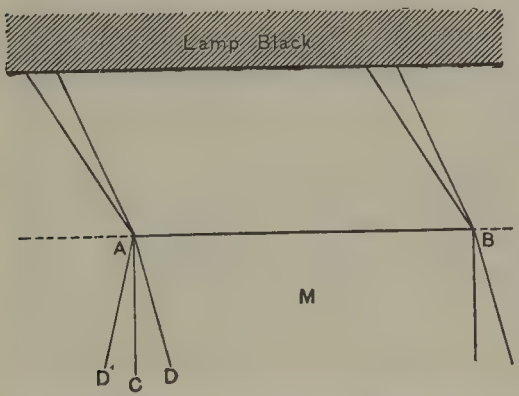
But for rays passing obliquely through such media, or for rays passing in any direction into substances such as metals, we must take account of reflection from the surface which will influence materially our results.

Thus, no substance is so opaque for heat as metals, but yet only a small portion of the heat falling on them is absorbed, the rest being reflected back; consequently for such bodies the radiation (which must be equal to the absorption) is very small.

It is also desirable, for another reason, to investigate the laws according to which the reflective nature of the surface of

a body influences its radiation. For the question arises, is the law of an equal and independent radiation of each particle of a body theoretically consistent with equilibrium of temperature? That is, suppose we have an irregularly-shaped inclosure walled round with a variety of substances, and each particle of each substance radiating into the inclosure, from the sides of which it is reflected many times backwards and forwards before it is finally absorbed; this being the case, will the law of equal and independent radiation, and those of reflection and refraction, so fit with one another, that every particle of the walls of the inclosure shall absorb precisely as much heat as it radiates? It will be endeavored to show that these laws are so adapted to each other; and I shall select for the proof a definite form and description of inclosure, the conclusions arrived at rendering it highly probable (if not rigidly demonstrating) that the same adaptation will hold good for every inclosure, however irregular or varied.

For these reasons, I shall now endeavor to investigate what connection the radiation of a substance has with the reflective power of its surface; and in doing so (in order to abstract entirely from the effects produced by the variable thickness of the radiating plate), I shall suppose it to be of indefinite thickness; so that all the heat which enters it is absorbed. Our consideration is, therefore, limited to the effects of *one* surface.



25. Let AB be a portion of the line of section of an indefinitely extended surface with the plane of the paper supposed perpendicular to the surface, and let this surface belong to a body (M) of indefinite thickness downwards; also let there be an indefinitely extended surface of lamp-black parallel to this lower surface, as in the figure. Lastly, let the whole be kept at uniform temperature. In order that the body (M) may be maintained at this temperature, it is necessary that the heat which has left the surface AB, having

come from the interior of (M), in the direction contained in any very small angle CAD, shall be replaced by an equal quantity of heat entering the surface AB, to diverge into the interior through the same small angle CAD. For, by this arrangement it is clear the particles in CAD get back as much heat as they give out.

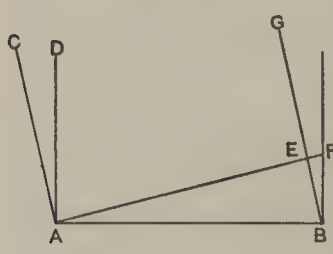
Part of the heat, no doubt, which fell on A in any direction DA, would be reflected back in the direction AD′, making the same angle with the surface as AD ; but this loss would be made up for by part of the heat falling upon A, in the direction D′A, being also reflected back in the direction AD.

The internal reflection at A being compensated for, if the heat that really leaves the medium be also compensated for, then as much heat will be passing at A in the direction AD as will be passing in the direction DA. It will be the same, therefore, as if the body, instead of having a surface at A, were indefinitely extended upwards from A, as well as downward ; in which case, as has been already shown (Art. 20), there will be equilibrium of temperature, provided that the radiation of a particle is equal to its absorption, and that for every description of heat.

Before proceeding further with this investigation, it will be necessary to establish some preliminary propositions.

26. *1st Preliminary Proposition.*

The heat which falls on the line AB in the directions contained in the very small angle CAD, is the same which falls on AE, perpendicular, EB, through the same very small angle. For every ray which fell on AB passed through AE, with the exception of a small quantity which passed through EF ; but the angle EBF being very small, EF is very small compared with AE, and consequently the heat falling on EF may be neglected in comparison with that falling on AE.



It is clear, also, that the heat falling on AB is proportional to AB, and to the size of the very small angle CAD.

The above will still hold if, instead of the substance of which AB is the surface being supposed below AB, and the rays fall-

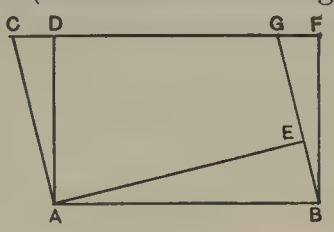
ing on it through a vacuum, we suppose a substance to be indefinitely extended upward and the rays to originate in the substance itself, and fall on its surface AB.

For, although any ray GE, which falls on E, will be partly absorbed between E and B, it will be as much recruited by the united radiation of the particles between E and B as it was absorbed; so far, indeed, as regards quality and intensity (from what has been already proved, Art. 20), we may consider such a ray to be traversing a vacuum, it being recruited just in proportion as it is absorbed.

It is evident, also, that in this case the quantity of heat falling on AB will be proportional to the size of the very small angle CAD.

27. *2d Proposition.*

First case.—If AB represent a surface (the substance being below AB), and CF a surface of lampblack indefinitely extended (as in Art. 25), from which rays fall on AB through a small angle CAD; then, if AE be drawn perpendicular to GB, the heat that falls on AB will = a const. × AE, whatever be the value of the angle CAB.



For, since the angle CAD is exceedingly small, CD may be considered very small in comparison with CF or CG; therefore the heat which impinges on AB through the angle CAD may be taken to be that which radiates from CG in direction between CA and DA; but since the radiative power of lampblack in any direction varies as the sine of the angle which that direction makes with the surface, this will = const. × AE. Hence, if R × CAD be the quantity of heat which falls on AB, when AB is perpendicular to GB, that which falls on it when GB makes any angle GBA with AB. will be R × CAD sin GBA.

If i denote the angle which GB makes with the perpendicular to AB, then the heat impinging on AB will be R cos i × CAD.

2d case.—If the substance be above AB, and the rays falling on AB originate in the substance, the same formula will hold, for it has been shown, in Prop. 1st, that in this case, the heat falling on AB through the small angle CAD = that which falls

on AE through the same small angle; but, since the radiation from the interior of the substance is the same in all directions (each particle radiating independently and equally in all directions), the amount falling on AE will not be affected by the angle which AE makes with the surface; hence the heat falling on AB = const. × AE = const. × sin GBA.

If R′ × CAD = quantity which falls on AB when AB is perpendicular to GB, that which falls on it when GB makes any angle GBA with AB, will be R′ × CAD sin GBA; also the expression corresponding to R cos i' × CAD will be R′ cos i'' × CAD.

28. *3d Proposition.* Let a ray strike the surface of a medium, at an angle of incidence = i; and another ray at an angle of incidence $i + \delta i$, it is required to find the difference between the two angles of refraction.

Let μ be the index of refraction, then,

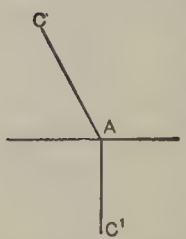
$$\sin i = \mu \sin i''$$

Hence,
$$\delta(\sin i) = \mu\, \delta(\sin i'')$$
$$\cos i\, \delta i = \mu \cos i''\, \delta i''$$

Hence,
$$\delta i'' = \frac{\cos i}{\mu \cos i''} \delta i$$

29. I shall also make the following supposition with regard to the laws of reflection and refraction.

1st. That if Q represent the quantity of heat falling on the surface of a medium in any direction CA, and aQ be the quantity reflected, then $(1 - a)$ Q is the quantity of heat refracted into the medium in the direction AC′. This follows from the law of the conservation of *vis viva.*



2d. That if the same heat Q originate in the medium, and strike A in the direction C′A, the quantity reflected back into the medium will be aQ, and the quantity refracted out in the direction AC will be $(1-a)$ Q.

30. These preliminary propositions being established, and suppositions made, let us suppose that heat from the surface of lampblack strikes the surface AB of the indefinitely thick medium (Fig. Art. 25) through a small angle δi (i being the angle of incidence), by Prop. 2d. the quantity of this heat will

be $R \cos i \delta i$; while the part of it which enters the substance we shall call $(1-a)$ $R \cos i \delta i$. These rays will diverge in the substance through an angle $\delta i' = \frac{\cos i}{\mu \cos i'} \delta i$ (Prop. 3).

But the quantity of heat that falls on AB from the interior through this angle will be

$$R' \cos i \delta i' = R' \cos i' \frac{\cos i}{\mu \cos i'} \delta i = \frac{R'}{\mu} \cos i \delta i,$$

and the portion of this which leaves the medium will be $\frac{(1-a) R' \cos i \delta i}{\mu}$

Equating this with $(1-a)$ $R \cos i \delta i$, which enters the medium, we have $\frac{R'}{\mu} = R$ or $R' = \mu R$. With this supposition, therefore, the law of an equal and independent radiation of each particle will give us equilibrium of temperature in the particular case under consideration. Had R′ been a function of i', it would have shown that the law of an equal an independent radiation was inconsistent with equilibrium of temperature.

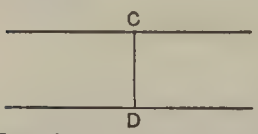
31. Only part, however, of the heat from the lampblack falling on AB entered into the medium, a portion of it $= a R \cos i \delta i$ being reflected back to the lampblack, hence the total quantity of heat radiated and reflected which leaves the surface AB through the small angle δi will be $= R \cos i \delta i$, the same as if the substance had been lampblack, the only difference being, that, in the case of lampblack, *all this heat is radiated*, whereas in other substances *only part is radiated, the remainder being reflected heat.*

32. Although we have considered only one particular case, yet this is quite sufficient to make the general principle plain. Let us suppose we have an enclosure whose walls are of any shape, or any variety of substances (all at a uniform temperature), the normal or statical condition will be, that the heat radiated and reflected together, which leaves any portion of the surface, shall be equal to the radiated heat which would have left that same portion of the surface, if it had been composed

of lampblack. And, indeed, we may see, from what has been already proved, that should such a state of things only once take place, it would always remain, there being no disposition to alter it.

Let us suppose, for instance, that the walls of this enclosure were of polished metal, then only a very small quantity of heat would be radiated ; but this heat would be bandied backwards and forwards between the surfaces, until the total amount of radiated and reflected heat together became equal to the radiation of lampblack.*

33. The equation $R' = \mu R$ must necessarily hold for every individual description of heat. We have, therefore, two laws necessary to the equilibrium of temperature—1st, That the absorption of a particle is equal to its radiation, and that for every description of heat ; 2d, That the flow of heat from the interior upon the surface of a substance of indefinite thickness, is proportional *caeteris paribus* to its index of refraction and that for every description of heat. It will, however, be borne in mind, that the former of these laws has been verified by ex-



* This will be clearly seen if we consider only those rays that are radiated perpendicular to the surface to the case of two parallel plates of polished metal of the same description radiating to one another. For let r be the common radiation of the point C in direction CD, and of the point D in the direction DC, then since these radiations are bandied backwards and forwards in the directions CD, DC, until they are extinguished, we have the total quantity of heat falling on D in the direction CD (if ar denote the proportion of r reflected after one single reflection) expressed as follows:

$$\left.\begin{array}{l}\text{Total heat radiated and reflected, falling on D,}\end{array}\right\} = \left\{\begin{array}{l} r+a^2r+a^4r+\text{etc,} \\ +ar+a^3r+a^5r+\text{etc.}\end{array}\right\} \begin{array}{l} = r(1+a+a^2+a^3) \\ = \frac{r}{1-a} \text{ (since } a < 1)\end{array}$$

But $1-a$ denotes the absorptive power of the metallic surface (all the heat not reflected being absorbed). Hence, since the radiative powers of bodies are proportional to their absorptive powers (Leslie's Inquiry) 1 being the absorptive power of lampblack, the perpendicular radiation of a lampblack point will be $= \frac{r}{1-a}$ which is the very same expression we have obtained for the total heat radiated and reflected together, falling on D, in the same perpendicular direction from the metallic point C.

periment, while the latter is only deduced from a theoretical investigation. It will also be seen, that by increasing the thickness of the radiating plate indefinitely, the radiation becomes ultimately independent of the diathermancy of the plate and is regulated only by its refractive index.

34. The connection which we have attempted to trace between the refractive and radiative power of a substance, presumes that those rays which we have been considering, have the power of forming wave lengths with the medium under consideration ; that is, of being capable of proper reflection and refraction.

It may be, however, that glass and other similar substances are so opaque, with respect to most of the rays of heat of low temperature, as to stop them almost entirely at the surface.

As such rays may, therefore, be conceived to be absorbed within the limit of physical surface of the medium, the corresponding radiation may be conceived to proceed from this physical surface. To such a case we may perhaps suppose reasoning similar to that of Fourier (as given by Professor Forbes in the *Philosophical Magazine* for Feb., 1833,) to be applicable ; the intensity of radiation being therefore proportional to the sine of the angle which the direction makes with the surface.

35. Let us now see, in conclusion, whether these investigations seem to point out any connection between internal radiation and conduction.

Now, without in the least confirming that these are identical there seem to be two points of similarity between them.

1st, Since the heat which enters metals is all absorbed at a very small depth, it follows that the flux of radiant heat from within upon the interior of metallic surface is derived from a very small depth.

Also, if we allow (what it has been endeavored to prove, Art. 30) that the flux of heat upon the interior of the surface is proportional to the index of refraction, this flux will be greatest in the case of metals which may be supposed to have a very high refractive power ; besides which, it will, as we have seen, be derived from a very small depth. The radiation of a metallic particle is therefore very great.

Now, if internal radiation be in any way connected with con-

duction, we might expect that good conducting substances should also be good internal radiators of heat, and we see they are so.

2d, The second bond of similarity is this. It seems to be a law that substances are almost invariably more diathermanous for heat of high temperature than for heat of low ; consequently, at high temperatures, the radiation of a thin plate or particle of a substance will bear a smaller proportion to the total lamp-black radiation of that temperature than at low temperatures. The internal radiations of particles of bodies would therefore diminish at high temperatures (not absolutely, but with respect to the proportion which they would bear to the total radiation of these temperatures). If the same rule holds for metals, and conduction be connected with internal radiation, we should expect that at high temperatures the conducting power of metals would be less than at low temperatures. Now this has been proved to be the case by Professor Forbes.

RESEARCHES ON RADIANT HEAT.

SECOND SERIES.

BY

BALFOUR STEWART.

Transactions of the Royal Society of Edinburgh.
Vol. XXII, Pt. I. pp. 59—73. *April,* 1859.

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VI.—RESEARCHES ON RADIANT HEAT.

SECOND SERIES.

By Balfour Stewart, M. A.

Communicated by Professor Forbes.

(Read April 18th, 1859.)

Division of Subject.

I. The first part of this paper describes the following groups of experiments :

Group I. On the effect which roughening the surface of a body produces on its radiation.

II. On the nature of that heat which is radiated by rock salt at 212° F.

III. On the radiation of glass and mica, at high temperature.

The second, or theoretical, portion of the paper, has reference to the law which connects the radiation of a particle with its temperature and to Dulong and Petit's experiments on this subject.

There is also an addition of a later date than the rest of the paper on General Diathermancy.

Instruments used, and Method of Investigation.

2. The instruments used, and the method of using them, were much the same as described in the first series of these researches, Art. 3. Should any difference occur in the method of conducting a particular experiment, it will be mentioned when the experiment so performed comes to be described.

First group of Experiments Described.

3. This group of experiments has reference to the effect of roughening the surface of a body upon its radiation. This was

suggested to the writer by Professor Forbes. The first substance tried was rock salt.

A. *Rock salt.*—It was found that roughening the surface by means of emery paper, until it became quite dim, had little or no effect in increasing the radiation, as will be seen from the following statement embodying the mean result of three sets of experiments.*

The pieces used were the middle piece (thickness = .36 inch) and the thickest piece (thickness = 77 inch), described in first series, Art. 6. These pieces were placed at a distance of about four inches from the mouth of the polished brass cone, and in order to increase the effect, no diaphragm was used. They were heated in the boiling-water apparatus already described. With this arrangement

The thick piece gave, when polished, a deviation of .21°.1
when roughened, ...21.8
The middle piece gave, when polished, a deviation of .13.6
when roughened, ...13.5

4. The next point was to ascertain if roughening had any effect upon the *quality* of heat radiated.

The following table will show that it does not alter the quality of the heat sensibly; its quality being tested by its capacity of penetrating a screen of rock salt.

TABLE I.

SOURCE OF HEAT.	Percentage of whole Heat which penetrates a Rock Salt Screen thickness .29 inch.
Rock salt, .77 inch thick, polished,	49
roughened,	51
Rock salt, .36 inch thick, polished,	42
roughened,	43

* In the experiments with roughened surfaces, only one of the surfaces of the substance was roughened, the other being left polished. In *radiation* experiments, therefore, the roughened surface was placed next the pile; while in *transmission* experiments it was placed furthest from the pile.

The trifling difference between polished and roughened salt in this table may fairly be attributed to error of experiment. We may therefore conclude, that roughening by emery paper neither alters the quantity nor the quality of the heat radiated by rock salt.

5. Again, the transmissive power of rock salt for lampblack heat of the temperature 212°, is not sensibly altered by roughening the surface. This will be seen from the following statement:

	The percentage of Lampblack heat transmitted was
With Screen of Rock salt, thickness .36 inch, polished,	77
With Screen of Rock salt, thickness .36 inch, roughened,	77

This result naturally follows from the previous one, for it has been shown (First Series, Art. 19) that the absorption of a plate equals its radiation and since roughening its surface does not influence the radiation it ought not to influence the absorption.

6. B. *Glass.*—It is already known that roughening the surface of a plate of glass does not sensibly increase its radiation. It is only necessary, therefore, to ascertain whether roughening the surface of a radiating plate of glass alters the capacity of its heat for penetrating a screen of glass. Accordingly, a plate of crown glass .05 inch thick, 3.75 inches square being placed before the cone as a screen, and a similar plate roughened, heated in the boiling-water apparatus, being used as the source of heat, and no diaphragm used,

The deviation was	1°.0
When the source of heat was a similar plate, .10 inch thick, the deviation became,	1.5
And lastly, when the source of heat was a plate covered with lampblack, the deviation was,	1.9

With the same sources of heat, only the glass polished instead of being roughened, these numbers were 0°.95, 1°.45, 1°.95. From the correspondence between these two sets of results, we may infer that the quality of the heat radiated by glass (at least in so far as transmission through a plate of glass

can test it) is not altered by roughening the surface of the glass.

7. And from all these experiments, we may infer (what has indeed been already remarked by Professor Forbes), that although roughening its surface with sand or emery paper renders a body dim for light yet it still remains specular for heat rays, which, possessing a greater wave length than those of light, are less liable than the latter to be influenced by scratches or furrows.

Second Group of Experiments Described.

8. The second group of experiments has reference to the nature of the heat which is radiated by rock salt at 212°. Its quality being tested by transmission through

α. A screen of mica.
β. One of mica split by heat.
γ. One of glass.

9. *α. Mica Screen.*—By the mean of three sets of experiments, a mica screen (thickness = .003 inch nearly) passed about 31 per cent of ordinary lampblack heat, while it only passed 18 per cent of rock salt heat. Or if we call the proportion of black heat passed by the mica 100, that of rock salt heat will be 58.

10. *β. Split Mica Screen.*—Two sets of experiments agreed in giving twenty per cent as the proportions of lampblack heat of 212°, transmitted through a screen of mica split by heat, while the proportion of rock salt heat transmitted by the same screen was only 15¼ per cent. These numbers are to one another as 100 to 76.

11. *γ. Glass Screen.*—In order to avoid secondary radiation from the screen, which, in this case, absorbs nearly all the heat, two screens of microscopic glass were used, the one behind the other, with an interval between.

Moreover, as in this case, the proportion of heat transmitted is exceedingly small, the following arrangement was adopted to make it measurable.

The experiment consisted of four parts,

1st. The effect of rock salt heat upon the pile without a screen was observed by the ordinary galvanometer.

2d. The effect of lampblack heat, also without a screen, was observed by the same galvanometer.

3d. The wires of the pile were then transferred to a more sensitive galvanometer, and the effect of lampblack heat observed, the glass screen being interposed.

4th. The sensitive galvanometer and glass screen being retained, the effect of rock salt heat was lastly observed.

By this method of experimenting, it was merely the relation between the diathermancy of the screen for lampblack heat and for rock salt heat that was measured; its absolute diathermancy for either of these heats not being determined. Two sets of experiments, conducted in this manner, gave the following result:

By the first set, calling the proportion of the whole lampblack heat which passed the screen 100, that of the rock salt heat which passed the same screen was 54. And by the second set, these numbers were 100 to 60.

12. As in these experiments with a glass screen the proportion of heat passed is very small, great numerical accuracy cannot be looked for and the results obtained are valuable rather as determining the direction and character of a fact, than as measuring the extent to which it holds.

13. It is already well known that rays of great refrangibility or small wave length pass through glass and mica more readily than those of an opposite character. The difficulty with which rock salt heat penetrates these substances as compared with ordinary heat might therefore lead us to infer that the wave length of this heat is greater than that of ordinary lampblack heat.

14. If, therefore, the heat radiated by rock salt is of great wave length since (First Series, Art. 19) the quality of the heat radiated is the same as that of the heat absorbed, it follows that the heat most absorbed by rock salt must be heat of great wave length; and this derives confirmation from a fact noticed by Professor Forbes, viz., that rock salt passes a somewhat greater proportion of heat of high temperature than of that of low; heat of high temperature possessing a less average wave length.

15. If we look now to the relative transmission of the two

descriptions of heat through mica split by heat, we see that the facility of transmission is yet in favor of ordinary heat, but not so strikingly as with a screen of common mica. This will be seen from the following table :

TABLE II.

NATURE OF SOURCE.	Transmission of Ordinary Heat, at 212° F.	Transmission of Rock salt Heat at 212° F.
Mica.....	100	58
Mica split by heat.....	100	76

Compare this with the following table deduced from the results given by Professor Forbes, in the Fourth Series of his Researches, Art. 9.

TABLE III.

NATURE OF SCREEN.	Transmission of Heat from Blackened Brass at 700° F.	Transmission of Black Heat at 212° F.
Mica .015 inch thick,.....	100	52
Mica split by heat,.....	100	64

From a comparison of these two tables, it will be seen that, as tested by the two substances, mica and mica split by heat, rock salt heat at 212° F bears to ordinary heat of that temperature a relation similar to that which ordinary heat at 212° F bears to heat at 700° F; that is to say, that just as heat of 212° F has a greater wave length than heat of 700° F, so rock salt heat at 212° F has a greater wave length than ordinary heat at that temperature. And the *surface stoppage* produced by splitting the mica, telling most powerfully upon heat of high temperature, or small wave length, while the *stoppage* by *substance* is in the opposite direction, we see how the one effect tends, to a certain extent, to neutralize the other, rendering the proportions of different kinds of heat passed by split mica more nearly alike than those passed by ordinary mica.

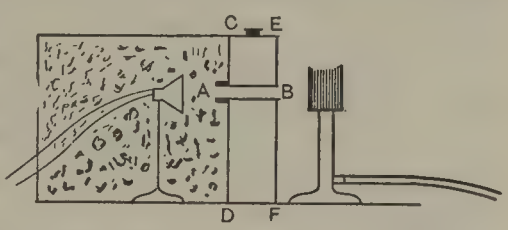
16. All these experiments concur in showing that heat from rock salt possesses very great wave length, and probably heat from a thin plate of this substance, at a low temperature, may

be found to possess a greater average wave length than any other description of heat which can be exhibited.

Third Group of Experiments Described.

17. I now proceed to describe the third group of experiments, or those on the radiation of glass and mica at high temperatures.

A. Glass.—For the experiments on glass, the following apparatus was used: The pile was placed within a box, and surrounded with cotton wadding. The orifice through which radiant heat was admitted into the box consisted of a brass tube AB, blackened in the inside. The diameter of this tube was ½ inch, its length 3 inches, and during the greater part of its length it passed through water, contained in the chamber CEFD. The side of the box (CAD) next the pile was lined with tin foil. Owing to the small divergence of the rays of heat which had to pass through the narrow tube, the cone might be placed several inches to the left of A without sensibly weakening the effect, and, on the other hand, the source of heat might be placed some distance to the right of D without ceasing to fill up the field of view. By this means, the distance between the pile and the source of heat being considerable, no currents of heated air from the latter would be able to reach the former; and as the tube AB was blackened in the inside, and passed through water, reflection and secondary radiation would both be avoided. By means of a lid fitting on the tube at A the aperture might be diminished at pleasure. The pile was connected with a very sensitive galvanometer.



When glass at a high temperature was the source of heat, a very small aperture was sufficient, and thus the advantage was gained of having the whole field covered with glass, all at a high temperature, which could not have been the case had the aperture been large.

Slips of glass about ¾ inch broad were used, and were set vertically, just touching a gas flame from a Bunsen's burner. When two slips—one behind the other—were used, the one

just touched that portion of the flame next the pile, and the other that portion furthest from it. A cross section of the arrangement is shown on page 59.

A single slip of glass about .1 inch thick thus heated gave a deviation of 16°.5, while two slips, the one behind the other, gave 18°.5. When the slips were .05 inch thick these numbers were 29°.1 and 36°.3.

18. From these experiments we may conclude, that at a high temperature, 700° or 800° F, the radiation from two plates of glass, one behind the other, is sensibly greater than that from one—a result which does not hold for glass at 212°. Or the fact may be stated thus:

The radiation of a single plate of glass bears a smaller proportion to the total radiation of 700° than at 212°.

19. It was next tried whether the capacity of a glass screen for passing heat from blackened copper at 700° was altered by its being heated.

In order to ascertain this, blackened copper at 700° F was placed behind a slip of glass, and the amount of heat from the copper which passed the glass was observed.

Firstly, When the glass was cold.

Secondly, When it was heated to between 700° and 800° F.

20. As in these experiments the considerably fluctuating temperature of the source of heat causes a somewhat large difference between successive observation, and renders necessary a great number in order to arrive at a correct result, it was thought desirable, instead of using momentary deviations, to employ permanent ones. This was done with complete success; the application of the heated copper, or its removal, causing an unmistakable alteration of the position of the needle.

21. The experiment was then varied in the following manner: The needle was kept permanently deviated by the heated glass, and the momentary swing due to the application or withdrawal of the heated copper was noticed, and was compared with that occasioned by the hot copper when the glass was cold and the needle at zero.

22. These experiments, which are not, perhaps, individually susceptible of very great exactness, agreed, however, in rendering it probable that glass, owing to its being heated up to

about 700° F. does not change its diathermancy for heat of 700° F.

23. B. *Mica.*—The experiments on mica were made with the ordinary galvanometer. A piece of mica, thickness about .008 of an inch, being used as a screen, and a diaphragm, .65 of an inch square, at the distance of three inches from the mouth of the pile, being employed, the mean of two sets of experiments made the proportion of black heat of 200° F passed by the mica to be 13 per cent. Placing an additional diaphragm of the same size 3½ inches beyond the first, and using as a source the temperature of 400° F, the mean of two sets of experiments made the proportion of heat passed by the mica screen to be 21 per cent.

In order to test whether the apparently greater diathermancy of the screen for heat of 400° F was owing to the difference in the nature of the heat, or to the heat at 400° F striking the screen more nearly at a perpendicular incidence, and thus experiencing less reflection as well as passing through a smaller thickness of mica, an experiment was made on heat at 200° F, with the arrangement and distance used for heat of 400° F, which seemed to show that this difference of distance does not affect sensibly the proportion transmitted. We may therefore conclude that the difference in proportions transmitted is owing to a difference of quality in the two descriptions of heat.

24. A cast-iron box was next constructed having this same plate of mica inserted as a window, so that, while one side of the box consisted merely of a moderately thin plate of cast-iron, the other, except round the edges, was composed of mica. The cast-iron side was then blackened, and the box filled with mercury. A thermometer inserted in the box measures the temperature. At 200° F, with the usual diaphragm three inches from the mouth of the pile, the proportion between the radiation of the blackened side and the mica window was, by the mean of three sets of experiments, as 100 to 87.8, while at 400° F, with the usual arrangement of two diaphragms, the same proportion was 100 : 84.1.

25. Let us endeavor to discuss these results. The radiation from the mica window consists of three portions :

a. The proper radiation of the mica plate.

β. That portion of the radiation of the mercury which has been able to penetrate the mica plate.

γ. That portion of the radiation of the mica which, striking upon the mercury, is reflected back by it and has penetrated the mica plate.

Now, supposing there was no mercury behind the mica, and that mica between 200° and 400° does not alter its diathermancy as a screen in any respect, let us inquire what ought to have been the result obtained. Then, since the radiation of a thin plate equals its absorption (First Series, Art. 19), and since the absorption of this mica plate was 8 per cent less at 400° than at 200° (Art. 23), its proportional radiation ought to be 8 per cent less at 400° than at 200°.

26. But the effect of the mercury behind the mica manifestly tends to diminish this difference. For we know that the mica (Art. 23) passes 8 per cent more of lampblack heat at 400° than at 200°; it will therefore no doubt pass a greater proportion of the heat from the mercury behind at 400° than at 200°. But we have reason to think that the radiation of mercury is nearly ¼ of that of lampblack.* Consequently we may suppose that owing to this action of the mercury, the proportional radiation of the mica window at 400° is increased about ¼ of 8, that is, 2 per cent. This reduces therefore the difference from 8 to 6 per cent.

27. But the mercury acts in another manner also in the same direction. Had mercury been a perfect reflector, its presence behind the mica would have been equivalent to doubling the thickness of the plate; for it would have sent the whole radiation of the mica that fell upon it back through the mica. But the difference between the proportional radiation at 200° and at 400° is less for a thick plate of mica than for a thin one (indeed, when the plate is indefinitely thick, this difference vanishes, and the proportional radiation is the same at all temperatures); this action of the mercury, therefore, would tend still further to diminish the already diminished difference of

* Provostaye and Desains estimated the proportion of heat reflected by mercury to be 77 per cent. The radiation, being complementary to this, may be reckoned to be 23 per cent nearly.

6 per cent. The amount of this action cannot be far from 2 per cent,* in which case the 6 per cent would be reduced to 4 per cent; now 3.7, or, in round numbers, 4 per cent is the observed difference between the proportional radiation of the mica window at the temperatures 200° and 400°.

28. We see thus that the behavior of the mica as a screen, compared with its behavior as a radiator, agrees very well with the supposition which we made in Art. 25; viz., the mica between the temperatures of 200° and 400° does not alter its diathermancy in any respect; a result similar to that which we have already deduced for glass (Art. 22) between somewhat wider limits.

29. Experiments with the same object in view, but of a more direct description, were made upon mica, similar to those already described as having been made upon glass, that is, it was endeavored to ascertain whether hot mica passed as much heat from hot copper as cold mica; but in these experiments the fluctuation was very considerable, probably owing to the small body of the mica. Nevertheless, they confirmed the results above obtained; viz., that mica does not change its diathermancy in any respect owing to its being heated.

30. We may therefore conclude that this property (at least within moderate limits) is common both to glass and to mica, and indeed, *a priori*, there appears no good reason why the mere heating of a substance should change its diathermancy. It is the theoretical importance of this property that has induced me to take pains to verify experimentally and its importance will be seen from some of the consequences which follow its establishment, which I shall now proceed to discuss.

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* It would have been better to have tested, by means of a direct experiment, to what extent the difference between the proportional absorption and radiation of mica at 200° F and at 400° F would have been diminished by doubling the thickness of the plate; but unfortunately the plate of mica was so much cut up by being used as a window, as to be unfit for being formed into a double screen.

We see, however, from Art. 37, that while the difference between the proportional radiation of a plate of glass (thickness 1mm) at 100° C and and 390° C is 9 per cent, the same difference for a plate of double the thickness is only 7 per cent, or 2 per cent less. We may, therefore, without much risk of error, adopt this difference of 2 per cent for mica under experiment.

On the Law which Connects the Radiation of a Body with its Temperature.

31. The experiments of Dulong and Petit upon the cooling of two thermometers, one naked, and the other covered with silver, seemed to show that the proportion between the radiations of these two substances was the same at the different temperatures of experiment.

Now I have endeavored to prove in these researches—1st, That the radiation of a thin plate at any temperature equals its absorption of black heat of that temperature. 2nd, That the diathermancy of glass and mica (and probably of other substances) is not altered by heating the substances. Again, it is well known that substances are generally more diathermanous for heat of high, than for heat of low temperature; it follows that the radiation of a thin plate of a substance at a high temperature should bear a less proportion to the total radiation of that temperature than at a low temperature.

32. While, therefore, it is likely that the radiation of a silvered thermometer (silver leaf being quite opaque for all heat) will bear a constant relation to that of a blackened thermometer at all temperatures, we should expect that for a naked thermometer, just as for the mica window, the radiation should bear a somewhat less proportion to the total radiation at a high temperature than at a low. We should therefore expect the radiation of the naked thermometer to increase somewhat less rapidly with the temperature than that of the silvered thermometer. Dulong and Petit, nevertheless, found the rate of increase to be the same for both.

33. Now, in the first place, since glass is exceedingly opaque for heat even of 300°C (the highest temperature experimented on), the difference we are in search of (analogous to the difference of four per cent in the mica window) would be exceedingly small. But, in the second place, Dulong and Petit had two thermometers, one of which, containing about three pounds of mercury, was used for high, and the other and smaller one for low temperatures. This latter circumstance will complicate or even vitiate their experiments so far as regards this peculiar difference we are treating of.

34. Although, for these reasons, attaching little importance to Dulong's and Petit's observations, so far as varying diathermancy is concerned, yet it may be well to state that they show, on the whole, a very small difference in the direction which would indicate a superior diathermancy of the glass at a high temperature.

35. Assuming it proved that the proportional radiation of a thin plate is less at a high than at a low temperature, I shall now endeavor to show that this difference increases as we diminish the thickness of the plate. To prove this, it is only necessary to exhibit the following table, given by Melloni :

TABLE IV.

NUMBER OF RAYS OUT OF 100 PASSED.				
Thickness of Glass Screen.	Locatelli Lamp.	Incandescent Platinum.	Black Copper at 390° C.	Blackened Copper at 100° C.
mm				
0.07	77	57	34	12
.5	54	37	12	1
1.0	46	31	9	0
2	41	25	7	0
4	37	20	5	0
6	35	18	4	0
8	33.5	17	3.4	0

36. We have already seen that glass does not change its properties with regard to heat, by being raised to the temperature of 390° C ; it is perhaps, however, too much to conclude, that when heated to the temperature of a Locatelli lamp, its properties would remain unchanged. At all events, in order to make use of the whole of the above table, we may suppose the properties of the glass to remain the same throughout, especially as the results we shall deduce from the supposition will be of the same nature as if we had only extended it to glass at 390° C.

37. Presuming, therefore, that the diathermancy of glass does not alter through its being heated, and allowing 4 per cent as the proportion of the heat striking it reflected from the first surface of a glass screen, and supposing also the same proportion of the heat which is able to reach the second surface to be

reflected from it, we may, on the principle that the proportional radiation of a plate equals its proportional absorption, construct the following table:

TABLE V.

PROPORTIONAL RADIATION OF GLASS PLATES AT DIFFERENT TEMPERATURES, (RADIATION OF LAMPBLACK = 100).

Thickness of plate.	Temp. of Locatelli Lamp.	Temp. of Incandescent Platinum.	390° C.	100° C.
mm				
0.07	16	37	61	84
.5	40	58	84	95
1.0	48	64	87	96
2	53	70	89	96
4	58	75	91	96
6	60	77	92	96
8	61.5	78	92.6	96

38. Let us call the proportional radiation of the glass plate at 100°C unity, and we derive the following table.

TABLE VI.

PROPORTIONAL RADIATION OF GLASS PLATES AT DIFFERENT TEMPERATURES, THEIR RESPECTIVE PROPORTIONAL RADIATIONS AT 100 C. BEING RECKONED UNITY.

Thickness of Plate.	100° C.	390° C.	Temp. of Incandescent Platinum.	Temp. of Locatelli Lamp.
mm				
0.07	1	.72	.44	.19
.5	1	.88	.61	.42
1.0	1	.91	.66	.50
2	1	.93	.73	.55
4	1	.95	.78	.60
6	1	.96	.80	.62
8	1	.965	.81	.64

39. We see thus that the radiation of thick plates of glass increases most rapidly, and that of thin plates least rapidly, as the temperature increases, and we may suppose, that if we could procure a plate of glass of sufficient tenuity, we might (without heating the plate at all), by finding its absorption for heat of

different temperatures find its radiation at those temperatures, which (if the plate were thin enough) would give us the law of radiation of a glass particle. This law would not increase nearly so fast with increasing temperatures as Dulong and Petit's law; it may even be that the radiation of glass particles is proportional to its absolute temperature.

40. But all substances (with the exception of black mica and black glass, whose peculiarity may perhaps be otherwise explained) have the same properties as glass with regard to heat; that is, they are more diathermanous for heat of high than for heat of low temperatures. The radiation of thin plates or particles of all substances will therefore increase less rapidly with temperature than that of black surfaces. It may therefore be, that the same law of radiation is common to very thin plates or particles of all bodies; this law (whatever it be) giving in all cases, a less rapid increase of radiation with temperature than is indicated by Dulong and Petit's law. Had, however, the diathermancy of thin plates of different substances in some cases diminished and in others increased for heat of high temperature, the law of radiation of a particle could not have been the same for all bodies.

The generality of this law of increased diathermancy of all bodies for heat of high temperatures seems, therefore, to me, to argue in favor of the universality of the unknown law of particle radiation which depends upon the former.

41. What, then, does Dulong and Petit's law express? The answer is, it expresses the law of radiation of indefinitely thick plates, and we have shown that it increases faster than the law of radiation of a material particle.

To facilitate the comprehension of this subject as much as possible, I have put it in the following shape. Suppose we have two substances opposite one another, the one having the temperature of 0°, and the other of 100°, the latter will of course lose heat to the former—let us call its velocity 100. Suppose, now, that (the first surface still retaining its temperature of 0°) the second has acquired the temperature of 400°; then we should naturally expect the velocity of cooling to be denoted by 400; but by Dulong and Petit's law, it is much greater. The reason of the increase may be thus explained.

At the temperature of 100° we may suppose that only the exterior row of particles of the body supplied the radiation, the heat from the interior particles being all stopped by the exterior ones as the substance is very opaque for heat of 100° ; while at 400° we may imagine that part of the heat from the exterior particles is allowed to pass, thereby swelling up the total radiation to that which it is by Dulong and Petit's law.

42. We have thus ascertained—1st, That Dulong and Petit's law is not the law of radiation of a material particle ; and, 2d, That this law increases less rapidly with the temperature than Dulong and Petit's law. But now the question arises, can any method be indicated of ascertaining, experimentally the law of radiation of a material particle ? Now, by continually diminishing the thickness of the plate whose radiation at different temperatures we are ascertaining, we certainly approach nearer and nearer to the desired law, and, by using the method indicated in Art. 37, we may avoid heating this plate at all and thus overcome one source of experimental difficulty. Yet the thinnest plate we can procure of a substance such as glass or mica acts, to all intents, as an indefinitely thick substance for a great many of the rays of heat—that is, it stops them all. The change, therefore, of the unknown law of particle radiation into Dulong and Petit's law will to a great extent, have taken place even within this very thin plate ; so that, in order to reach the desired law or even approximate to it, we should have to use much thinner plates than we could possibly procure ; and, even without the necessity of heating the films, the experimental difficulty and labor of such an investigation would be very great.

On the other hand, we may suppose that, since a thin film stops so much heat, a portion may be stopped in the physical surface of the body, and the absorption might thus influence the law of reflection of heat from the surface. The amount of this influence depending on the absorptive nature of the particles, we might be able to measure the absorption, and, consequently, the radiation of the physical surface, that is, of a very thin plate. But, in the first place, the difficulties of such an investigation would be even greater than in the previ-

ous case ; and, in the second place, the true law of reflection is not yet finally settled.

I am therefore induced to think that it is nearly hopeless to attempt to ascertain the true law of radiation of a material particle, at least by any method of experimenting depending upon the use of thin plates, or on the change which absorption may be presumed to cause in the amount of heat reflected from the surface of a body.

Edinburgh, March 22, 1859.

On General Diathermancy (added 15th June).

43. Circumstances having occurred which may interfere in the meantime with my further experiments on heat, I annex to this paper an account of some experiments made since the day of reading. These were proposed with the view of ascertaining whether diathermancy is confined to rock salt or whether bodies partake of this property. If the latter be the case, the reason why we have not hitherto ascertained it to be so is evidently the difficulty of obtaining crystals of many bodies sufficiently large to operate upon ; and if we wish to prove these diathermanous we must do so in a way that does not render necessary the use of large crystals.

44. Now, a body that is transparent for light, forms, when pounded, a white powder or one that reflects a great deal of light. It will be granted that the reason of this is because we have not only the reflection from the outer surfaces of the crystals, but also from many interior surfaces. Now the same remark is applicable to heat. A body that is diathermanous or transparent for heat should, as a powder, be white for heat, or, in other words, reflect it. But (First Series, Art. 31) the reflection *plus* the radiation of the body at any temperature equals the lampblack radiation at that temperature. Hence a powdered diathermanous substance ought to radiate less than lampblack. Accordingly, different substances having been pounded into a fine crystalline powder, made into a paste with water, spread on the two sides of parallelopipedons of wood, dried and one of the sides, when dry, rubbed over with lampblack, the following result was obtained :

TABLE VII.

Radiation at 212°		
Name of Substance.	White Side.	Black Side.
Table salt	83.1	100
White sugar	98.7	100
Alum	100.0	100
Sulphate of potash	88.1	100
Nitrate of potash	86.7	100

45. Thus we see that table salt being white for heat, the radiation of the white side is less than that of the black side; and further, white sugar and alum being both nearly black for heat, the radiation of the one side is nearly equal to that of the other. We see, moreover, that sulphate of potash and nitrate of potash, especially the latter, are white for heat, although not quite so much so as table salt. May we not therefore presume that these substances are diathermanous? There is, moreover, the following method of confirming the testimony in favor of the diathermancy of these substances as derived from this experiment.

46. Table salt being white for heat, part of the reflected heat will be composed of rays which have been reflected from the internal surfaces of crystals. Such rays have therefore been sifted, having left behind that description of heat which passes with difficulty through rock salt and also (Art. 9) through mica. The whole reflected heat from a surface of table salt should therefore be of a nature which passes more easily through mica than ordinary heat, and (First Series, Arts. 31 and 33) since the sum of the reflected and the radiated heat is equal both in quantity and quality to that from lampblack, it follows that the radiated heat from table salt (and probably from other substances white for heat) should, in order to make up the average quality, have a somewhat greater difficulty in passing through mica than ordinary lampblack heat. Accordingly, it was found that the diathermancy of a mica screen for heat from table salt was less than that for ordinary lampblack heat in the proportion of 92 to 100, while it was less for heat from pounded sulphate of potash in the proportion of 93 to 100, thus confirming the analogy between

rock salt and sulphate of potash. No such difference was observed for heat from sugar.

47. We see also from the above table that the radiation and therefore the absorption of table salt is 83.1 per cent, leaving 16.9 per cent for the reflected heat. Now Melloni found that chalk absorbed 56.6 per cent, and consequently reflected 43.4 per cent, of heat from a Locatelli lamp; and if we suppose table salt to be at least as white as chalk for heat of that temperature, we must conclude that table salt is less white for heat of 212°, than for heat from a Locatelli lamp, following in this respect the same law as chalk, which, from being nearly black for heat at 212°, becomes comparatively white for heat from a Locatelli lamp. There is also little doubt that table salt reflects more than 16.9 per cent of the light that falls upon it. Hence we may conclude generally that powders even of diathermanous bodies are less white for heat of low temperature than for heat of high temperature and for light.

48. It would also seem, that, although comparing one powder with another, there is no relation between apparent whiteness and whiteness for heat, since it was found that very white surfaces of pounded sugar and alum (the particles compressed, not made into a paste with water) reflected little or no heat; yet, comparing powdered surfaces of the same diathermanous body together, there seems to be some relation between their apparent whiteness and their whiteness for heat, insufficient pounding, or any circumstance which diminishes the apparent whiteness, diminishing also its whiteness for heat.

Biographical Sketch.

Balfour Stewart was born Nov. 1, 1828 at Edinburgh. He studied at the Universities of St. Andrews and Edinburgh, and later entered upon commercial life. His particular taste for physical science soon developed itself, however, and we have in 1858 a couple of his earliest papers. He became associated with Kelland and Forbes at this time and in 1858 contributed his most important work on the extension of Prévost's Theory of Exchanges in radiation. With the elaborate (at that time) facilities at his command he was able to demonstrate the equality of the radiating and absorbing power of every substance. For this and other work he was awarded the Rumford Medal some years later. In 1859 he was appointed director of the Kew Observatory, where for eleven years he directed the important studies and investigations carried on there. During this period he was seriously injured in an accident from which he never recovered. In 1870 he was appointed to the chair of Physics in Owens College, Manchester, which he occupied until his death Dec. 19, 1887. During this time he issued his well-known texts in Physics. His "*Conservation of Energy*," "*The Unseen Universe*" (in conjunction with Tait), his experiments on the viscosity of ether, etc., all illustrate the comprehensiveness of his mind and the originality of his genius.

ON THE RELATION BETWEEN THE EMISSIVE AND THE ABSORPTIVE POWER OF BODIES FOR HEAT AND LIGHT.

BY

G. R. KIRCHHOFF.

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ON THE RELATION BETWEEN THE EMISSIVE AND THE ABSORPTIVE POWER OF BODIES FOR HEAT AND LIGHT.[1]

§ 1

HEAT rays have the same nature as light rays ; these constitute a special class of the former. The invisible heat rays are distinguished from light rays only by the period of vibration or the wave length. All heat rays follow the same laws in their propagation, which are known for light rays. A luminous body in space sends out light rays that are independent of the bodies on which they fall ; similarly all heat rays which a body sends out are independent of the bodies which form its environment.

Of the heat rays that are sent out to a body by its surroundings a part are absorbed, the others are sent on in directions which are varied by reflection and refraction. The rays refracted and reflected by it pass off along with those sent out by it, without any mutual disturbance taking place.

Through the radiations which a body sends out, the quantity of heat which it contains will, according to the law, sustain a loss which is equivalent to the *vis viva* of those rays and, through the heat rays which it absorbs, a gain which is equivalent to the *vis viva* of the absorbed rays. But in certain cases an exception to this rule may occur, in that the absorption and the radiation produce other changes in the body, as for example in bodies which are chemically changed by light, and light absorbing media which lose their power of shining

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[1] Investigations on the solar spectrum and the spectra of the chemical elements, 2d. Edition, Berlin, Ferd. Dummler's publishing house 1862.

through the radiation of the light which they have absorbed. Such cases should be excluded on the assumption that *neither by means of the rays which it radiates or absorbs, nor by means of other influences to which it is exposed, does the body possess the power to undergo a change, if its temperature is kept constant by the addition or the subtraction of heat.* Under these conditions, according to the law of equivalence of heat and work, the amount of heat which must be transferred to a body in a given time to prevent cooling, which would occur in consequence of its radiation, is equivalent to the *vis viva* of the emitted rays ; and the amount of heat which must be withdrawn in order to counterbalance the heating from absorption of radiations, is equivalent to the *vis viva* of the absorbed rays.

Let a body which satisfies these conditions be surrounded by an enclosure, having the same temperature, through which no heat rays can penetrate, whose temperature is kept constant and which satisfies the same conditions. The body sends out heat rays and is encountered by such heat rays, which, in part, proceed from the enclosure, in part, are thrown back to the same by reflection from it, absorbing a part of them. Its temperature must thus remain the same, unless heat is withdrawn from it or communicated to it as follows on the principle from which Carnot's law results. For this reason, the *vis viva* of the rays, which it sends out in a certain time, must equal the *vis viva* of the rays which it absorbs in the same time.

The proof which rests upon this conclusion requires the accurate investigation of the rays that travel back and forth between the body and the enclosure. This investigation will be much simplified if we imagine the enclosure to be composed, wholly or in great part, of bodies which, for infinitely small thickness, completely absorb all rays which fall upon them.

I will call such bodies *perfectly black*, or more briefly *black*. A *black body*, in this sense of the word, must have the same refractive index as the medium in which the radiation takes place ; then there will be no reflection at its surface, and all incident rays will be wholly absorbed. Thick iodine vapour in contact with air, or pitch in contact with glass, may be treated as black bodies, approximately, but not iodine vapour in contact with glass or pitch in contact with air. Next, the radia-

tion in empty space will be investigated ; the *black bodies* referred to must therefore have a refracted index which differs only infinitely little from 1.

The assumption that such *black bodies* are conceivable forms an important aid in the proof which will be presented here. Further, it will be assumed that perfectly diathermanous bodies are conceivable, that is, such which will absorb none of the incident heat rays of whatever nature these may be, and finally, that a perfect mirror is conceivable, i.e., a body which reflects completely all heat rays. A perfect mirror, like every diathermanous body, can itself send out no rays ; for if it did (confined in an enclosure of like temperature) it would warm this enclosure more and more and cool itself more and more.

§ 2.

Definitions.

Before a body C, Figure 1, imagine two screens, S_1 and S_2 placed in which are two openings 1 and 2, whose dimensions are infinitely small with respect to their distance apart, and each of which has a center.

Through these openings passes a pencil of rays sent out by the body C. Of this pencil of rays, let us consider the part, whose wave length lies between λ and $\lambda + \delta\lambda$, and let this be divided into two polarized components, whose planes of polarization are the planes a and b perpendicular to each other, passing through the axis of the ray pencil.

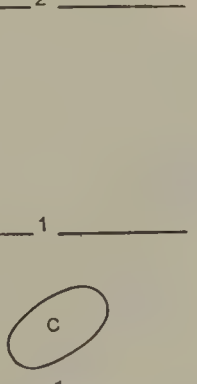

FIG. 1.

Let $E\delta\lambda$ be the intensity of the component polarized in a, or, what is the same thing, the increase, which the *vis viva* of the ether beyond the screen S_2 experiences through this component in the unit of time. The quantity E is called the *emissive power* of the body C.

Conversely, upon the body C there falls through the openings 2 and 1 a pencil of rays having the wave length λ, polarized in the plane a; of this, the body absorbs a part while it reflects or transmits the remainder ; let the ratio of the inten-

sity of the absorbed rays to the incident rays be A and let this be called the *absorptive power* of the body C. The quantities E and A depend upon the nature of the condition of the body C, besides also upon the form and position of the openings 1 and 2, the wave length λ and the direction of the plane a.

§ 3.

Under these conditions the following law holds : *The ratio between the emissive and the absorptive power is the same for all bodies at the same temperature.*

This law will be proven, first, for the case where only black bodies are compared with each other, that is, those whose absorptic power $= 1$; i. e., it will be shown that the radiating power of all black bodies is the same at the same temperature.

The proof of this special law is similar to that of the general law, but simpler ; it will therefore facilitate the understanding of the latter. Moreover, conclusions which are drawn from the special law will be used in the proof of the common law.

§ 4.

Proof of the Law § 3 *for black bodies.*

Let C be a black body ; let its emissive power, which is commonly indicated by E, be called e ; it will be proven that e remains unchanged, when C is replaced by any other black body of the same temperature.

Imagine the body C enclosed in a black covering, of which the screen S_2 forms a part, let the second screen S_2, like the first, be made of black substance and let both be united with each other on all sides by black walls, as shown in Figure 2. Suppose the opening 2 to be closed at first by a black surface, which I will call surface 2. The whole system must have the same temperature and the covering be maintained at a constant temperature throughout. According to the statements made in Figure 2, § 1, the *vis viva* of the rays which the body C sends out in the given time, must then equal the vis viva of the rays, which it absorbs in the same time ; in other words : the sum of the intensities of the rays which it sends out must equal the sum of the intensities of the rays which strike it, since according to

supposition it completely absorbs the latter. Now suppose the surface 2 removed, and the opening closed by a portion of a perfectly reflecting spherical surface, placed directly back of it and having its center at the middle point of the opening 1. Equilibrium of temperature will then exist. There must also be equality between the intensity of the rays which the body C sends out, and of those incident upon it. Since the body C now sends out the same rays as in the cases previously considered, it follows that the intensities of the rays incident upon C are the same in both cases. By the removal of surface 2 the rays are withdrawn from C which pass through opening 1; therefore the concave mirror placed at opening 2 throws just the same rays back to C which this sends out itself through the openings 1 and 2, for the concave mirror forms from opening 1 an image which coincides with itself.[1]

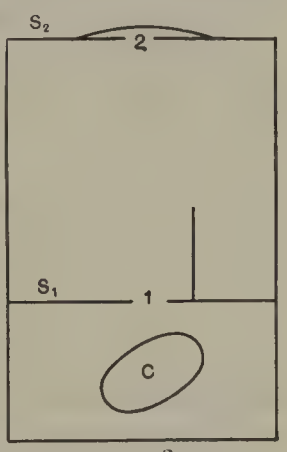

FIG. 2.

The law given would therefore be proved if all rays of the two pencils compared have the wave length λ and are polarized in the plane a. Both pencils of rays, however, are made up of different components and form the equality of the intensity of the whole pencil. We may not directly infer the equality of the intensity of corresponding parts.

The necessary completion of the proof may easily be given when a plate is supposed to exist, having the property of transmitting undiminished rays whose wave length lies between λ and $\lambda + \delta\lambda$ and whose plane of polarization is parallel to the plane a; but which completely reflects rays of other wave

[1] The diffraction of the rays at the edges of opening 2 may be neglected, since the openings 1 and 2 may be assumed infinitely small with respect to their distance and yet infinitely great with respect to the wave length, that is, so great, that the defraction may be inappreciable.

From this it follows that the intensity of the pencil of rays, which the body C sends out through openings 1 and 2, equals the intensity of the pencil of rays which the black surface 2 sends out through the opening 1. Since this intensity is independent of the form and further character of the black body C, so, likewise, is the former.

lengths or of an opposite polarization. If we should imagine the arrangement shown in Figure 2 modified by bringing such a plate before opening 1, then we may immediately arrive at the law to be proved by the treatment employed in respect to this figure.

The assumption that such a plate is possible is in no wise justified. On the contrary, a plate is possible which, of the rays striking it at the same angle, transmits and reflects them in different degrees according to their wave length and plane of polarization. A plate, which is so thin that the colors of thin films are visible and which is placed obliquely in the path, shows this.

Such a plate is required for the investigation under consideration in order to compare them. Besides this, it is necessary to make such an arrangement that both pencils of rays do not pass through the plate, but are reflected from it at the polarizing angle, the plane of reflection coinciding with the plane *a*. This is advantageous in as much as the rays polarized perpendicularly to *a* need not be considered. Further, the plate must be made of a perfectly diathermanous medium, it will then absorb no rays and send out none.

§ 5.

In the arrangement described in Figure 2 imagine a plate of the kind described and designated as *P*, brought between the openings 1 and 2 (Fig. 3). Let it be so placed that the pencil of rays passing through the openings 1 and 2 is incident at the polarizing angle and the plane of incidence is the plane *a*. Let the wall which unites the screens S_1 and S_2 be so shaped that the image, which the plate *P* casts from the opening 2 lies within it; in the place and of the form of this image imagine an opening which I will call opening 3. Let opening 2 be closed by a black surface of the temperature of the whole system, and let opening 3 be closed in the first place by a similar surface, and in the second place by a perfect concave

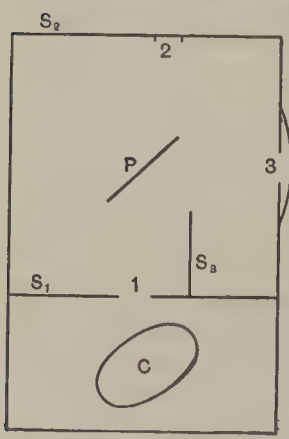

FIG. 3.

mirror having its center where the plate P forms the image of the middle of opening 1. In both cases the equilibrium of temperature is maintained; through consideration given in the preceding paragraph, it follows therefore that the sum of the intensities of the rays, which the body C is deprived of through the removal of surface 3 equals the sum of the intensities of the rays which are brought to it through the agency of the concave mirror. Let a black screen S_3 (of the temperature of the whole system) be so placed that none of the rays which surface 3 sends out are directly incident upon opening 1. The first sum, then, is the intensity of the rays which proceed from surface 3, and are reflected by plate P and pass through opening 1; they will be designated by Q. The second sum is made up of two parts; one component comes from C and is:

$$= \int_0^\infty d\lambda e r^2$$

where r represents a quantity dependent upon the nature of the plate P and the wave length λ; the second part consists of rays which have come from a portion of the black wall which unites the screen S_1 and S_2, have passed through the plate P and been reflected from the concave mirror, and then from the plate P; this part will be designated as R. It is unnecessary to examine further the value of R; it suffices to notice that R, as well as Q, is independent of the nature of C. Between the magnitudes introduced there exists the equation:

$$\int_0^\infty d\lambda e' r^2 + R = Q$$

If we now imagine the body C replaced by another black body of the same temperature, letting e' indicate for this what e has represented for the other, there exists the equation

$$\int_0^\infty d\lambda e' r^2 + R = Q$$

From this it follows that

$$\int_0^\infty d\lambda \, (e - e') \, r^2 = 0$$

Let us now assume that the index of refraction of the plate P

differs but little from unity. From the theory of the colors of thin plates it follows then that we can place

$$r = \rho \sin^2 \frac{p}{\lambda}$$

where p represents a quantity proportional to the thickness of the plate P, independent of λ, and a quantity independent of this thickness. From this follows the deduced equation :

$$\int_0^\infty d\lambda \, (e-e') \, \rho^2 \sin^4 \frac{p}{\lambda} = 0$$

Since this equation must hold for every thickness of the plate P, and hence for every value of p, it follows that for every value λ we may conclude that

$$e-e' = o$$

To prove this, substitute in that equation for $\sin^4 \frac{p}{\lambda}$:

$$\tfrac{1}{8}(\cos 4 \frac{p}{\lambda} - 4 \cos 2 \frac{p}{\lambda} + 3)$$

and differentiate twice with respect to p : we then have

$$\int_0^\infty d\lambda \frac{(e-e') \rho^2}{\lambda^2} (\cos 4 \frac{p}{\lambda} - \cos 2 \frac{p}{\lambda}) = 0$$

In place of λ let us introduce a new quantity into the equation ; where

$$\frac{2}{\lambda} = a$$

and set

$$(e-e') \rho^2 = f(a)$$

we thus obtain

$$\int_0^\infty da f(a) (\cos 2p a - \cos p a) = 0$$

If we consider that when $\phi(a)$ represents any arbitrary function of a

$$\int_0^\infty da \phi(a) \cos 2pa = \tfrac{1}{2} \int_0^\infty da \phi(\frac{a}{2}) \cos pa$$

from which we may conclude that if we substitute $\frac{a}{2}$ for c, we may therefore write

$$\int_0^\infty da [f(\frac{a}{2}) - 2f(a)] \cos p a = 0$$

Multiply this equation by $dp \cos xp$, where x represents an arbitrary quantity, and integrate it from $p = 0$ to $p = \infty$. According to Fourier's formula which is expressed by the equation

$$\int_0^\infty dp \cos px \int_0^\infty da\phi(a) \cos pa = \frac{\pi}{2} \phi(x)$$

we have

$$f\left(\frac{x}{2}\right) = 2f(x) \text{ or } f\left(\frac{a}{2}\right) = 2f(a)$$

From this it follows that $f(a)$ either vanishes for all values of a, or becomes infinitely great when a approaches zero. When a approaches zero λ becomes infinite. If we remember the meaning of $f(a)$ and consider that ρ is a proper fraction, and that neither e nor e' can become infinite when λ increases to infinity, then it is evident that the second case cannot exist and therefore, that for all values of λ, $e = e'$.

In a similar way we may treat the case when C is not a black body but is an arbitrary one. We shall not assume for the same that it is homogeneous ; partly on its surface, partly in its interior will the rays therefore, which are incident upon it from the black envelope, experience the most manifold modifications. On these grounds, there must be, as a preliminary to the proposed proof, a study made of the radiation which takes place between black surfaces of the same temperature, for arbitrary bodies. To this investigation, which depends upon the formula just proved, the following paragraphs are devoted.

§ 6.

Mutual Radiations of Black Surfaces.

If the pencil of rays which the body C sends out through openings 1 and 2 should be partly linearly polarized, the plane of polarization of the polarized portion must rotate when C is rotated around the axis of the pencil. Such a rotation must therefore change the value of e. Since, according to the equation proved, such a change cannot take place, the pencil of rays can have no linearly polarized portion. It can be proved also, that it can have no circularly polarized part. But the proof for this will not be given here.

We will also grant, without this, that black bodies are con-

ceivable in whose structure there is no reason why they should send out in any direction more right handed circularly polarized rays than left handed circularly polarized rays.

Of this character will the black bodies, concerned in the further treatment, be assumed; they send out in all directions unpolarized rays.

§ 7.

The quantity represented by e depends, aside from the temperature and wave length, on the form and the relative position of the openings 1 and 2. If w_1, w_2 represent the projections of the openings upon planes perpendicular to the axis of the pencil, and if s is called the distance of the openings, then

$$e = I \frac{w_1 w_2}{s^2}$$

where I is a function of the wave length of the temperature only.

§ 8.

Since the form of a body C is arbitrary, a surface may be substituted, which exactly fills opening 1 and which I will call surface 1; the screen S_1 may then be imagined removed. Further the screen S_2 may be considered removed if the pencils of rays which e covers, is defined as that which falls from surface 1 upon surface 2, which the opening 2 exactly fills.

§ 9.

A consequence of the last equation, which immediately follows and which will later be used, is that the value e remains unchanged if we imagine the openings 1 and 2 interchanged.

§ 10.

We will now establish a law which may be treated as a generalization of the law presented in the last paragraph.

Between the two black surfaces of the same temperature 1 and 2, is placed a body which may refract, reflect, or absorb in any way the rays which one sends to the other. Several pencils of rays may pass from surface 1 to surface 2; choose one of these, and consider the part of the one at 1 whose wave length lies between λ and $\lambda + d\lambda$, and divide this into two components

whose planes of polarization are the planes of a_1 and b_1 which are perpendicular to each other (otherwise arbitrary). Let the part of the first component which enters 2 be divided into two components whose planes of polarization are the planes a_2 and b_2 perpendicular to each other (otherwise arbitrary). Let the intensity of the component polarized in a_2 be $Kd\lambda$. Of the pencil of rays which passes over the same path as the preceding one, from 2 to 1, let us consider the part at 2 whose wave length lies between λ and $\lambda + d\lambda$, and divide this into two components polarized in a_2 and b_2. Divide the portion, which reaches 1 from the first component, into two parts whose planes of polarization are a_1 and b_1. Let the intensity of the components polarized in a_1 be $K'd'\lambda$. Then

$$K = K'.$$

The proof of this law will be made upon the assumption that the rays under consideration undergo no weakening in their path, and also upon the assumption that refraction and reflection occur without loss, that there is no absorption and that the rays, coming from 1, polarized in a_1, reach 2 polarized in a_2, and *vice versa*.

Through the middle point of 1 pass a plane perpendicular to the axis of the pencil of rays, either incident or emergent at this point, and imagine in this a right-angled coördinate system, whose origin is that middle point. Let x_1 y_1 be coördinates of any point in the plane, Figure 4. At the distance of unity from this plane, imagine a second, parallel to it, and in this, a coördinate system whose axes are parallel to each of those, and whose origin lies in the axis of the pencil of rays. Let x_3 y_3 be coördinates of any point in this plane. In a similar manner pass through the middle point of 2, a plane perpendicular to the axis of the bundle of rays, incident or emergent at this point, and introduce in this, a rectangular system of coördinates whose origin is the middle point mentioned. Let x_2 y_2 be coördinates of a point in this plane. Finally, at a distance of unity from this plane and parallel to it imagine a fourth, and in it a system of coördinates whose axes are parallel to the axis of x_2, y_2 and whose origin lies in

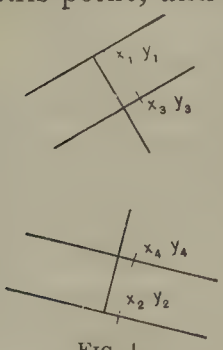

FIG. 4.

the axis of the pencil of rays. Let x_4, y_4 be coördinates of any point in this fourth plane.

From an arbitrary point let a ray pass to any other point (x_2, y_2); let T be the time required to pass from one point to the other; we will suppose it to be a known function of x_1, y_1, x_2, y_2. If the points (x_3, y_3) and (x_4, y_4) lie in the path of the ray referred to, (and if for the sake of brevity the velocity of the ray *in vacuo* be taken as unity) the time which the ray requires to pass from (x_3, y_3) to (x_4, y_4) will be

$$= T - \sqrt{1 + (x_1 - x_3)^2 + (y_1 - y_3)^2}$$

$$- \sqrt{1 + (x_2 - x_4)^2 + (y_2 - y_4)^2}$$

Assuming the points (x_3, y_3), (x_4, y_4) given, and the points (x_1, y_1), $(x_2\ y_2)$ required, we could find these from the condition that the above expression is a minimum. If we assume that the eight coördinates x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4 are infinitely small, the following equations express the condition that the four points (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4) lie in one ray:

$$x_3 = x_1 - \frac{\partial T}{\partial x_1}, \; x_4 = x_2 - \frac{\partial T}{\partial x_2},$$

$$y_3 = y_1 - \frac{\partial T}{\partial y_1}, \; y_4 = y_2 - \frac{\partial T}{\partial y_2}$$

Now let (x_1, y_1) be a point in the projection of surface 1 on the plane x_1, y_1 and let dx_1, dy_1 be an element of this projection in which the point x_1, y_1 lies and which is infinitely small with respect to the surfaces 1 and 2. Let (x_3, y_3) be a point in a ray proceeding from (x_1, y_1) to surface 2, dx_3, dy_3, a surface element in which the point (x_3, y_3) lies, of the same order as dx_1, dy_1. The intensity of the rays of the required wave length and of the given plane of polarization, which, proceeding from dx_1, dy_1, pass through dx_3, dy_3, is then according to § 7;

$$d\lambda \; I \, dx_1 \, dy_1 \, dx_3 \, dy_3.$$

According to the supposition, this amount of rays reaches surface 2 undiminished and forms an element of a quantity designated by $Kd\lambda$. K is the definite integral

$$I \int\int\int\int dx_1\, dy_1\, dx_3\, dy_3.$$

The integration here with respect to x_3, y_3 is to be taken over those values which these quantities have according to the above equations, while x_1 and y_1 remain constant and x_2, y_2 have all the values which correspond to the projections of surface 2 upon the plane x_2, y_2. The integration with respect to x_1, y_1 is then to be taken over the projection of surface 1. The double integral

$$\int\int dx_3\, dy_3 \text{ so limited,}$$

is, however,

$$= \int\int \left(\frac{\partial x_3}{\partial x_2} \frac{\partial y_3}{\partial y_2} - \frac{\partial x_3}{\partial y_2} \frac{\partial y_3}{\partial x_2} \right) dx_2\, dy_2$$

or from the equations for x_3, y_3

$$= \int\int \left(\frac{\partial^2 T}{\partial x_1\, \partial x_2} \frac{\partial^2 T}{\partial y_1\, \partial y_2} - \frac{\partial^2 T}{\partial x_1\, \partial y_2} \frac{\partial^2 T}{\partial x_2\, \partial y_1} \right) dx_2\, dy_2,$$

where the integration is taken over the projections of surface 2 Whence

$$K = I \int\int\int\int \left(\frac{\partial^2 T}{\partial x_1\, \partial x_2} \frac{\partial^2 T}{\partial y_1\, \partial y_2} - \frac{\partial^2 T}{\partial x_1\, \partial y_2} \frac{\partial^2 T}{\partial x_2\, \partial y_1} \right) dx_1 dy_1\, dx_2\, dy_2$$

where the integration is taken over the projections of surfaces 1 and 2.

If the magnitude of a K' be treated in the same way, remembering that a ray requires the same time to pass over a path between two points in either direction, the same expression will be found for K' as for K. Thus, the enunciated law is proved, subject to the limiting conditions under which it will next be proved. This limitation may, however, be immediately disposed of by an observation made by Helmholtz in his "Physiological Optics," p. 169. Helmholtz says here (with somewhat different notation): "A light ray passes from point 1 to point 2 after any number of refractions, reflections, etc. Through 1 suppose two arbitrary planes a_1 and b_1 passed in the direction of the ray perpendicular to each other, in which its vibrations are supposed to be resolved. Two similar planes, a_2 and b_2 are also passed through the ray at 2. Then the following

may be proved: If a quantity i of light polarized in the plane a_1 proceeds from 1 in the direction of the ray mentioned and of this, the quantity k of light polarized in the plane a_1 reaches 2, so will *vice versa* the quantity k of light polarized in a_2 reach point 1 if the quantity i of light polarized in a_2 proceed from point 2." [1]

Applying this law, and representing by γ the value of the ratio $\frac{k}{i}$ for the two rays which pass in either direction between the points (x_1, y_1) and (x_2, y_2), then an expression is obtained for K as well as for K' which differs from that formed only in that now γ appears as a factor under the integral sign.

The equality of K and K' exists accordingly when γ has a different value for the rays into which one of the pencils compared may be divided; for example, it is unaffected if a part of the pencil is intercepted by a screen.

§ 11.

Of the same pencils which were compared in the preceding paragraphs, the following law also holds: of the pencil passing from 1 to 2, consider the part at 2 whose wave lengths lie between λ and $\lambda + d\lambda$ and resolve this into two components polarized in a_2 and b_2; let the intensity of the first component be $Hd\lambda$. Of the pencil which passes from 2 to 1, consider at 2 the portion whose wave lengths lie between λ and $\lambda + d\lambda$, and resolved this into two components polarized in a_2 and b_2. Let the portion of the first component reaching point 1 be $H'd\lambda$.

Then $H = H'$. The proof of this law is the following; K and K' are to have the same meaning as in the preceding paragraph; let L and L' be the quantities which arise form K and K' when plane a_1 is interchanged with plane b_1.

---

[1] The law of Helmholtz, as he himself noted, does not hold good when the plane of polarization of a ray undergoes any rotation, such as magnetic force produces according to Faraday's discovery; therefore, in the following considerations magnetic force must not be considered as present. Helmholtz limited his law also by supposition that light undergoes no change of refrangibility such as occurs in fluorescence; this limitation is unnecessary in the application of the law, if rays of only a given wave length are regarded.

Then $L = L'$, similarly $K = K'$, further $H = K + L$, because rays polarized perpendicularly to each other do not interfere, when they are brought back to a common plane of polarization in case they are a part of an unpolarized ray; and according to § 6 the surface 1 sends out unpolarized rays.

Finally $H' = K + L'$, because two rays, whose planes of polarization are perpendicular to each other, do not interfere. From these equations it follows that $H = H'$.

§ 12.

Let Fig. 2 have the same meaning as in § 4, only let the body C be not a black body but an arbitrary one. Let opening 2 be *closed* by surface 2. This surface sends out a pencil of rays through opening 1 to the body C, which is partly absorbed by this body, and partly scattered in different directions by reflection and refraction. Of this pencil consider the part between 1 and 2 whose wave lengths lie between λ and $\lambda + d\lambda$ and resolve this into two components polarized in plane a and the plane perpendicular to this. Let that part of the first component which escapes absorption by C, and hence strikes the black covering in which the body C is inclosed, be $M' d\lambda$. A certain portion of the rays which a part of the covering sends out to the body C, fall upon surface 2 through opening 1; thus by means of the body C a pencil of rays is produced which passes through opening 1 to the surface 2. Of this, consider the part whose wave lengths lie between λ and $\lambda + d\lambda$ and divide it into two components polarized in plane a and the plane perpendicular to it. Let the intensity of the first components be $M d\lambda$.

Then $M = M'$. The truth of this law follows from the proposition from the preceding paragraph, if we apply this to all pencils of rays, which surface 2 and all the elements of the black cover surrounding the body C interchange with each other by means of the body C, and then form the sum of the equations so obtained.

§ 13.

Proof of the proposition § 3 for any body.

Let the arrangement shown in Fig. 3 and described in § 5 be

taken, only let the body C no longer be a black body, but arbitrary. In both cases described there, the equilibrium of heat subsists; then the *vis viva* which is drawn out from the body C by the removal of the black surface 3, must therefore be equal to the *vis viva* which is supplied to this by the presence of the concave mirror. The symbols used in § 5 will be used here with the same meaning. The letters E and A will have the meaning given them in § 2.

If surface 3 is removed, then the rays are withdrawn from the body C which this surface sends to it; the intensity of the part of these rays which it absorbed is $= \int_0^\infty d\lambda e r A$.

Now the rays must be examined which are transmitted to the body by the presence of the concave mirror. All these rays must be reflected from the concave mirror to plate P and from this to opening 1, and these must pass in the same direction as if they came from opening 2. Before they strike the concave mirror, they have either experienced a reflection from it or not. In the first case they can only be sent back again to the concave mirror by means of the body C over the path which is the reverse of that already described. It must next be premised that the body C has such a position that, of the rays which pass to it through 2 and 1, only an infinitely small part will be reflected back again through opening 1 to opening 2. Then, of the rays in question, only an infinitely small fraction have suffered multiple reflection at the concave mirror, and it is sufficient to consider those which are reflected only once at the mirror. Of these, a part proceed from the body C, the rest from the black covering. The first part has experienced a double reflection at plate P; the *vis viva* which the body C absorbs from it is

$$= \int_0^\infty d\lambda E r^2 A.$$

The second part which proceeds from the black enclosure may again be considered as consisting of two parts; one which passes to the concave mirror without the mediation of the body C., and a second, by means of it. Each arises from rays which proceed from black partition opposite the concave mirror, and have passed through the plate P, have been reflected from the concave mirror to the plate P and from this to opening 1.

Without examining from which part of the black wall these rays have proceeded, their intensity may be found by the law established in § 11.

By the application of this, the intensity of those rays which were absorbed by the body C is shown to be

$$= \int_0^\infty d\lambda e r\,(1-r)\,A.$$

Finally in order to find the intensity of the rays which proceed from the black covering by means of the body C to the concave mirror, and pass back from this to the body C and are here absorbed, let N designate the quantity which the quantity indicated by M in § 12 becomes in consequence of plate P being brought into its place and the surface 3 removed; the intensity is then $= \int_0^\infty d\lambda N r^2 A.$

The difference between M and N arises only from the variation which the rays, falling upon the body C from the black covering through opening 1, undergo by the introduction of plate P and the removal of surface 3. Suppose the plate P brought into its position, without removing surface 3, then M can undergo no change, since all the pencils of rays, which go to the opening 1 remain unchanged; the pencil proceeding from surface 2, for example, suffers a loss through reflection at plate P which will be exactly replaced by the reflection of the rays going out from the surface 3. The difference $M-N$ is therefore only produced by the removal of surface 3 and is also equal to the part of M which arises from the rays sent out by surface 3 to opening 1 by means of plate P. According to the supposition made in these paragraphs concerning the position of the body C, $M-N$ is infinitely small in comparison with the intensity of the rays of equal wave length which surface 3 sends to opening 1 by means of plate P, as well as infinitely small in comparison with the intensity of rays of equal wave length and polarized in the plane a, which surface 2 sends to opening 1 by the absence of the plate P, and therefore finite and also infinitely small with respect to the quantity represented by M' in § 12 (assuming that $1-A$ is not infinitely small). Since, however, as already pointed in the places cited, $M' = M$, we may also place

$$N = M = M'.$$

But according to the definition given of M'

$$M' = e\,(1-A) \text{ and therefore}$$

$$\int_0^\infty d\lambda\, Nr^2\, A = \int_0^\infty d\lambda e\ (1-A)\, r^2\, A.$$

The proposition presented at the beginning of this paragraph would then be expressed by the equation :

$$\int_0^\infty d\lambda\, er\, A = \int_0^\infty d\lambda\, Er^2\, A + \int_0^\infty d\lambda\, er\,(1-r)\, A + \int_0^\infty d\lambda\, e\,(1-A)\, r^2\, A,$$

or by the equation $\int_0^\infty d\lambda\,(E-Ae)\, Ar^2 = 0.$

By the same treatment employed in § 5 with reference to a similar equation, we may conclude that for every value of λ

$$\frac{E}{A} = e,$$

or substituting for e its value in § 7.

$$\frac{E}{A} = I\,\frac{w_1\, w_2}{s^2}.$$

Thus, the law § 3 is proved under the assumption that, of the pencil which falls from surface 2 through the opening 1 upon the body C, no finite part is reflected by this back to the surface 2 ; further, that the law holds without this limitation, if we consider that when the condition is not fulfilled, it is only necessary to turn the body C infinitely little in order to satisfy it, and that by such a rotation the quantities E and A undergo only and infinitely small change.

§ 14.

A Generalization of the Law § 3.

The discussions given assume that the space in which the radiation occurs is a vacuum. But the same treatment also obtains when this space is filled with any perfectly diathermanous medium ; only the function I will then be different than in the former case. The symbol I may then be retained for a vacuum and I' may be called the corresponding function of temperature and wave length for a certain diathermanous medium ; if n is the index of refraction of the same for the

temperature and wave length to which I and I' refer, then a simple relation exists between I', I, and n; the same follows from the law already demonstrated as will be here shown.

Imagine a layer of a diathermanous medium bounded by two parallel planes, and with one side in contact with the black surface F. Let the thickness of the layer $= 1$. For this body, the absorptive power of A, and the emissive power E, in relation to a certain pencil of rays will be investigated. The opening 1 and 2 which determine the form of the pencil will be in screens S_1 and S_2, of which the first covers the surface of the layer hitherto supposed to be free, and the second is parallel to it; let the line joining the middle points of the openings be perpendicular to the screens. Of any pencil of rays of a definite wave length and direction of polarization, which passes from the opening 2 to the opening 1, a fraction will be reflected at the latter which may be designated by ρ; the rest passes to the surface F and is here completely absorbed; therefore

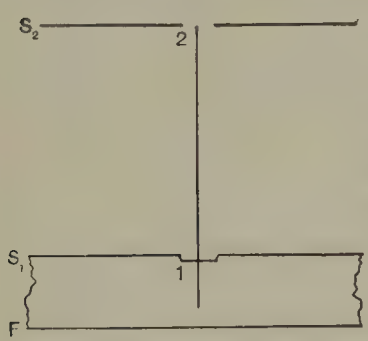

FIG. 5.

$$A = 1 - \rho.$$

To find E, represent by x, y; x_1, y_1; and x_2, y_2 the coördinates of a point of the surface F, the opening 1, and the opening 2, reckoned from those points which are found in the axis of the pencil. If these points lie in a ray, then if s again represents the distance of the two openings,

$$n\left(1 + \frac{(x_1 - x)^2}{2} + \frac{(y_1 - y)^2}{2}\right) + \left(s + \frac{(x_2 - x_1)^2}{2s} + \frac{(y_2 - y_1)^2}{2s}\right)$$

must be a minimum with respect to x_1 and y_1 : i. e.,

$$x = x_1 - \frac{x_2 - x_1}{ns}, \quad y = y_1 - \frac{y_2 - y_1}{ns}$$

if w_1 and w_2 are the surfaces of the two openings, we find by a treatment, which is given in a more general form in § 10, the intensity of the rays (polarized in a and of wave lengths between λ and $\lambda + d\lambda$) which, falling from F upon opening 1, in part, pass to opening 2,

$$= I' \, d\lambda \, w_1 \, w_2 \left(\frac{\partial x}{\partial x_2} \frac{\partial y}{\partial y_2} - \frac{\partial x}{\partial y_2} \frac{\partial y}{\partial x_2} \right),$$

that is

$$= \frac{I' \, d\lambda \, w_1 \, w_2}{n^2 \, s^2}$$

Of these rays the fraction $1-\rho$ goes through the opening 1 and arrives at the opening 2.

Thus $E = (1-\rho) \frac{I' \, w_1 \, w_2}{n^2 \, s^2}$

If these values of E and A are substituted in the equation

$$\frac{E}{A} = \frac{I \, w_1 w_2}{s^2}$$

then

$$I' = n^2 \, I.$$

§ 15.

Some Results of the Law § 3.

When any given body—a platinum wire for instance—is gradually heated, it emits, up to a certain temperature, only rays whose wave lengths are greater than that of the visible rays. At a certain temperature, rays of the wave length of the extreme red begin to be visible; as the temperature rises higher and higher, rays of a shorter and shorter wave length are added, so that for every temperature, rays of a corresponding wave length come into existence, while the intensity of the rays of greater wave lengths increase. If the law proven be applied to this case, it will be seen that the function I for any wave length vanishes for all temperature below that of a certain temperature, depending on the wave length for higher temperatures increases with the same. From this it follows, when the same law is applied to other bodies, that all bodies, whose temperature is gradually raised, begin to send out rays of the same wave length at the same temperature, and begin to glow with red rays at the same temperature, and at a higher common temperature, yellow, and so on. The intensity of rays of a certain wave length which different bodies send out at the same temperatures may, however, be very different; it is proportional to the absorptive powers of bodies for rays of that particular wave length. At the same temperature accordingly metal glows more brightly than glass, and this more brightly than a gas. A body that remains perfectly transparent at the

highest temperature would never become incandescent. Into a ring of platinum wire of about 5mm diameter, I introduced some phosphate of soda and heated it in the nonluminous flame of the Bunsen burner. The salt melted and formed a fluid lens and remained perfectly clear; but it emitted no light, while the platinum ring in contact with it radiated the most brilliant light.

Draper[1] has drawn the conclusion from investigations that all solid bodies begin to glow at the same temperature. From his researches he has, however, noted that certain bodies like chalk, marble, fluor spar glow at a lower temperature than they should according to this law; he called this luminosity *phosphorescence*, and said it clearly differed from glowing, by the color. But whatever name may be given to this luminosity, it is in contradiction with law, § 3, and a body which shows it must therefore not satisfy the assumptions which were made in the proof of the law, that, at a constant temperature, it must remain unchanged; phosphorescence is not purely an effect of heat, and is not exclusively determined by the temperature, but is produced by changes in the body; if these changes—whether they are chemical or of another nature—have ceased then the phosphorescence must vanish.

§ 16.

From the law, § 3, it follows that a body, which absorbs more rays from one plane of polarization than from another, sends out in the same ratio more rays from the first plane of polarization than from the second. Consequently, as is known to happen, a glowing opaque body having a smooth surface sends out light in directions oblique to this surface which is partially polarized perpendicularly to the plane passing through the ray and the normal to the surface; of the incident rays, which are polarized perpendicularly to the plane of incidence, the body reflects less, but also absorbs more than it does of rays whose plane of polarization is the plane of incidence. According to this law the state of polarization of the rays sent out may easily be given if the law of the reflection of the incident rays is known.

[1] Phil. Mag. XXX. p. 345; Berl. Ber. 1847.

A tourmaline plate, cut parallel to the optic axis, absorbs, at ordinary temperatures, more of the rays which strike it normally, if the plane of polarization of these is parallel to the axis than when it is perpendicular to it. Assuming that the tourmaline plate retains this property when it is at a glowing heat, it must give out rays in a direction normal to it, which are partially polarized in the plane passing through the optic axis and which is the plane perpendicular to that which is called the plane of polarization of the tourmaline. I have proved this striking deduction from theory by experiment and it confirmed the same. The tourmaline plates employed withstood a considerable temperature, glowing for a long time in the flame of the Bunsen burner without undergoing any permanent change; only after cooling did they appear dull at the edges. The property of polarizing transmitted light was retained even at an incandescence, although in a considerable less degree than at a lower temperature. This appeared on observing, through a double refracting prism, a platinum wire made incandescent in the flame and placed behind a tourmaline plate. The two images of the platinum wire were of unequal brightness, although the difference was much less than when the tourmaline plate was outside of the flame. The double refracting prism was then given the position in which the difference of the intensities of the two images was a maximum ; suppose the upper image were the brighter ; then, after removal of the platinum wire the two images of the tourmaline plate were compared. The upper image was not strikingly, but, unmistakably, darker than the other; both images appeared exactly like to two similar incandescent bodies, of which the upper had a less temperature than the lower one.

§17.

Still another result of the law established may, in conclusion, be admitted here. When a space is surrounded by bodies of the same temperature, and no rays can penetrate through these bodies, every pencil in the interior of the space is so constituted, with respect to its quality and intensity, as if it proceeded from a perfectly black body of the same temperature, and is therefore independent of the nature and form of the

bodies, and only determined by the temperature. The truth of this statement is evident if we consider that a pencil of rays, which has the same form but the reverse direction to that chosen, is completely absorbed by the infinite number of reflections which it successively experiences at the assumed bodies. In the interior of an opaque glowing hollow body of given temperature there is, consequently, always the same brightness whatever its nature may be in other respects.

Biographical Sketch.

Gustav Robert Kirchhoff, the son of Counselor-at-law Kirchhoff, was born March 12, 1824, at Königsberg. He took his degree of doctor of philosophy at the University in 1847. The following year he became private-docent at the University of Berlin. He early showed those rare mathematical faculties which later distinguished him. As early as his eighteenth year he decided upon physics as the branch to which he should devote his life's work. By 1845 he had investigated electric currents, and established the two so-called Kirchhoff's laws for current conduction. Other important papers on electricity followed in rapid order. In 1854 he was called to Breslau, where he became associated with Bunsen. He went to Heidelberg in 1854 where Bunsen had preceded him. Here in his prime he wrought and sought for twenty years, and in connection with Bunsen achieved some of the most important discoveries in the history of physical science. In 1875 he accepted, after twice declining an invitation to the University, a call to the chair of theoretical physics at Berlin where he became associated with his former colleague von Helmholtz. Here he delivered for eleven years (with serious interruption in the last two years) his famous courses of lectures on theoretical physics. It is during this period that we find the most brilliant aggregation at Berlin of scholars in the faculty of mathematics and physics during the century. His contributions extend over optics, heat, fluid, motion, electricity, elasticity, etc., and all bear the imprint of the great genius he was. He died unexpectedly Oct. 17, 1887, after many months of disability. His papers and lectures have been collected and edited and now form one of the enduring monuments in physical science.

CHEMICAL ANALYSIS BY SPECTRAL OBSERVATIONS.

BY

G. KIRCHHOFF AND R. BUNSEN.

Poggendorf's Annalen, Band 110, 1860 ; *Gesammelte Abhandlungen von G. Kirchhoff. pp.* 598-625, 1882.

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CHEMICAL ANALYSIS BY SPECTRAL OBSERVATIONS.[1]

It is well known that many substances have the property when they are brought into a flame of producing in the spectrum certain bright lines. We can found on these lines a method of qualitative analysis which greatly enlarges the field of chemical reactions and leads to the solution of problems unsolved heretofore. We shall confine ourselves here only to the extension of the method to the detection of the metals of the alkalis and the alkali earth and to the illustration of their value in a series of examples.

The lines referred to show themselves the more plainly, the higher the temperature and the weaker the natural illuminating power of the flame. The gas lamp[2] described by one of us gives a flame of very high temperature and very small luminosity; this is consequently especially adapted to investigations on those substances characterized by bright lines.

In Figure 1 the spectra are represented which the flames referred to give when the salts, as pure as possible, of potassium, sodium, lithium, strontium, calcium, and barium are vaporized in it. The solar spectrum is annexed in order to facilitate the comparison.

The potassium compound used for the investigation was obtained by heating chlorate of potassium which had been six to eight times recrystallized beforehand.

The chloride of sodium was obtained by combining pure carbonate of sodium and hydrochloric acid and purifying the same by repeated crystallization.

The lithium salt was purified by precipitating fourteen times with carbonate of ammonium.

For the production of the calcium salt a specimen of marble

[1] Kirchhoff and R. Bunsen, Pogg. Ann. Vol. 110. 1860.

[2] Bunsen, Pogg. Ann. Vol. 100 p. 85.

FIGURE 1.

as pure as possible, and dissolved in hydrochloric acid, was used. From this solution the carbonate of calcium was thrown down by a fractional precipitation with carbonate of ammonium in two portions, of which only the latter, precipitated in calcium nitrate, was used. The calcium salt thus obtained we dissolved several times in absolute alcohol and converted it finally into the chloride by evaporating the alcohol and by precipitation with carbonate of ammonium in hydrochloric acid.

In order to obtain the pure chloride of barium we extracted it from the commercial salt by pulverizing and boiling repeatedly in nearly absolute alcohol. The residue thus extracted and freed from alcohol was dissolved in water and thrown down by fractional precipitation in two portions, only the second being dissolved in hydrochloric acid, and the barium chloride thus obtained being further purified by repeated crystallizations.

In order to obtain chloride of strontium, as pure as possible, the commercial salt was crystallized out from alcohol, and fractionally precipitated in two portions with carbonate of ammonium, the second part being dissolved in nitric acid and the nitrate freed from the last traces of calcium by pulverizing and boiling with alcohol. From the product thus purified the chloride of strontium was obtained finally by precipitating with carbonate of ammonium and dissolving the precipitate in hydrochloric acid. All these purifications were made in platinum vessels as far as it was possible.

Figure 2 represents the apparatus which we have used mainly in the observation of the spectra. *A* is a box blackened on the inside the bottom of which has the form of a trapezium and rests on three feet; the two inclined sides of the same form an angle with one another of about 58° and carry the two small telescopes *B* and *C*. The ocular of the first is removed and replaced by a plate in which is a slit formed of two brass cheeks which are placed at the focus of the objective. The

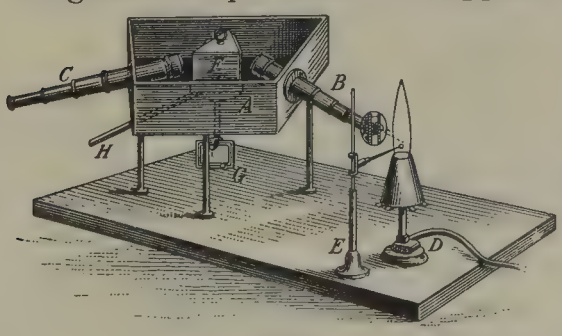

FIG. 2.

lamp D is so placed before the slit that the mantle of the frame is intersected by the axis of the tube B. Somewhat beneath the point where the axis meets the mantle the end of a very fine platinum wire bent into a small hook and carried by the holder E passes into the same; on this hook is melted a globule of the chloride previously dried. Between the objective of the telescopes B and C is placed a hollow prism F with a reflecting angle of 60° and filled with carbon disulphide. The prism rests on a brass plate which can be rotated on a vertical axis. This axis carries on its lower end the mirror G and above it the arm H which serves as the handle to rotate the prism and the mirror. A small telescope is adjusted before the mirror which gives an image of a horizontal scale placed at a short distance. By rotating the prism we can cause to pass before the vertical thread of the telescope C the entire spectrum of the flame and bring every portion of the spectrum into coincidence with this thread. To every reading made on the scale there corresponds a particular portion of the spectrum. If the spectrum is very weak the cross hair of the telescope C is illuminated by meaus of a lens which throws some of the rays from a lamp through a small opening which is placed laterally in the ocular of the telescope C.

The spectra in Fig. 1. obtained by means of the pure chloride above mentioned we have compared with those which we obtained if we introduce the bromides, iodides, hydrated oxides, sulphates, and carbonates of the several metals into the following flames:—

into the flame of sulphur,
" " " " carbon disulphide,
" " " " aqueous alcohol,
" " non-luminous flame of coal gas,
" " flame of carbonic oxide,
" " " " hydrogen and
" " oxyhydrogen flame.

From these comprehensive and lengthy investigations whose details we may be permitted to omit, it appears that the difference in the combinations in which the metals were used, the multiplicity of the chemical processes in the several flames, and the enormous differences of temperatures of the latter *exert no influence on the position of the spectral lines corresponding to the individual metals.*

How considerable the differences of temperature mentioned are, is shown by the following treatment.

We may arrive at an approximation of temperature of a flame by means of the equation $t = \frac{\Sigma gw}{\Sigma ps}$, in which t is the temperature of the flame sought, g the weight of the substance burning in oxygen, w the heat of combustion of the same, p the weight and s the specific heat of one of the products of combustion.

If we take as the heat of combustion

of sulphur	as 2240° C
" carbon disulphide	" 3400
" hydrogen	" 34462
" marsh gas	" 13063
" elayle	" 11640
" ditetryle	" 11529
" carbonic oxide	" 2403

and place according to Regnault the specific heat at constant pressure

for sulphurous acid	=0.1553
" carbonic acid	=0.2164
" nitrogen	=0.2440
" aqueous vapor	=0.4750

we find accordingly the temperature

of the sulphur flame	1820° C
" " bisulphide of carbon flame	2195
" " coal gas flame[1]	2350
" " carbonic oxide flame[2]	3042
" " hydrogen flame in air[3]	3259
" " oxyhydrogen flame[4]	8061.

It appears that the same metal compound gives in one of these flames a spectrum as much more intense as the temperature is higher. Of the compounds of these metals, those give the greatest intensity in a flame which have the greatest volatility.

In order to obtain a further proof that each of the severally mentioned metals always give the same bright lines in the spectrum, we have compared the spectra referred to with those

[1] Liebig's Ann. Vol. CXI. p. 258

[2] Gasometric Methods by R. Bunsen. p. 254.

[3] Ibid.

[4] Ibid.

which an electric spark produces which passes between electrodes made from these metals.

Small pieces of potassium, sodium, lithium, strontium, and calcium were fastened on a fine platinum wire and so melted in pairs within glass tubes that they were separated by a distance of 1 to 2mm from one another the wires piercing the sides of the tubes. Each of these tubes was placed before the slit of the spectroscope; by means of a Ruhmkorff's induction apparatus, we caused electric sparks to pass between the metal pieces mentioned and compared the spectrum of the same with the spectrum of a gas flame in which the chloride of the corresponding metal was brought. The flame was placed behind the glass tube. When the Ruhmkorff apparatus was thrown alternately in and out of action it was easy to be convinced, without any accurate measurement, that, in the brilliant spectrum of the spark, the bright lines of the spectrum of the flame were present undisplaced. In addition to these there appeared other bright lines in the spark spectrum a part of which must be attributed to the presence of foreign metals in the electrodes, others to nitrogen which filled the tubes after the oxygen had partly oxidized the electrodes.[1]

It appears, accordingly, beyond a question that the bright lines of the spectra indicated may be considered as certain proof of the presence of the metal in consideration. They can serve as reactions by means of which this material may be detected more certainly, and quickly and in smaller quantities than by any other analytical method.

The spectra, represented, refer to the case when the slit is wide enough so that only the most prominent of the dark lines of the solar spectrum are visible, the magnifying power of the observing telescope being small (about four-fold) and the intensity of the light moderate. These conditions seem to us

[1] In one investigation with strontium electrodes we used a tube filled with hydrogen instead of nitrogen, and the stream of sparks was transformed very soon into an arc, while the sides of the tube were covered with a gray precipitate. On opening the tube under rock-oil it appeared that the hydrogen had vanished and a vacuum existed. The gas appears, therefore, at the enormous temperature of the electric spark, to have dissociated the strontium oxide which had not been completely removed from the surface of the metal.

most advantageous when it is necessary to carry out a chemical analysis by spectral observations. The appearance of the spectrum may under other conditions be quite different. If the purity of the spectrum is increased, many of the lines appearing as single, resolve themselves into several, the sodium line, for example, into two ; if the intensity is increased new lines appear in many of the spectra shown and the relation of the brightness of the old ones becomes different. In general the brightness of a darker line increases with greater luminosity more rapidly than the brighter ones, but not so much that the former exceed these. A clear example of this is given by the two lithium lines. We have observed only one exception to this rule, namely, with the line *Ba* η, which, with low luminosity, is barely visible while *Ba* γ appears very distinct, and, with greater luminosity, much brighter than the former. This fact appears of importance, and we shall make a further study of the same.

We will now consider more closely the characteristics of the several spectra, the knowledge of which is of importance from a practical standpoint, and indicate the advantage which the chemical analytical method founded upon it furnishes.

Sodium.

Of all the spectral reactions that of sodium is the most sensitive. The yellow line *Na* α, the only one which is shown in the sodium spectrum coincides with Fraunhofer's line *D* and is characterized by its peculiarly sharp boundary and its extraordinary brilliancy. If the temperature of the flame is very high and the quantity of the substance used very great, traces of a continuous spectrum are seen in the immediate neighborhood of the line. Lines of other substances, in themselves very weak, lying near it appear still weak and will, therefore, often first be visible after the sodium reaction has begun to disappear.

In the oxygen, chlorine, iodine and bromine compounds in sulphuric acid and carbonic acid the reaction is most evident. But it is present also in the silicates, borates, phosphates and other non-volatile salts.

Swan[1] has already called attention to the minuteness of the

1 Pogg. Ann. Vol. C, p. 311.

quantity of common salt which can produce the sodium line clearly.

The following investigation shows that chemistry produces no single reaction which in the remotest degree can compare in sensitiveness with this analytical spectral determination of sodium. We detonized in one corner of the experiment room which contained about 60 cubic meters of air and as far as possible from our apparatus three milligrams of chlorate of sodium with milk sugar while the non-luminous flame was observed before the slit. After some minutes, the flame, becoming gradually colored pale yellow, gave a strong sodium line, which, after ten minutes, again completely vanished. From the weight of the detonized salt and the air contained in the room it is easy to calculate that in a unit weight of the latter not a $\frac{1}{20000000}$th part of sodium smoke could have been suspended. As the reaction can be readily observed in a second, and as, in this time, according to the rate of flow and the composition of the gases in the flame, only about 50 ccm or 0.0647 grams of air which contained less than $\frac{1}{200000000}$th of sodium salt, reach the state of incandescence in the flame, it follows that the eye is capable of detecting less than $\frac{1}{3000000}$th of a milligram of sodium salt with the greatest distinctness. With such a sensibility of the reaction it is evident that only rarely is a sodium reaction not visible in glowing atmospheric air. The earth is covered over more than two-thirds of its surface with a solution of chloride of sodium, which, by the waves breaking into foam, is transformed continually into spray; the particles of sea-water, which reach the atmosphere in this way, evaporate and leave behind them motes of salt which vary in magnitude, but, as it appears, are rarely absent from the atmosphere, and, perhaps, serve to supply the small organisms the salt which the larger plants and animals secure from the ground. The presence in the air of salt, easily shown by spectral analysis, is yet of interest from another standpoint. If, as we yet can scarcely doubt, there are catalytic influences which are the cause of the miasmic spread of disease, it is possible that an antiseptic substance, such as salt, even in vanishingly small quantities, may indeed not be without definite influence upon such processes in the air. From daily and long continued

spectrum observation it would be easy to learn whether the variation in the intensity of the spectral line *Na a*, produced by the sodium combination in the air, is related in any degree to the appearance and the spread of endemic diseases.

In the exceedingly delicate sodium reaction may also be sought the reason why all bodies exposed to the air show the sodium line after a time when heated in the flame, and why it is possible with only a few compounds to eliminate the last trace of the sodium line *Na a* by crystallizing it out ten or more times from water which has come in contact with platinum vessels only. A hair wire of platinum, which has been freed, by heating, from every trace of sodium, shows the reaction most vividly again, if it is exposed some hours to the air. Dust which settles in the room from the air shows it in the same degree, so that, for example, the slapping of a dusty book is quite sufficient to produce at a distance of several spaces the most brilliant flashes of the *Na a* line.

Lithium.

The incandescent vapors of the lithium compound give two sharply defined lines, one a very weak yellow *Li β* and a red brilliant line *Li a*. In certainty and delicacy this reaction exceeds all those known heretofore in analytical chemistry. It approximates in sensibility that of the sodium reaction perhaps because the eye is more sensitive for yellow rays than for red. On detonizing nine milligrams of carbonate of lithium with a large excess of milk sugar and potassium chlorate in the room which contained about 60 cubic meters of air, the line became quite evident. The eye can therefore in this way, as a calculation similar to the one made above will show, perceive less than $\frac{9}{10000000}$th of a milligram of carbonate of lithium with the greatest distinctness. 0.05 grams of the same salt, detonized in the way already mentioned, made it possible to observe the *Li a* line in the air of the same room during more than an hour.

The oxygen, chlorine, iodine and bromine compounds are most suitable for observing lithium. But the carbonate, sulphate, and even phosphate are almost as well suited for this purpose. Minerals containing lithium, as triphyllin, triphan, petalit, lepidolith, need only to be held in the flame in order

to give the line *Li a* with an intense lustre. In this way it is possible to show the presence of lithium in many feld-spars, for example in orthoclase from Baveno. The line is seen only momentarily immediately after the insertion of the specimen in the flame. Thus mica from Altenberg and Penig indicates the presence of lithium while on the contrary mica from Miask, Ashaffenburg, Modum, Bengal, Pennsylvania, etc., is free from lithium. When in naturally deposited silicates only a vanishingly small quantity of lithium is present, it escapes immediate observation. The test in such cases is then best made in the following way: we digest and evaporate a small quantity of the substance for examination with hydrofluoric acid or fluoride of ammonium, moisten the remainder with sulphuric acid, and dissolve the dry mass with absolute alcohol. The alcoholic solution is then evaporated to dryness, again dissolved with alcohol, and the fluid, thus obtained, evaporated in as shallow a dish as possible. The product which remains can be easily scraped together by means of an erasing knife and brought into the flame on platinum wire. $\frac{1}{10}$th of a milligram of the same is usually quite sufficient for the experiment. Other compounds than the silicates, in which we may wish to detect the least traces of lithium, may be transformed into sulphates by evaporation with sulphuric acid or in any other way and then treated as above.

By means of these experiments, the unanticipated conclusion is readily drawn that lithium belongs to those substances which are most widely distributed in nature. This is easily shown by means of 40 cubic centimeters of sea-water which was collected in the Atlantic ocean in latitude 41° 41′ and longitude 39° 14′. Ashes of Fucoids (kelp) which was driven on to the Scottish coast from the Gulf Stream contained appreciable traces of it. All orthoclase and quartz from the granite of the Oldenwald which we have tested show a lithium content. A very pure drinking water from a spring on the western granitic declivity of the Neckar valley in Schlierbach near Heidelberg contained lithium, while the spring rising in the red sandstone which supplies the water pipes of this chemical laboratory was free from it. Mineral water, in a litre of which lithium can scarcely be detected by the ordinary analytical methods, shows

the *Li a* line frequently if we put a drop of the same into the flame on a platinum wire.[1] All the ashes of woods in the Oldenwald which grow on granite soil, as well as Russian and other commercial potashes examined by us, contain lithium. Neither, even, in the ashes of tobacco, vine leaves, vine-wood and grapes,[2] as well as in the ashes of crops which were cultivated in the Rhine plain near Waghausel, Deidesheim and Heidelberg on non-granitic earth, was lithium lacking, nor in the milk of the animals which were fed upon these crops.[3]

It will be scarcely necessary to remark that a mixture of volatile sodium and lithium salts shows, along with the reaction of sodium, that of lithium with a scarcely less preceptible sharpness and distinctness. The red line of the last appears still quite distinct when a small bead containing the $\frac{1}{1000}$th part of lithium salts is introduced into the flame, where the eye, unaided, perceives in the same, nothing more than yellow light of sodium without any indication of red coloration. On account of the greater volatility of lithium salts, the sodium reaction lasts somewhat longer. When, therefore, it is desired to detect very small traces of lithium along with sodium, the bead for testing must be introduced into the flame whilst we are observing through the telescope. We then often observe the lithium line only for a few moments during the first products of volatilization.

In the production of lithium compounds on a commercial scale spectrum analysis is a means of inestimable value in the selection of the raw material used and the determination of an efficient method of manufacture. Thus for example, it is only necessary to evaporate a drop of the different mother-liquors in

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[1] When it is required to introduce a liquid into the flame we bend in the end of a horse-hair platinum wire, a ring of suitable diameter and hammer the same flat. If we let a drop of the fluid fall into the ring thus formed a sufficient quantity for the investigation remains hanging within.

[2] Lithium is concentrated so much in the mother-liquors in the manufacture of tartaric acid that we can obtain considerable quantities from them.

[3] Dr. Folwarczny has even been able to show with the lithium line *Li a* the lithium compounds in the ash of human blood and of muscular tissue.

the flame and observe through the telescope, in order to show at once, that, in many of these saline residues, a rich and hitherto overlooked lithium source exists. Thus in the process of preparation, we can follow any loss of lithium in the associated products and wastes by means of the spectral reaction, and thus easily seek more efficient methods of production than those heretofore used.[1]

Potassium.

The volatile potassium compounds produce in the flame a very extended continuous spectrum which only show two characteristic lines; the first $K\,\alpha$, in the outermost red bordering on the ultra red rays falls exactly on the dark line A of the solar spectrum; the second $K\,\beta$ far in the violet toward the other end of the spectrum, corresponds likewise to a Fraunhofer's line. A very weak line, coinciding with the Fraunhofer's line B, which, however, is only visible with an intense flame, is less characteristic. The blue line is somewhat weak but is almost as well suited for detecting potassium as the red line. The position of both lines, in the neighborhood of the limits of the rays perceptible by the eye, renders the reaction somewhat less sensitive.

In the air of our room it became first visible when we burned about one gram of chlorate of potassium mixed with milk sugar. We can, therefore, make clear to the eye in this way about $\frac{1}{1000}$th of a milligram of chlorate of potassium.

Potassium hydrate and all compounds of potassium with volatile acids, show the reaction without exception. Potassium silicate and similar non-volatile salts, on the contrary, produce it only when the potassium is present in large quantities. With small amounts, the test bead may be melted together with some carbonate of sodium in order to make the characteristic lines visible. The presence of the sodium does not prevent the reaction and hardly affects the sensibility. Ortho-

[1] We obtained by such an approved method from two jars of mineral water (about four litres) a mother-water, which gave on evaporation with sulphuric acid a residue of 1.2 K, half an ounce of carbonate of lithium of the purity of the commercial, whose cost would be about 140 fl. per pound. A great number of other mother-waters which we examined showed a like wealth in lithium compounds.

clase, sanidine, and adularia may easily be distinguished in this way from albite, oligoclase, Labradorite, and anorthite. In order to detect traces of potassium, vanishingly small, we need to heat to a feeble incandescence the silicate, with a large excess of fluoride of ammonium, in a platinum crucible and introduce the residue into the flame on a platinum wire. In this way we find that almost every silicate contains potassium. The lithium salts disturb the reaction but little. Thus, for example, it is only necessary to hold the ash end of a cigar in the flame before the slit, in order to produce at once very distinctly the yellow line of the sodium and the two red ones of potassium and lithium, the last metal being scarcely ever absent in tobacco ash.

Strontium.

The spectra of the alkali earths are not so simple as those of the alkalis. That of strontium is characterized, particularly, by the absence of green bands. Eight lines of the same are quite remarkable namely six red, one orange and one blue. The orange line *Sr a* which appears close to the sodium line toward the red, the two red lines *Sr β*, *Sr γ* and finally the blue line *Sr δ* are the most important in their position and intensity. In order to test the sensibility of the reaction we heated quickly in a platinum dish, over a large flame, an aqueous solution of chloride of strontium of known concentration until the water was evaporated and the dish began to glow. The salt then began to decrepitate into microscopic particles which were thrown into the air in the form of white smoke. A weighing of the salt residue in the dish showed that in this way 0.077 grams of chloride of strontium had passed out into the 77,000 grams' weight of air of the room in form of a fine dust. After the air of the room had been thoroughly mixed, by means of an open umbrella moved rapidly about, the characteristic lines of the strontium spectrum were very beautifully outlined. We can according to this experiment estimate the amount of chloride of strontium preceptible at $\frac{6}{100000}$th of a milligram.

The chlorine and the other haloid compounds of strontium give the most distinct reaction. Strontium hydrate and carbonate of strontium show them much more feebly; the sulphate

still less distinctly; the compounds, with the non-volatile acids, the weakest or not at all. We must therefore introduce into the flame, first, the bead for testing by itself, and then again, after previously moistening with hydrochloric acid. If we assume sulphuric acid in the bead, we must hold it some moments in the reducing part of the flame before moistening with hydrochloric acid, in order to transform the sulphate into the sulphide which is decomposed by hydrochloric acid. To detect strontium in compounds of silicic, phosphoric, boracic or other non-volatile acids we proceed best in the following manner: for fusing with carbonate of sodium a conical spiral of platinum wire is used instead of a platinum crucible. The same is made white hot in the flame and dipped into dry fine pulverized carbonate of sodium which, when possible, contains enough water so that the necessary quantity of the salt remains hanging to the same on the first immersion. Fusion can be effected in this spiral much quicker than in the platinum crucible, since the mass of the platinum heated is small and the salt to be fused comes into immediate contact with the flame. If we transform the fine pulverized substance to be tested into the glowing fluid soda by means of a small platinum spatula, and maintain it in a glowing state for a few minutes, we need only to knock the spiral, inverted with its vertex upward, on the edge of the lamp stand in order to obtain the contents of the same in the form of a large solidified bead. We then cover the bead with a sheet of writing paper and press it by means of an elastic knife blade, which we also use after removing the paper, in order to reduce the mass still farther to the finest powder. This is collected on the edge of a plate slightly tilted and carefully covered with hot water which is allowed to flow backwards and forwards over the substance, heaped up by gentle tipping of the plate and finally, the fluid, remaining over the sediment, is decanted. It is easy, by repeated heating of the plate, to draw off the soluble salt after several repetitions of this process without stirring up the sediment and losing an appreciable amount of the same. If instead of water we use a common salt solution, the operation may be conducted more quickly and certainly. The residue contains the strontium as carbonate, of which a few tenths of a

milligram, moistened with a little hydrochloric acid on a platinum wire, give a brilliant reaction. In this way, without platinum crucible, mortar, evaporating dish, and without funnel and filter, it is possible to carry out, in a few minutes, all the necessary operations of fusing, powdering, digesting and washing.

The reaction of potassium and sodium is not affected by the presence of strontium. The lithium reaction takes place along with the three mentioned with perfect distinctness, if the quantity of lithium is not too small with respect to that of the strontium. The lithium line *Li α* then appears as a narrow intensely red and sharply defined band upon the weaker red background of the broad strontium band *Sr β*.

Calcium.

The spectrum of calcium can be immediately distinguished at the first observation from the four spectra already considered in that a very characteristic and intense line *Ca β* is present in the green. Also a second not less characteristic feature is the very brilliant orange line *Ca α* which lies considerably farther toward the red end of the spectrum than the sodium line *Na α* and the orange line of strontium *Sr α*. By burning a mixture of calcium chloride, chlorate of potassium and milk sugar we obtain a smoke whose reaction is approximately of the same sensibility as that of the fumes from the chloride of strontium under the same conditions. It follows from an examination made in this way that $\frac{6}{1000000}$ of a milligram of calcium chloride can be detected easily and with absolute certainty. Only the calcium compounds, volatilized in the flame, show this reaction, and the more volatile they are the more distinct it is. Chloride of calcium, iodide of calcium, and bromide of calcium are best in this respect. Sulphate of calcium gives a spectrum only after it has become basic but then very brilliantly and long continued. In the same way the reaction of the carbonate becomes distinct after the acid has been driven off.

Compounds of calcium with non-volatile acids remain indifferent in the flame, but if they are attacked by hydrochloric acid, the reaction may be easily obtained in the following way:

we introduce a few milligrams, or perhaps only a few tenths of a milligram, of the finely pulverized substance on the flat platinum ring, somewhat moistened, into the less heated portion of the flame until the powder is frittered without being melted. If we allow a drop of hydrochloric acid to fall on the ring the greater part of it will remain hanging. If we pass this drop before the slit of the spectroscope into the hottest part of the flame it volatilizes without boiling on account of its spheroidal condition. If during the volatilizing of the drop we look into the telescope there appears at the instant when the last portion of the fluid has been evaporated a brilliant calcium spectrum which flashes out but for a moment with a small amount, but continues a longer or a shorter time with considerable quantities of metal.

Only in silicates which are attacked by hydrochloric acid can the calcium be found in this way; in silicates which are not attacked by hydrochloric acid the test is best obtained in the following way:—a few milligrams of the substance to be tested are pulverized as fine as possible and placed on a flat platinum crucible cover with about a gram of half-dissolved fluoride of ammonium and the cover held in the flame until it volatilizes the fluoride of ammonium. We moisten the salt residue remaining on the cover with one to two drops of sulphuric acid, and drive off the excess of the same by gently heating over the flame. If the residue of the sulphates now remaining on the cover be scraped together with the finger-nail or a spatula and about a milligram of the same be introduced into the flame by means of a wire, we obtain, if *K*, *Na* and *Li* are present, the characteristic reaction of these three bodies simultaneous or successively. If calcium and strontium be also present their spectra usually first appear after *K*, *Na* and *Li* have been vaporized. The reaction of these metals fails with weak contents of calcium and strontium; we obtain it, however, immediately if we introduce the wire for a few moments into the reducing part of the flame, moisten it with hydrochloric acid, and bring it again into the flame.

All these tests, as the heating of it alone, or with hydrochloric acid, the treatment with ammonium fluoride alone, or with sulphuric and hydrochloric acid, provide the mineralogist

and still more the geologist with a series of highly simple tests for determining many substances occurring in nature even in the smallest particle, such, for example, as the minerals so similar to one another, consisting of double silicates, containing lime, with a certainty which is scarcely attainable with an abundant supply of material by means of an extended and protracted analysis. Some examples will illustrate this best.

1. A drop of sea-water evaporated on a platinum wire showed a strong sodium reaction, and after volatilizing the chloride of sodium a weak calcium reaction which, by moistening the wire with hydrochloric acid, became for a moment very brilliant. If we treat a few decigrams of the residue of sea-water, in the way described for lithium, with sulphuric acid and alcohol we easily obtain the reaction of potassium and lithium. The presence of strontium in sea-water can be observed best in the boiler crusts of steamships. The filtered hydrochloric acid solution of the same leaves, on evaporation and solution in the smallest quantity of alcohol, a dull yellow coloring from the basic iron salt which is deposited after some days and collected on a filter and washed with alcohol. The filter burnt on a fine platinum wire gives, along with the calcium line, a complete and bright strontium spectrum.

2. Mineral waters often show at once the potassium, sodium, lithium, calcium, and strontium reactions. For example, if we introduce a drop of Durkheim or Krauznach mineral water into the flame we obtained the lines *Na* α, *Li* α, *Ca* α, and *Ca* β. If we use instead of the mineral water a drop of the mother liquid the same lines appear with great brilliancy. In proportion as the chloride of sodium and lithium are volatilized and the chloride of calcium has become more basic, the characteristic lines of the strontium spectrum gradually develop themselves and, becoming brighter, finally are seen in all their extent. We obtain here also, by a mere glance at a single drop vaporized in the flame, the complete analysis of the mixture of five substances in a few moments.

3. The ash of a cigar moistened with some HCl and held in the flame gives the lines *Na* α, *K* α, *Li* α, *Ca* α, *Ca* β.

4. Potash glass of a combustion tube gave, both with and without hydrochloric acid, *Na* α and *K* α, and treated with

fluoride of ammonium and sulphuric acid *Ca α*, *Ca β* and traces of *Li α*.

5. Orthoclase from Baveno gives either alone or with hydrochloric acid only *Na α* with traces of *K α* and *Li α*; with fluoride of ammonium and sulphuric acid the intense line *Na α*, *K α* and somewhat less distant *Li α*. After volatilizing the constituents thus observed the bead introduced, into the flame with *HCl*, gives only a scarcely distinguishable flash of the lines *Ca α* and *Ca β*. The residue remaining on the platinum wire after this test showed, when moistened with cobalt solution and heated, the characteristic color of alumina. If we employ the well-known reaction of silicic acid also it follows from this examination, made in a few minutes, that the orthoclase from Baveno contains silicic, alumina, potash with traces of soda, lime and lithia whilst every trace of baryta and strontia fail.

6. Adularia from the Gotthard conducted itself quite similar to the orthoclase from Baveno only that the lithium reaction failed entirely and the calcium reaction nearly so.

7. Labradorite from St. Paul gives, by itself, only the sodium line *Na α* and not the calcium spectrum. But the sample moistened with hydrochloric acid gives the calcium lines *Ca α* and *Ca β* very brilliantly. With the test by means of fluoride of ammonium we still obtain a weak potassium reaction and very faint traces of lithium.

8. Labradorite from the Diorite of Corsica comported itself similarly only that the traces of the lithium reaction were wanting.

9. Mosanderite from Brevig and Tscheffkinite from the Ilmen mountains gave by itself only the sodium reaction, but the calcium line *Ca α* and *Ca β* when treated with hydrochloric acid.

10. Melinophane from Lamoe gave by itself only *Na α* but with hydrochloric acid *Ca α*, *Ca β* and *Li α*.

11. Scheelite and Sphene gave, on treatment with hydrochloric acid, the very brilliant calcium reaction.

12. If small quantities of strontium are present with calcium we employ the line *Sr δ* most advantageously to detect the former. By means of the same it is easy to detect a small content of strontium in very many sedimentary limestones. *Na α*, *Li α* *K α* particularly *Li α* are shown immediately on heating the

limestone in the flame. Those minerals, converted into calcium chloride by hydrochloric acid and introduced into the flame in this form, give the same lines and besides frequently the line *Sr δ* quite distinctly. But this appears only for a short time and most distinctly whilst it is being developed in the course of the volatilization in the flame and shortly before the fading out of the calcium spectrum.

In this way the lines *Na α*, *Li α*, *K α*, *Ca α*, *Ca β*, *Sr δ* were found in the following limestones :—

Silurian limestone[1] from Kugelbad near Prague,
Shell limestone from Rohrbach near Heidelberg,
Lias limestone from Malsch in Baden,
Chalk from England.

The following limestones showed the lines *Na α*, *Li α*, *K α*, *Ca α*, *Ca β*, without the blue strontium line:—

Marble from the granite of Auerbach,[2]
Devonian limestone from Gerolstein in the Eifel,
Carboniferous limestone from Planite in Saxony,
Dolimite from Nordhausen in the Hartz,
Jura limestones from the Streitberg in Franconia.

We now see from these few experiments that extended and careful spectral analysis of the lithium, potassium, sodium, and strontium content of various limestone formations are of the greatest geological interest with respect to their order of formation and their local disposition and may possibly lead to unexpected conclusions on the nature of the earlier ocean and sea basins in which the formation of these minerals took place.

Barium.

The spectrum of barium is the most complicated of the spectra of the alkalis and alkaline earths. It is distinguished at the first glance from those heretofore examined by

[1] The lithium line could not be detected with certainty in this class of minerals, the line *Sr δ* on the contrary was very strong.

[2] By means of the experiment with alcohol above described enough nitrate of strontium was obtained from twenty grams of marble to produce a bright and complete spectrum of strontium. Whether the remaining limestones treated in this way show a strontium content we have not investigated.

the green lines *Ba α* and *Ba β*, which exceed all the others in brilliancy, appearing first and disappearing last in weak reactions. *Ba γ* is less distinct but is still always to be treated as a characteristic line. The relatively great extension of its spectrum is the reason why the spectral reaction of the barium compounds is somewhat less delicate than those of the substances heretofore examined. 0.3 grams of chlorate of barium burned in our room with milk sugar gave, after the air had been thoroughly mixed by moving an open umbrella, the line *Ba α* most distinctly, for a long time. We may therefore conclude from a calculation made similar to that for sodium, that the reaction will show, with perfect distinctness, not less than $\frac{1}{1000}$th of a milligram.

Chloride, bromide, iodide, and fluoride of barium, the hydrated oxide, the sulphate, and the carbonate, give the reaction most markedly and can therefore be determined by immediate heating in the flame.

Silicates decomposable by hydrochloric acid containing barium give the reaction, if, as indicated in the case of lime, they are introduced into the flame with a drop of hydrochloric acid. Thus, for example, barytharmotome treated in this way gives the line *Ca α Ca β* along with the lines *Ba α*, *Ba β*.

Compounds of barium with non-volatile acids, which are indifferent with or without hydrochloric acid in the flame, we may fuse best, in the way given for strontium, with carbonate of sodium and then test the carbonate of barium thus obtained. If in such compounds *Ca*, *Ba* and *Sr* occur together in very unequal amounts, we dissolve in a drop of sulphuric acid the carbonates obtained by fusion and extract the salt with alcohol from the evaporated residue. The residue then contains only barium and strontium both of which may be easily detected if they do not occur in too unequal quantities. When it is desired to test for the smallest traces of *Sr* or *Ba*, we transform the residue, by heating with sal ammoniac, into chlorides, from which the chloride of strontium can be easily extracted in a sufficiently concentrated state for detection by means of alcohol. If neither of the substances to be tested is present in very small quantities all such methods of separation are quite unnecessary, as the following experiment shows:—a mixture of sodium, potassium,

lithium, calcium, strontium, and barium chlorides which contained $\frac{1}{10}$th of a milligram of each of these six substances at the most, was introduced into the flame and observed. At first the brilliant sodium line *Na α* appeared on the background of a weak continuous spectrum. As soon as this began to fade away, the sharply defined brilliant red line of lithium *Li α* appeared and on the same side of the sodium line, still farther away, the faint potassium line *K α* whilst the barium lines *Ba α* and *Ba β* appeared very distinctly in their characteristic position and peculiar shade. Whilst the compounds of potassium, lithium, and barium were slowly volatilized their lines faded away, or vanished again gradually in succession until, after a few minutes, the lines *Ca α Ca β* and *Sr α Sr β Sr λ Sr δ* became visible out of the less and less prominent lines of strontium, as from a dissolving view, in all their characteristic form, shade and position, and then faded away and entirely vanished after a very long time.

The absence of any one or more of these components could be instantly detected, in the observation, by the absence of the corresponding lines.

For those who have become familiar with the individual spectra by repeated observation, an accurate measurement of the individual lines is unnecessary; their color, their relative position, their characteristic definition and shade, the gradation in their brilliancy, are criterions which are quite sufficient for definite recognition even for the inexperienced. These characteristics may be compared with the distinguishing features which the various precipitates present in their outward appearance, which we use as a reaction test. Just as the character of a precipitate determines whether it be gelatinous, pulverulent, flocculent, granular or crystalline, so also the spectral lines indicate their characteristics in the sharpness of their edges, in the shading off uniformly or irregularly on one or both sides, or in their broader or narrower appearance, as the case may be. And just as we use only those precipitates in analysis which can be produced by the greatest possible dilution, so we also use in spectrum analysis for this purpose only those lines which require for their production the smallest amount of the

substance and only a moderately high temperature. In such characteristics therefore the two methods are quite similar. On the contrary spectrum analysis furnishes, in the color phenomena used in the reaction, a property which gives it unlimited advantage over every other method of analysis. Most of the precipitates which are used for the detection of substances are white and only a few colored. Further the tint of the latter is not very constant and considerably differentiated according to the greater or less condensed state of the precipitate. Often the smallest mixture of a foreign substance is sufficient to obliterate completely a characteristic color. Small differences of color of the precipitate can therefore be no longer used as a chemical test. In spectrum analysis, on the contrary, the colored bands remain undisturbed by such foreign influences and are undisturbed by the presence of other bodies. The positions which they have in the spectrum determine a chemical characteristic which is of as unalterable and fundamental a nature as the atomic weight of the substance, and therefore, permits us to determine it with an almost astronomical exactness. What, however, gives to the spectral analytical method a peculiar importance, is the fact that it almost infinitely exceeds the limits to which chemical analysis of matter has heretofore reached. It predicts for us the most valuable conclusions on the distribution and arrangement of geological substances in their formation. Already the few investigations, which this memoir contains, lead to the unexpected conclusion that not only potassium and sodium but also lithium and strontium must be counted among the substances of the earth most widely scattered, though only in minute quantities.

Spectrum analysis will also play a not less important part in the discoveries of elements not yet detected. For if there are substances which are so sparsely scattered in nature that the methods of analysis heretofore used in observing and separating them fail, we may hope to detect and determine many of them, by the simple examination of their spectra in flames, which would escape the ordinary method of chemical analysis. That there are actually such elements heretofore unknown we have already had an opportunity of showing. We believe that we shall be able yet to declare with absolute certainty, supported

by the unquestioned results of spectral analytical methods that besides potassium, sodium and lithium, there is still a fourth metal belonging to the alkali group which will give quite as characteristic a spectrum as lithium—a metal which shows, with our spectral apparatus, only two lines, a weak blue line, which almost coincides with the strontium line *Sr δ* and another blue line, which lies only a little farther toward the violet end of the spectrum, rivaling in intensity and distinctness the lithium line.

On the one hand spectrum analysis offers, as we believe we have already shown, a means of wonderful simplicity for detecting the slightest traces of certain elements in terrestrial substances, and on the other, it opens up to chemical investigation a field heretofore completely closed, which extends far beyond the limit of the earth even to our solar system itself. Since, by the analytical method under discussion, it is sufficient simply to see the gas in an incandescent state in order to make an analysis, it at once follows that the same is also applicable to the atmosphere of the sun and the brighter fixed stars. A modification with respect to the light which the nucleus of these heavenly bodies radiate must be introduced here. In a memoir "On the Relation between the Emission and the Absorption of Bodies for Heat and Light"[1] one of us has proven, by theoretical considerations, that the spectrum of an incandescent gas is *reversed,* that is, that the bright lines are transformed into dark ones when a source of light of sufficient intensity, which gives a continuous spectrum, is placed behind the same. From this we may conclude that the sun's spectrum, with its dark lines, is nothing else than the reversal of the spectrum which the atmosphere of the sun itself would show. Hence the chemical analysis of the sun's atmosphere requires only the examination of those substances which, when brought into a flame, produce bright lines which coincide with the dark lines of the solar spectrum.

In the article mentioned, the following examples are given as experimental proof of the theoretically deduced law referred to:

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[1] Kirchhoff, Pogg. Ann. Vol. CIX p. 275. See previous memoir.

The bright red line in the spectrum of a flame in which a bead of chloride of lithium is introduced is changed into a black line when we allow full sunlight to pass through the flame.

If we substitute for the bead of lithium one of sodium chloride, the dark double line *D* (which coincides with the bright sodium line) shows itself in the sun's spectrum with unusual brilliancy.

The dark double line *D* appears in the spectrum of the Drummond's light if we pass its rays through the flame of aqueous alcohol, into which we have introduced chloride of sodium.[1]

It will not be without interest to obtain still further confirmations of this remarkable theoretical law. We may arrive at this by the investigation which will now be described.

We made a thick platinum wire incandescent in a flame and by means of an electric current brought it nearly to its melting point. The wire gave a brilliant spectrum without any trace of bright or dark lines. If a flame of very aqueous alcohol in which common salt was dissolved were introduced between the wire and the slit of the apparatus, the dark line *D* showed itself with great distinctness.

We can produce the dark line *D* in the spectrum of a platinum wire which has been made incandescent by a flame if we merely hold before it a test tube into which some sodium amalgam has been introduced, and then heat it to boiling. This investigation is important, on this account, in that it shows that far

[1] In the March number of the Philosophical Magazine for 1860 Stokes calls attention to the fact that Foucault had made already an observation in 1849 which is similar to that mentioned above. In the examination of the electric arc between two carbon points he observed (1, Institut 1849, p. 45) that in the spectrum the same bright lines were present in the position of the double line D of the solar spectrum, and that the dark line D of the arc is intensified, or produced, if we allow the rays of the sun or one of the incandescent points to pass through it and then resolve them in the spectrum. The observation mentioned in the text gives the explanation of this interesting phenomena already observed by Foucault eleven years before and shows that the same is not influenced by the peculiarity of the electric light, which is still, from many points of view, so enigmatical, but arises from a sodium compound which is contained in the carbon and is transformed by the current into incandescent gas.

below the point of incandescence of sodium vapor, its absorbent effect is exercised exactly in the same parts of the spectrum as with the highest temperatures which we are able to produce and at which that of the solar atmosphere exists.

We have been able to reverse the bright lines of the spectra of *K*, *Sr*, *Ca*, *Ba* by the employment of sunlight and mixtures of the chlorates of these metals with milk sugar. Before the slit of the apparatus a small iron trough is placed; into this the mixture was introduced, and the full sunlight passed along the trough to the slit and the mixture ignited on one side by an incandescent wire. The telescope was set with the intersection of its cross hairs, which were mounted at an acute angle with one another, on the bright line of the flame spectrum, the reversal of which was to be tested; the observer concentrated his attention on this point in order to judge whether at the moment of ignition a dark line was visible, passing through the intersection of the cross hairs. In this way it was quite easy with the proper proportion of the mixture, to be burnt, to establish the reversal of the lines *Ba α* and *Ba β* and the line *K β*. The last of these coincided with one of the most distinct lines of the solar system, although not indicated by Fraunhofer; this line appeared much more distinctly at the moment of ignition of the potash salt than otherwise. In order to observe the reversal of the bright lines of the strontium spectrum in the way described, the chlorate of strontium must be dried in the most careful manner; a slight trace of moisture causes the sun's rays to be weakened and produces the positive spectrum of strontium on account of the flame becoming filled with salt particles which have been spattered about by the ignition.

We have limited ourselves in this memoir to the investigation of the spectra of the metals of the alkalis and alkaline earths, and these only in so far as was necessary for the analysis of terrestrial matter. We reserve for ourselves the further extension of these investigations which are desirable in connection with the analysis of terrestrial substances and the analysis of the atmospheres of the stars.

Heidelberg, April, 1860.

MEMOIRS ON RADIATION AND ABSORPTION.

Biographical Sketch.

Robert Wilhelm Bunsen was born in Göttingen March 15, 1811. He received his doctor's degree in 1830 and became private-docent in 1833 in that University. In 1836 he became Professor of Chemistry in the Polytechnic School at Cassel and in 1838 accepted a similar position in the University of Marburg. In 1851 he went to Breslau and the following year to Heidelberg, where he remained until his death. Both as a teacher and as an investigator, he was one of the most eminent of his generation, making many important contributions to both Physics and Chemistry. His spectroscope, photometer, calorimeter, gas burner, filter pump, battery, etc., on the one hand, and his early contributions to organic chemistry, the methods of gas analysis, photochemical action, etc., on the other, illustrate the versatility of his genius. His work in connection with his colleague, Kirchhoff, resulted in one of the most brillant achievements of the century,—the application of the spectroscope to the analysis of terrestrial substances,—which revealed at once several new elements and showed the common constitution of all bodies in the stellar system. His final contribution was made in 1887 after an illustrious scientific career of nearly sixty years. During his last years he was a familiar figure on the streets of Heidelberg, in which city he died Aug. 15, 1899.

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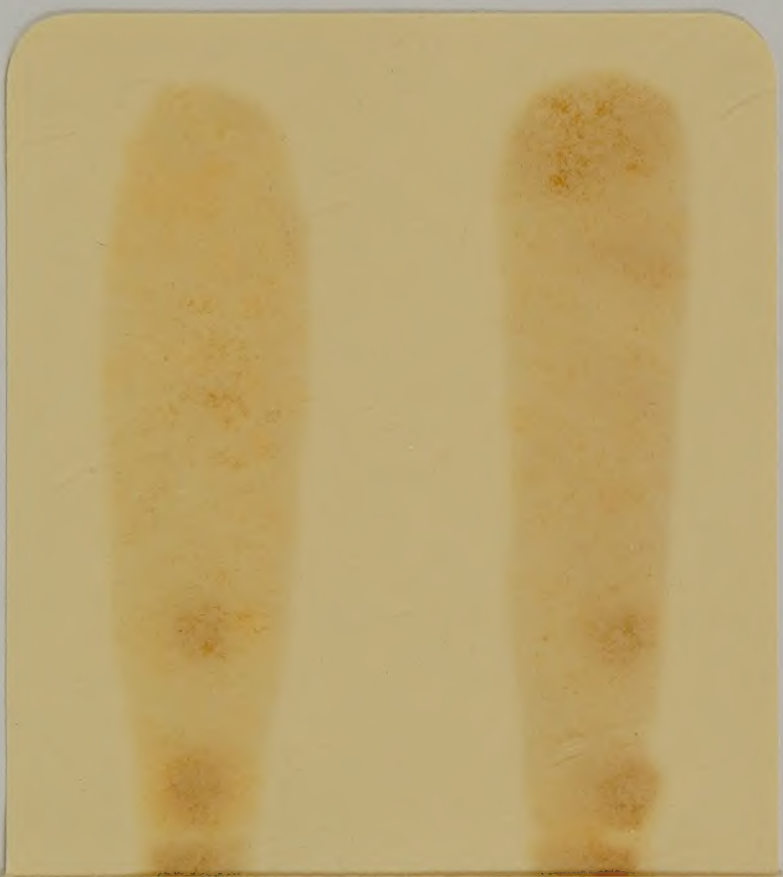

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